

Dependence of instability strain upon damage in thermoviscoplastic materials

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*Dedicated to Professor Piotr Perzyna
on the occasion of his 70th birthday*

BASED ON THE FIELD EQUATION for the number density of voids and the expression for the expansion of a spherical void in a perfectly plastic infinite body subjected to a uniform hydrostatic tensile stress, an expression for the rate of dilatation of voids is derived. Damage is defined as the volume density of voids. The flow stress of the material is assumed to decrease affinely with an increase in the damage. It is used to find the instability strain in a thermoviscoplastic body deformed in simple shear and simultaneously subjected to a uniform hydrostatic tensile stress. The instability strain is determined by two methods: (i) the Considère condition, i.e., when the shearing traction becomes maximum, and (ii) by studying the stability of a slightly perturbed homogeneous solution of equations governing thermomechanical deformations of a thermoviscoplastic body. Both techniques give essentially the same value of the instability strain. Assuming that failure occurs when the accumulated damage equals 0.3, the failure strain is computed. For a 4340 steel, values of the instability and the failure strains as a function of the nominal strain rate and the hydrostatic pressure are computed.

1. Introduction

TYPICAL DAMAGE MECHANISMS that have been studied are the development of micro-cracks, micro-voids, and adiabatic shear localization. Many investigations [1-4] have revealed that the ductile failure of a body deformed at a high strain rate generally involves the initiation and development of adiabatic shear bands (ASBs), nucleation of micro-voids either within an ASB or by the separation of the matrix material from inclusions or both, growth and coalescence of micro-voids to form micro-cracks, the coalescence of micro-cracks to form cracks, and the propagation of cracks to the boundaries of the body. McCLINTOCK [5] has analyzed the expansion of a long circular cylindrical cavity embedded in a non-hardening material that is pulled along the cavity axis and also subjected to

transverse tensile stresses. He found that the relative void expansion per unit applied strain increment increases exponentially with the transverse normal stress. RICE and TRACEY [6] studied the effect of stress triaxiality on the growth of a spherical void embedded in a perfectly plastic infinite body and found that the relative void volume grows exponentially with the stress triaxiality. HANCOCK and MACKENZIE [8] postulated that the failure process in ductile metals involved the nucleation, growth and coalescence of voids. Thus the effective cross-sectional area is gradually reduced and the load carrying capacity of the member is decreased. They did not incorporate the reduction in the elastic moduli and the flow stress caused by the voids. GURSON [7] has proposed a plastic potential or a yield criterion for an isotropic microporous solid that accounts for the decrease in the flow stress of the material induced by voids. BATRA and JIN [9], BATRA and JABER [10] and BATRA *et al.* [34] used Gurson's flow potential coupled with the reduction in the elastic moduli, caused by the porosity to study the development of ASBs and the transition of the failure mode from brittle to ductile in plane strain deformations of prenotched thermoviscoplastic plates. PERZYNA and coworkers [11–13] have developed a theory of heat conducting microporous thermoviscoplastic solids that accounts for various dissipative mechanisms. Constitutive relations are derived by exploiting the Clausius-Duhem inequality. The material moduli degrade with the damage evolved which is equated to the density of voids.

A few models based on continuum damage mechanics (CDM) theory have been developed to account for the nucleation, the coalescence, and the growth of voids in a ductile body. In the CDM, these phenomena are generally represented by a macroscopic damage variable whose growth rate is taken to be a function of measurable macroscopic variables such as the stress triaxiality, effective plastic strain etc. The material moduli are presumed to decrease with an increase in the damage and the material is assumed to fail when the damage attains a critical material-dependent value. LEMAITRE [14] has summarized damage mechanics for elastoplastic deformations. Except for the degradation of material moduli, the theory is similar to that of internal variables developed by COLEMAN and GURTIN [15]. With the porosity regarded as the damage variable, PERZYNA and coworker's theory [11–13] and equations used by BATRA *et al.* [9, 10, 34] describe CDM models.

Statistical approach has been employed, amongst others, by CURRAN *et al.* [16] and BAI *et al.* [17] to derive macrolevel damage relations. From the conservation law of micro-cracks in phase-space, BAI *et al.* [17–20] derived a damage model for ideal cracks. LI *et al.* [21] adopted this method to study the damage due to void expansion in a ductile metal tube with the inner surface subjected to explosive loads.

Adiabatic shear banding is an important failure mechanism in dynamic deformations of ductile materials. An ASB is a narrow region, usually a few microns wide, of intense plastic deformation. TRESCA [22] observed these during the hot forging of a platinum bar. The activity in the field grew rapidly subsequent to their observations by ZENER and HOLLOMON [23] during the punching of a hole in a low carbon steel plate. They proposed that ASBs form when thermal softening overcomes the combined hardening due to strain and strain rate effects. Subsequent experimental [24] and numerical investigations [25] have revealed that an ASB develops in earnest after the load has attained its peak value. Whereas earlier investigations [1] employed the CONSIDÈRE criterion [26] to find the instability strain, BAI [27] used the perturbation method to find the strain when the homogeneous solution upon perturbation will become unstable. BATRA and CHEN [28] showed that these two techniques give essentially the same value of the instability strain. Here we prove that this holds even when damage evolution is considered and the effective stress required to deform the material plastically decreases affinely with the damage evolved.

2. Damage evolution equation due to growth of voids

Following BAI *et al.*'s work [17] on ideal microcracks, we make the following simplifying assumptions: (i) microvoids are spherical and are sparsely distributed, thus the interaction among them is negligible, (ii) no new voids nucleate but the volume of existing voids can change, (iii) the growth of a void is governed by macroscopic deformations, and (iv) the material is rigid perfectly plastic. Since voids of all shapes occur in a material, our assumption of voids being spherical will necessarily give us an approximate expression for the damage. Let v represent the volume of a microvoid and $n(v, t, \sigma_{eq})$ the number density at time t of voids of volume v . Here σ_{eq} is the effective or the von Mises stress in the matrix surrounding a void. Then the number of microvoids at time t in a unit volume of physical space of volume between v and $v + dv$ equals $n(v, t, \sigma_e)dv$. Thus the total volume, V_v , of microvoids is given by

$$(2.1) \quad V_v = \int_0^{\infty} n(v, t, \sigma_e) v dv.$$

The number density of voids of volume v changes with time according to the relation [17–19]

$$(2.2) \quad \frac{\partial n}{\partial t} + \frac{\partial(n\dot{v})}{\partial v} = 0,$$

where a superimposed dot indicates the material time derivative. This equation is analogous to the continuity equation for an incompressible body.

We define a damage variable, D , by

$$(2.3) \quad D = \frac{V_v}{V} = \frac{V - V_m}{V},$$

where V and V_m equal, respectively, the total volume of the body and the matrix or the solid phase. Assuming that the body as a whole is incompressible, Eq. (2.3) gives

$$(2.4) \quad \dot{D} = \frac{\dot{V}_v}{V}.$$

Assumptions $\dot{V} = 0$ and $\dot{V}_v \neq 0$ imply that the mass density of the matrix will change. However, in [9, 10] it was assumed that the matrix is incompressible and thus its mass density does not change. Accordingly, the evolution equation for \dot{V}_v derived here differs from that used in Refs. 9 and 10. Substitution from (2.1) and (2.2) into (2.4) yields

$$(2.5) \quad \dot{D} = \frac{1}{V} \int_0^\infty \frac{\partial n}{\partial t} v dv = -\frac{1}{V} \int_0^\infty \frac{\partial(nv)}{\partial v} v dv, \\ = -\frac{1}{V} \left[nvv \Big|_0^\infty - \int_0^\infty n v dv \right] = \frac{1}{V} \int_0^\infty n v dv,$$

since there are no voids of zero volume and no voids of infinite volume.

RICE and TRACEY [6] derived the following expression for the rate of change of radius, r , of a spherical void in a rigid perfectly plastic infinite body subjected to hydrostatic tension at infinity:

$$(2.6) \quad \dot{r} = Cr \dot{\epsilon}_e \exp\left(\frac{\sigma_{kk}}{2\sigma_y}\right)$$

where $\dot{\epsilon}_e$ equals the effective plastic strain rate, σ_{ij} is the Cauchy stress tensor, a repeated index implies summation over the range of the index, σ_y is the static yield stress of the material in simple tension and the value of the constant C depends upon the loading conditions at infinity. Within about one per cent error, C can be taken to be 0.279. For a spherical void, $v = 4\pi r^3/3$. Thus

$$(2.7) \quad \frac{\dot{v}}{v} = 0.837 \dot{\epsilon}_e \exp\left(\frac{\sigma_{kk}}{2\sigma_y}\right).$$

Equations (2.5) and (2.7) give

$$(2.8) \quad \dot{D} = 0.837 \dot{\epsilon}_e D \exp\left(\frac{\sigma_{kk}}{2\sigma_y}\right),$$

for the rate of evolution of the damage D .

For a two-dimensional problem the damage variable, D_s , is usually determined in terms of the surface area of voids divided by the total area of cross-section. Since for a spherical void of surface area s , $\dot{s}/s = (2/3)\dot{v}/v$, therefore, for a two-dimensional problem we take

$$(2.9) \quad \dot{D}_s = 0.558\dot{\epsilon}_e D_s \exp\left(\frac{\sigma_{kk}}{2\sigma_y}\right).$$

When σ_{kk}/σ_y is independent of ϵ_e and $D_{s0} = D_s(0)$, we can integrate Eq. (2.9) to obtain

$$(2.10) \quad D_s = D_{s0} \exp(k\epsilon_e),$$

where

$$(2.11) \quad k = 0.558 \exp\left(\frac{\sigma_{kk}}{2\sigma_y}\right).$$

Thus the failure strain, ϵ_f , corresponding to the critical value, D_{sc} , of the damage is given by

$$(2.12) \quad \epsilon_f = \frac{1}{k} \ln\left(\frac{D_{sc}}{D_{s0}}\right).$$

It is clear that the failure strain decreases exponentially with an increase in the hydrostatic tension. Damage given by (2.10) depends upon the effective plastic strain ϵ_e and the hydrostatic tension. The only material parameter appearing in (2.10) is the yield stress of the material in a quasistatic simple tension test because the matrix material has been assumed to be rigid perfectly plastic.

For a thermoviscoplastic material JOHNSON and COOK [29] postulated that

$$(2.13) \quad D = \sum \Delta\epsilon_e / (D_1 + (D_2 \exp(D_3 \sigma_{kk} / 3\sigma_{eq})) (1 + D_4 \ln(\dot{\epsilon}_e / \dot{\epsilon}_{e0})) (1 + D_5 \theta^{*b}))$$

where $\Delta\epsilon_e$ is the increment in the effective plastic strain which occurs at the effective plastic strain rate $\dot{\epsilon}_e$, σ_{eq} the effective stress, $\theta^* = (\theta - \theta_r) / (\theta_m - \theta_r)$, θ is the current temperature, θ_m the melting temperature, θ_r the room temperature, and b , D_1 , D_2 , D_3 , D_4 , D_5 and $\dot{\epsilon}_0$ are material parameters; $\dot{\epsilon}_0$ is generally taken to equal $1/s$. For $D_1 = D_4 = D_5 = 0$, the expression in the denominator of (2.13) reduces to the expression for the failure strain proposed by HANCOCK and MACKENZIE [8].

In Secs. 3 and 4 we will use Eq. (2.10) with σ_y replaced by σ_{eq} to ascertain the instability strain in a thermoviscoplastic body. Since this equation has been derived for mechanical deformations of a rigid perfectly plastic body, its application to thermoviscoplastic deformations necessarily involves unproven approximations. Furthermore, σ_{kk}/σ_{eq} will not, in general, be independent of ϵ_e . Nevertheless, the instability strain derived in Secs. 3 and 4 highlights the importance of considering damage evolution.

3. Instability strain derived from the Considère condition

We study locally adiabatic, simple shearing, and quasistatic thermomechanical deformations of an isotropic and homogeneous thermoviscoplastic body also subjected to uniform hydrostatic tensile tractions σ_∞ , and assume that the shear strain rate is constant. In the absence of body and inertia forces, the balance of linear momentum requires that the shear stress, τ , be uniform throughout the body. We assume that

$$(3.1) \quad \tau = \tau(\gamma, \dot{\gamma}, \theta, D_s)$$

where γ is the plastic shear strain, and D_s the damage parameter. Here elastic deformations have been neglected which is reasonable since at the onset of instability, elastic shear strain will be very small as compared to the plastic shear strain. Recall that the area of the face on which tangential tractions act does not change, and uniform hydrostatic stresses do not cause any volume change. Thus Considère's condition, which states that a structure becomes unstable when the load reaches a peak value, in this case implies that an instability will occur when the shear stress given by Eq. (3.1) reaches a maximum value. That is, an instability will occur when

$$(3.2) \quad \frac{d\tau}{d\gamma} = \frac{\partial\tau}{\partial\gamma} + \frac{\partial\tau}{\partial\dot{\gamma}} \frac{d\dot{\gamma}}{d\gamma} + \frac{\partial\tau}{\partial\theta} \frac{d\theta}{d\gamma} + \frac{\partial\tau}{\partial D_s} \frac{dD_s}{d\gamma} = 0.$$

In Eq. (3.2) $\partial\tau/\partial\gamma$ represents work hardening or strain hardening of the material, $\partial\tau/\partial\dot{\gamma}$ its strain-rate hardening, $\partial\tau/\partial\theta$ its thermal softening and $\partial\tau/\partial D_s$ its softening due to the damage evolution.

For locally adiabatic simple shearing deformations, the balance of internal energy gives

$$(3.3) \quad \frac{d\theta}{d\gamma} = \frac{\beta\tau}{\rho c}$$

where β is the Taylor-Quinney parameter that equals the fraction of the plastic work converted to heat, ρ is the mass density and c the specific heat.

For constant shear strain rate $\dot{\gamma}$, $d\dot{\gamma}/d\gamma = 0$. Recalling that $\varepsilon_e = \gamma/\sqrt{3}$, and substituting from (2.10) and (3.3) into (3.2), we conclude that an instability will occur when

$$(3.4) \quad \frac{\partial\tau}{\partial\gamma} + \frac{\partial\tau}{\partial\theta} \left(\frac{\beta\tau}{\rho c} \right) + \frac{\partial\tau}{\partial D_s} D_{s0} k^* e^{k^*\gamma} = 0,$$

where $k^* = k/\sqrt{3}$. In terms of the effective stress σ_{eq} and the effective plastic strain ε_e , equation (3.4) becomes

$$(3.5) \quad \frac{\partial\sigma_{eq}}{\partial\varepsilon_e} + \frac{\partial\sigma_{eq}}{\partial\theta} \frac{\beta\sigma_{eq}}{\rho c} + \frac{\partial\sigma_{eq}}{\partial D_s} D_{s0} k e^{k\varepsilon_e} = 0.$$

We now assume that the thermoviscoplastic behavior of the material is represented by the JOHNSON-COOK relation [30], viz.,

$$(3.6) \quad \sigma_{eq} = (1 - D_s)(A + B\varepsilon_e^n) \left(1 + C \ln \frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_{e0}} \right) (1 - \theta^{*m})$$

where $A, B, n, C, \dot{\varepsilon}_{e0}$ and m are material parameters. For most materials, $m \simeq 1$; here we study materials for which $m = 1$. We substitute for σ_{eq} from (3.6) into (3.5) and obtain

$$(3.7) \quad \frac{\varepsilon_e^{n-1}}{(A + B\varepsilon_e^n)^2} = \frac{\beta(1 - D_{so}e^{k\varepsilon_e})}{n\rho Bc(\theta_m - \theta_r)} \left(1 + C \ln \frac{\dot{\varepsilon}_e}{\dot{\varepsilon}_{e0}} \right) + \frac{kD_{so}}{nB(A + B\varepsilon_e^n)(e^{-k\varepsilon_e} - D_{so})}.$$

For a given value of the prescribed strain rate, the hydrostatic tension and the initial damage D_{so} , equation (3.7) gives the value of the effective plastic shear strain when the material will become unstable. In deriving (3.7), $k = \sigma_\infty/\sigma_{eq}$ has been assumed to be independent of γ or ε_e . Thus the uniform hydrostatic tension has been assumed to vary so that k is essentially constant during the deformation process. The variation of k with γ can be readily incorporated into the analysis; it will make equation (3.7) more complicated.

4. Instability strain derived by the perturbation method

In the absence of body forces, the equations governing simple shearing deformations of the thermoviscoplastic body are

$$(4.1) \quad \rho\ddot{\gamma} = \frac{\partial^2 \tau}{\partial y^2},$$

$$(4.2) \quad \rho c \dot{\theta} = \kappa \frac{\partial^2 \theta}{\partial y^2} + \beta \tau \dot{\gamma},$$

$$(4.3) \quad \dot{\gamma} = v_{,y} = \sqrt{3} \dot{\varepsilon}_{e0} \exp \left(\left(\frac{\sqrt{3} \tau}{(1 - D_s)(A + B\varepsilon_e^m)(1 - \theta^{*m})} - 1 \right) / C \right),$$

$$(4.4) \quad D_s = D_{s0} \exp(k^* \gamma).$$

Here v is the velocity of a material particle in the direction of shearing, κ is the thermal conductivity, and y is the position of a material particle. Equations (4.1) and (4.2) express, respectively, the balance of linear momentum and the balance of internal energy. Equation (4.3) is the Johnson-Cook relation (3.6). As in the previous section, we have neglected elastic deformations. Since we study

the stability of an infinitesimally perturbed homogeneous solution of equations (4.1)–(4.4), initial conditions are not specified.

Let the homogeneous solution, $\mathbf{s}^0 = [\gamma^0, \tau^0, \theta^0, D_s^0]$, of Eqs. (4.1)–(4.4) at time t_0 be given an infinitesimal perturbation

$$(4.5) \quad \delta \mathbf{s}(y, t, t_0) = \delta \mathbf{s}^0 e^{\eta(t-t_0)} e^{i\xi y}, \quad t \geq t_0,$$

where

$$(4.6) \quad \delta \mathbf{s}^0 = [\delta \gamma^0, \delta \tau^0, \delta \theta^0, \delta D_s^0]^T$$

is a small disturbance, ξ is the wave number and η the initial growth rate of the perturbation. Implicit in equation (4.5) is the assumption that surface tractions are prescribed on the bounding surfaces $y = \pm \text{const}$; otherwise only those perturbations are admissible for which $\mathbf{s}^0 + \delta \mathbf{s}$ satisfies the prescribed essential boundary conditions. $\text{Re}(\eta) > 0$ implies that the homogeneous solution at time t_0 is unstable; otherwise it is stable. Substituting $\mathbf{s} = \mathbf{s}^0 + \delta \mathbf{s}$ into equations (4.1)–(4.4) and linearizing the resulting equations in $\delta \mathbf{s}^0$, we obtain $\mathbf{A}(\mathbf{s}^0, \xi, \eta, t_0) \delta \mathbf{s}^0 = \mathbf{0}$ which has a nontrivial solution only if $\det(\mathbf{A}) = 0$. This gives the following cubic equation for the growth rate η :

$$(4.7) \quad \rho^2 c \eta^3 + \rho(\beta \dot{\gamma}^0 P_0 + \kappa \xi^2 + c R_0 \xi^2) \eta^2 + (-\beta \tau^0 P_0 + \rho c(Q_0 - \hat{D}_s^0 S_0) + \kappa R_0 \xi^2) \xi^2 \eta + \kappa(Q_0 - \hat{D}_s^0 S_0) \xi^4 = 0,$$

where

$$(4.8) \quad P_0 = -\left. \frac{\partial \tau}{\partial \theta} \right|_{\mathbf{s}=\mathbf{s}^0}, \quad Q_0 = \left. \frac{\partial \tau}{\partial \gamma} \right|_{\mathbf{s}=\mathbf{s}^0}, \quad R_0 = \left. \frac{\partial \tau}{\partial \dot{\gamma}} \right|_{\mathbf{s}=\mathbf{s}^0},$$

$$S_0 = -\left. \frac{\partial \tau}{\partial D_s} \right|_{\mathbf{s}=\mathbf{s}^0}, \quad \hat{D}_s^0 = \left. \frac{dD_s}{d\gamma} \right|_{\mathbf{s}=\mathbf{s}^0},$$

and $\dot{\gamma}^0$ is the nominal or the average shear strain rate. For materials exhibiting strain hardening, strain-rate hardening and thermal softening, $P_0 \geq 0$, $Q_0 \geq 0$, $R_0 \geq 0$. We presume that the material softens because of the damage evolved and the damage is a nondecreasing function of the plastic strain, i.e., $S_0 \geq 0$ and $\hat{D}_s^0 \geq 0$. Hence if $P_0 = 0$ and $\hat{D}_s^0 = 0$, then the homogeneous solution will be always stable. For perturbations of the homogeneous solution to grow, the material must soften either due to heating or due to damage evolution or both.

In terms of non-dimensional variables

$$(4.9) \quad \bar{\eta} = \frac{\kappa \eta}{c Q_0}, \quad \bar{\xi} = \frac{\kappa \xi}{c \sqrt{\rho} Q_0}, \quad I = \frac{c R_0}{\kappa}, \quad J = \frac{\beta \tau^0 P_0 + \rho c \hat{D}_s^0 S_0}{\rho c Q_0},$$

$$\Gamma = \frac{\beta \kappa P_0 \dot{\gamma}^0}{\rho c^2 Q_0}, \quad E = 1 - \frac{\hat{D}_s^0 S_0}{Q_0},$$

equation (4.7) becomes

$$(4.10) \quad \bar{\eta}^3 + [\Gamma + (1 + I)\bar{\xi}^2]\bar{\eta}^2 + (I\bar{\xi}^2 + 1 - J)\bar{\xi}^2\bar{\eta} + E\bar{\xi}^4 = 0.$$

For given values of t_0 and $\bar{\xi}$, equation (4.10) has three roots. The root with the largest real part will make the homogeneous solution most unstable; this root is denoted by $\bar{\eta}_d$. For fixed t_0 , $\bar{\eta}_d$ depends upon $\bar{\xi}$. We seek the wave number $\bar{\xi}_m$ for which $\bar{\eta}_d$ assumes the maximum value $\bar{\eta}_m$. Thus $\bar{\eta}_m$ and $\bar{\xi}_m$ satisfy Eq. (4.10) and

$$(4.11) \quad 0 = \left. \frac{d\bar{\eta}}{d\bar{\xi}} \right|_{(\bar{\eta}=\bar{\eta}_m, \bar{\xi}=\bar{\xi}_m)}.$$

Equations (4.10) and (4.11) give

$$(4.12) \quad \bar{\xi}_m^2 = \bar{\eta}_m \frac{(J - 1) - (1 + I)\bar{\eta}_m}{2(I\bar{\eta}_m + E)}.$$

Since $\bar{\xi}_m^2 \geq 0$, therefore,

$$(4.13) \quad 0 \leq \bar{\eta}_m \leq \frac{(J - 1)}{(I + 1)}.$$

Substitution for $\bar{\xi} = \bar{\xi}_m$ from (4.12) into (4.10) yields

$$(4.14) \quad 4(I\bar{\eta}_m + E)(\bar{\eta}_m + \Gamma) = [(J - 1) - (1 + I)\bar{\eta}_m]^2.$$

Thus, whenever

$$(4.15) \quad J > 1 + 2\sqrt{E\Gamma},$$

or

$$(4.16) \quad \frac{\beta\tau^0 P_0}{\rho c Q_0} + \frac{\hat{D}_s^0 S_0}{Q_0} > 1 + 2 \left[\left(1 - \frac{\hat{D}_s^0 S_0}{Q_0} \right) \frac{\beta\kappa P_0 \dot{\gamma}^0}{\rho c^2 Q_0} \right]^{1/2},$$

equation (4.10) will have a solution $\bar{\eta}_m$ with a positive real part. Equation (4.16) generalizes BAI'S [27] criterion to materials in which the dependence of the flow stress upon the damage is accounted for. If D_s is interpreted as an internal variable and the dependence of τ upon D_s is through the factor $(1 - D_s)$, then equation (4.16) can be deduced from equation (20) of BATRA and CHEN [28]. For $\hat{D}_s^0 = 0$ or $S_0 = 0$, equation (4.16) reduces to Bai's instability criterion. For isothermal deformations, $P_0 = 0$, and the instability will occur when the softening induced by damage evolution exceeds the strain hardening of the material.

DODD and ATKINS [31] have shown that flow localization in shear is possible under isothermal conditions if voids are present within the shear band. For plane strain thermomechanical deformations of a typical steel studied by BATRA and JIN [9], material softening due to void nucleation and growth was found to be greater than that induced by the temperature rise.

For locally adiabatic deformations, $\kappa = 0$, and the instability condition (4.16) becomes

$$(4.17) \quad \frac{\beta\tau^0 P_0}{\rho c} + \hat{D}_s^0 S_0 > Q_0.$$

Thus the material becomes unstable when the material softening due to the combined effects of thermal heating and damage evolution exceeds the work hardening of the material. For a unit increment in the shear strain γ , the first term on the left-hand side of (4.17) represents the magnitude of the decrease in τ due to the thermal softening of the material, and the second term equals the decrease in τ due to the magnitude of the softening of the material caused by the damage evolution, and the term on the right-hand side of (4.7) equals the increase in τ due to work hardening of the material. Even though strain rate hardening does not explicitly appear in equations (4.16) and (4.17), it affects the value of τ^0 and hence of P_0 , Q_0 and S_0 . In the presence of heat conduction, higher values of \hat{D}_s^0 and S_0 reduce the shear strain at instability but higher values of the nominal strain rate $\dot{\gamma}^0$ delay it. BATRA and CHEN [28] have delineated the effect of strain rate on the instability strain.

Let

$$(4.18) \quad \delta = 2 \left[\left(1 - \frac{\hat{D}_s^0 S_0}{Q_0} \right) \frac{\beta\kappa P_0 \dot{\gamma}^0}{\rho c^2 Q_0} \right]^{1/2}.$$

In order to estimate the magnitude of δ we use the following values, taken from Batra and Kim [32], of material parameters for a 4340 steel and set

$$3\sigma_\infty/2\sigma_{eq} = 1, \quad D_{s0} = 10^{-5}, \quad \beta = 0.9, \quad \theta \simeq \beta A \varepsilon_e / \rho c, \quad \sigma_{eq} = A.$$

Table 1. Values of material parameters for a 4340 steel.

$A(\text{MPa})$	$B(\text{MPa})$	C	n	m	$\rho(\text{kg/m}^3)$
792.2	509.5	0.014	0.26	1.0	7,860
$c(\text{J/kg K})$	$\kappa(\text{W/m}^2\text{K})$	$\theta_m(^{\circ}\text{C})$	$\theta_r(^{\circ}\text{C})$	$\dot{\varepsilon}_{e0}(\text{s}^{-1})$	
477	49.73	1520	25	10^{-3}	

Figures 1a and 1b show, respectively, the variation of δ with $\dot{\varepsilon}_e^0$ for $\varepsilon_e^0 = 0.4$ and the variation of δ with ε_e^0 for $\dot{\varepsilon}_e^0 = 10^3/\text{s}$. For the range of values of $\dot{\varepsilon}_e^0$ and

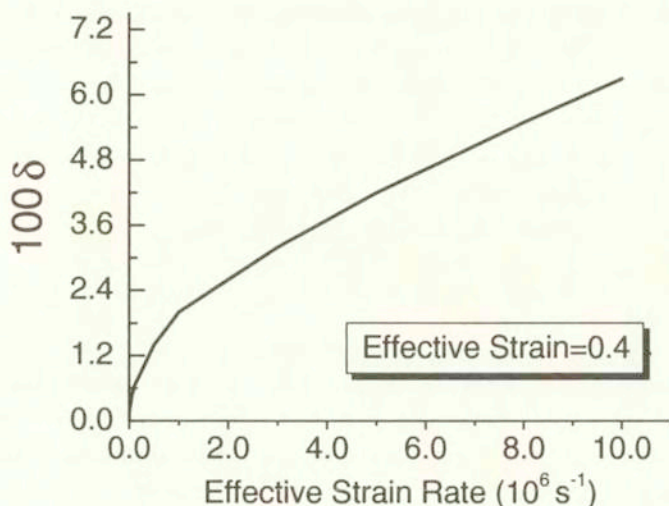


FIG. 1a. At an average effective strain of 0.4, variation of δ defined by Eq. (4.18) with the nominal effective strain rate.

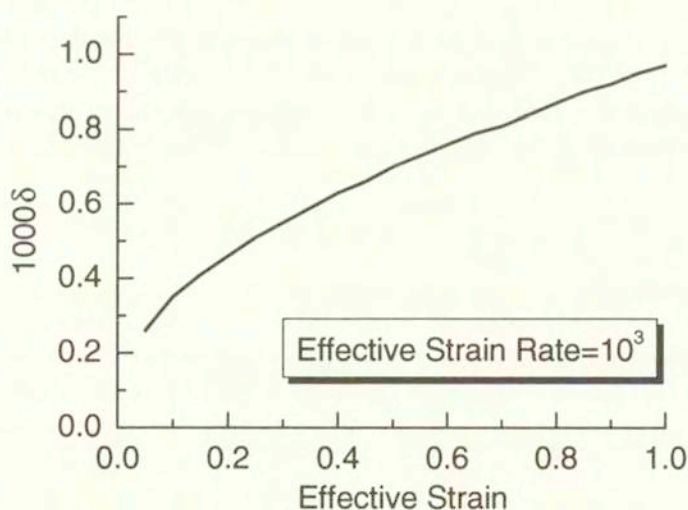


FIG. 1b. For a nominal effective strain rate of $10^3/s$, variation of δ defined by equation (4.18) with the effective strain.

ϵ_e^0 considered, the maximum value 0.06 of δ occurs for $\dot{\epsilon}_e^0 = 10^7/s$ and $\epsilon_e^0 = 0.4$. Thus $\delta \ll 1$ for typical values of strains and strain rates within a shear band, and the instability criterion (4.16) can be simplified to (4.17) even in the presence of heat conduction. This is also supported by the numerical experiments of BATRA and KIM [35] who found that thermal conductivity had a negligible effect on

the onset of an ASB but influenced significantly the subsequent deformations. Since the instability strain, γ_i , equals the minimum value of the shear strain for which inequality (4.17) holds, thus the value of γ_i may be found by replacing inequality in (4.17) by equality. A comparison of (4.17) with (3.5) reveals that the perturbation analysis and the Considère condition give essentially identical values of the instability strain. We note that for a homogeneous solution of equations (4.1)–(4.4), heat conduction plays no role.

5. Results and Discussion

For the 4340 steel deformed in simple shear with a superimposed hydrostatic tension, Fig. 2 shows the dependence of the effective instability strain ε_i upon the nominal strain rate for three values of the initial damage. It is evident that ε_i is an almost affinely decreasing function of $\log \dot{\varepsilon}_e$. Also, the initial value of the damage significantly affects the instability strain; it is because the accumulated damage is directly proportional to the initial damage. Figure 3 exhibits that the instability strain decreases exponentially with an increase in the initial damage. However, when the evolution of damage is neglected, then the instability strain is an increasing affine function of the initial damage. To elucidate this we neglect the third term on the left-hand side of (3.4) since it represents a contribution from the damage evolution. For the 4340 steel, the instability strain is then given by (e.g. cf. equation (3.7))

$$\frac{\varepsilon_i^{-0.74}}{(1 + 0.643\varepsilon_i^n)^2} = 1.0437\beta(1 - D_{s0}).$$

It is clear that a higher value of D_{s0} results in a lower value of ε_i . A positive value of D_{s0} may be viewed as decreasing β which equals the fraction of plastic working converted into heating. A lower heating rate reduces the temperature rise and hence the thermal softening effect which in turn increases the instability strain.

The effect of the hydrostatic tension on the instability strain ε_i and on the failure strain ε_f is shown in Fig. 4; ε_f is computed from equation (2.12) by setting $D_{sf} = 0.3$. It implies that the material ruptures when the surface area of voids equals 30% of the area of cross-section of the specimen. Plate impact experiments of SEAMAN *et al.* [33] suggest that copper specimens fail at a point where the porosity equals 0.3. Results plotted in Fig. 4 show that $\log(\varepsilon_f)$ decreases affinely with an increase in the hydrostatic tension. However, $\log(\varepsilon_i)$ is insensitive to the hydrostatic tension so long as it is small and below a certain value which depends upon the initial damage. Beyond this value of the hydrostatic tension, $\log(\varepsilon_i)$ decreases rather rapidly with an increase in the hydrostatic tension. Should the

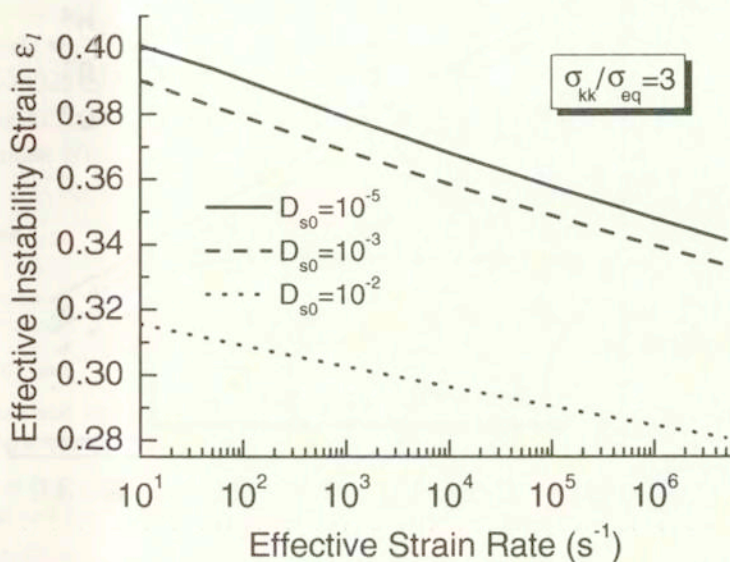


FIG. 2. For three values of the initial damage, variation of the effective instability strain with the nominal effective strain rate.

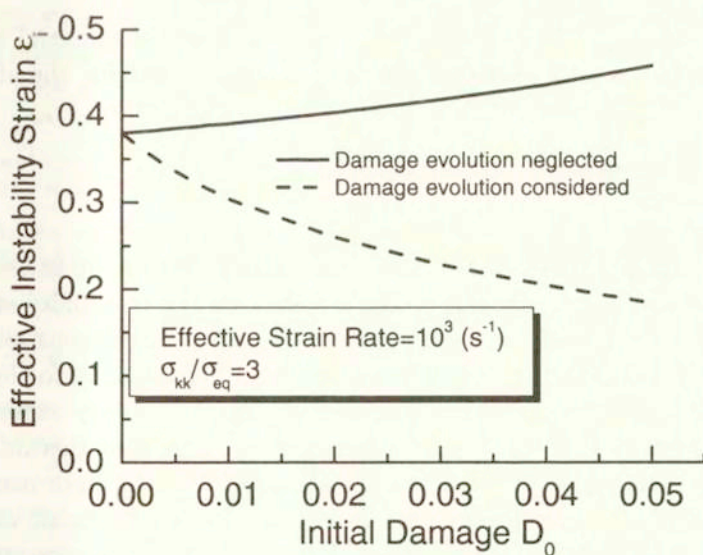


FIG. 3. For nominal strain rate of $10^3/s$, variation of the effective instability strain with the initial damage.

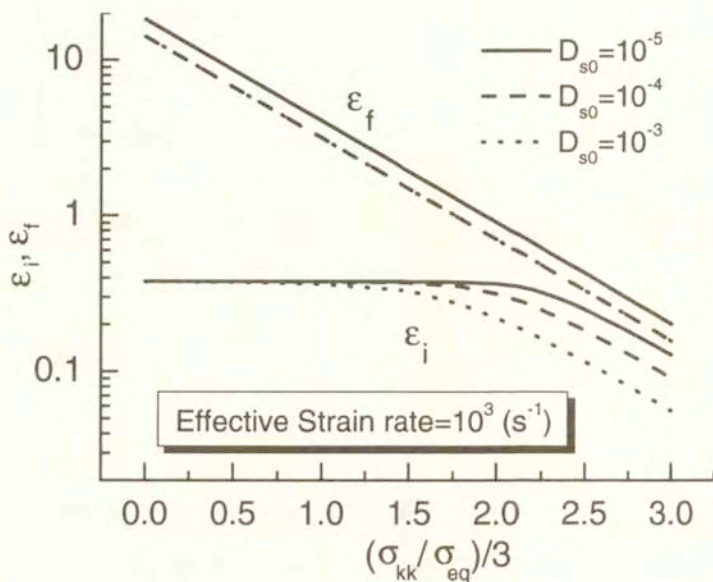


FIG. 4. For the effective nominal strain rate of $10^3/s$ and a prescribed value of the initial damage, variation of the instability and the failure strains with the normalized hydrostatic tension.

plots of ε_i vs. σ_{kk}/σ_{eq} and ε_f vs. σ_{kk}/σ_{eq} intersect, then the material will fracture before it becomes unstable. Results in Fig. 4 evince that for the steel studied here and $D_{sf} = 0.3$ it will happen only for large values of σ_{kk}/σ_{eq} .

6. Conclusions

For simple shearing deformations of a thermoviscoplastic body also subjected to a uniform hydrostatic tension, we have computed the instability strain by using the Considère criterion and also by examining if a homogeneous solution when perturbed will become unstable. The evolution of damage due to the growth of existing spherical voids has been considered. The instability strain found by the two methods is essentially the same. In the absence of thermal softening, the softening induced by the void and hence the damage growth may make the material unstable. The instability strain is not sensitive to small values of the hydrostatic tension but for large values of the hydrostatic tension, the instability strain decreases rather rapidly with an increase in the hydrostatic tension. However, the logarithm of the failure strain decreases rapidly with an increase in the hydrostatic tension. For the 4340 steel studied and assumed to fail when the accumulated damage equals 0.3, the material becomes unstable before it fails.

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