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Plastic strain in metals by shear banding.
II. Numerical identification and verification of plastic flow law

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Dedicated to Professor Piotr Perzyna
on the occasion of his 70^{th} *birthday*

THE PAPER IS DEVOTED to the identification and verification of the constitutive description of plastic flow accounting for the hierarchy of shear banding, which is proposed in the companion paper [1]. The numerical identification was carried out with use of the experimental results obtained in channel-die test for polycrystalline copper [2]. To verify the identified model, numerical simulation of the forging process was made and the calculated values of load subjected to the punch versus its displacement were compared with pertinent experimental data given in [11].

Key words: plastic flow with an account of shear bands, channel-die test

1. Introduction

THE AIM OF THE PAPER is to present the results of identification procedure of constitutive equations of plasticity accounting for the hierarchy of shear banding proposed in the companion paper [1]. As a model example, the problem of constrained plain strain compression is considered, which approximates the known channel-die test [2, 3]. Numerical calculations were carried out with application of the finite element program ABQUS/Standard [4]. Preliminary results of the mentioned above identification were discussed in [5-7]. The results presented in this paper have been obtained with the application of new algorithms developed in [8, 9]. The algorithms are based on the solution of nonlinear regression problem with use of the method of global optimization of C. G. BOENDER *et al.* [10]. The automatic procedure of the identification of the unknown scalar function, which describes the contribution of shear bands in plastic shear strain rate, was elaborated. An attempt of verification of the proposed constitutive description is also presented. To verify the plastic flow law with the identified contribution function, the experimental results of forging, presented in [11], were used. The discussed verification shows that the proposed description of plastic flow with an account of shear banding is well suited for the application of numerical simulation of a certain class of metal shaping operations.

2. Discussion of the available experimental data and the results of numerical simulations of channel-die test

In the mentioned above papers [2, 3] the experimental results of the channel-die test for polycrystalline copper were presented. The main subject of the study was the evolution of texture and development of micro-shear bands. The geometry of the matrix and experimental setting are depicted in Fig. 1. The following dimensions of the sample were assumed:

- the height corresponding to the direction of compression: $e_3 - 6.35$mm,
- the width corresponding to the direction of the free plastic flow:$e_2 - 9.53$mm,
- the length corresponding to the direction of matrix constraints: $e_1 - 14.73$mm.

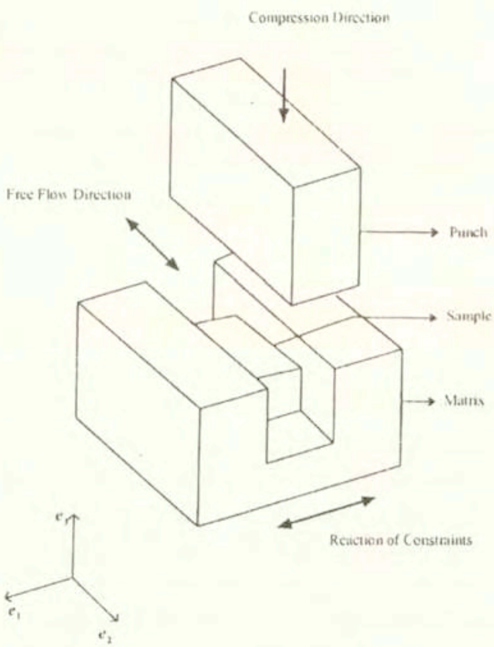

FIG. 1. Scheme of constrained plane strain compression in channel-die test.

The effects of friction were reduced with application of teflon plates covering the contact surfaces of the matrix and the sample. This motivated the authors to assume in numerical calculations that the contact between the sample and the matrix is frictionless. In the experimental investigations described in [2, 3], four series of experiments in which the samples were compressed for the value of: –0.21; –0.52; –1.0 and –1.54 of logarithmic strain ε_3, with the strain rate 0,001 s^{-1}, were conducted. The surface of deformed samples was subjected to metallographic observations. The results were summarized in [3], p. 234, in the following way: "... *at a true strain of -0.21, the grains are still reasonably equiaxed. Figure 10a shows the microstructure at a true strain of -0.52. At this stage the grains have been flattened, and localized bands, which are 0.1÷0.5 μm in thickness and inclined at* $\sim \pm 30 \div 40^{\circ}$ *to the horizontal can be observed within individual grains. The initiation of such micro-shear bands occurs somewhere between true strain levels of -0.21 and -0.52. The intensity of these micro-shear bands continues to increase as deformation progresses, and by a strain level of -1.00, Fig. 11a, macro-shear bands which cross grain boundaries have formed.*"

In works [2, 3] the results of calculations were also presented. The authors simulated numerically the similar process of constrained plaine strain compression, approximated by plane strain state, for polycrystalline aggregate with the application of finite element program ABAQUS. The aggregate consists of a large number of grains with assumed crystal lattice orientations, in which the three-dimensional activation of all potential slip systems is allowed. The viscoplastic flow law at the level of a single slip system was assumed. A separate finite element corresponds to a single crystal or a part of it. In the analysis of constrained plane compression problem an aggregate of 400 grains was assumed, which was represented by 400 quadrilateral continuum plane strain 4-node elements - CPE4. In order to reduce the computational time, the lack of friction between the surfaces of sample, matrix and punch was assumed. The detailed information concerning the algorithm and its implementation into finite element program ABAQUS one can find in [2]. The confrontation of the results of numerical calculations and experimental measurement is given in Fig. 2, which displays the relation between the absolute value of compression stress $|\sigma_3|$ and the absolute value of logarithmic strain $|\varepsilon_3|$. The experimental results are represented by points, taken from the plot of experimental data presented in [2], p. 457 - Fig. 7, which shows also the comparison with the results of numerical simulation - presented in our Fig. 2 as a curve labelled by 2. Let us observe, following the remark in [2], that for strain values taken from the range 0.21-0.52, in which the metallographic observations reveal intensive development of micro-shear bands, the discrepancy between the experimental points and the curve displaying the results of numerical calculations becomes visible. The discrepancy increases in the increase of strain, reaching about 22% for the value of logarithmic strain $|\varepsilon_3|=1.4$. The observed inconsistency of numerical simulation and experimental observations was attributed in [2] to the development of micro-shear banding and increasing contribution of this new mechanism in the process of constrained compression in channel-die test. The mechanism of micro-shear banding has not been taken into account in the constitutive model applied for numerical simulation of channel-die test presented in [2]. This might be the reason, according to our opinion, of the observed discrepancy.

3. Identification of plastic flow law accounting for micro-shear banding with application of numerical calculations of channel-die test

The discussed results presented in [2, 3] made a basis for identification of the plastic flow law accounting for micro-shear bands, proposed in the companion paper [1]. Using finite element program ABAQUS/Standard [4], the homogeneous plane strain compression process was calculated, which simulates the idealized, frictionless, channel-die test. The continuum plane strain eight node finite element of type CPE8 was implemented. Plastic flow law accounting for the contribution of symmetric system of micro-shear bands was used (cf. [1]). The specification of Eq. (35) in [1] for infinitesimal elastic strains gives the following rate-type equation:

$$\overset{\circ}{\sigma}_{ij} = C_{ijkl} D_{kl}, \qquad C_{ijkl} = 2G\left(\delta_{ik}\delta_{jl} + \frac{\nu}{1+\nu}\delta_{ij}\delta_{kl} - \frac{1}{\alpha\sigma_Y^2}\sigma'_{ij}\sigma'_{kl}\right), \tag{3.1}$$

where

$$\alpha = \frac{2}{3}\left[1 + \frac{h\,(1-f_{SB})}{3G}\right], \tag{3.2}$$

σ_{ij} and σ'_{ij} denote, respectively, the components of the Cauchy stress tensor and its deviator in an orthonormal basis, whereas h is plastic modulus. Let us observe that the contribution of shear banding $f_{SB} \in [0, 1]$ is accounted in the scalar parameter α, affecting elasto-plastic moduli and, at the same time, the stiffness matrix of the considered numerical scheme. The symbol $\sigma^{\circ}{}_{ij}$ denotes the components of Zaremba-Jaumann derivative of the Cauchy stress tensor. The results of calculations based on the classical model of elastic-plastic deformation with isotropic hardening (called in [4] J_2-theory) are represented in Fig. 2 by curve 1. One can observe the large discrepancy between the curve 1 and the experimental data shown in Fig. 2. For the strain $|\varepsilon_3|$=1.4 the difference reaches about 53%. The results of numerical simulation for elasto-plasticity model defined by Eq. (1) are represented by curve 3. In this case the calculations were conducted for such a form of function $f_{SB} = F_{SB}(\varepsilon^p)$, describing the dependence of shear banding contribution versus equivalent plastic strain ε^p, which assures possibly close fitting of the curve 3 with respect to the experimental points. The possibility provided by the program ABAQUS/Standard was used, which enabled modification of material description by the user procedure UMAT. In the procedure UMAT the simple version of the known algorithm of radial return was implemented, which is used typically for integration of constitutive equations of elasto-plasticity with the Huber-Mises yield condition and isotropic hardening (cf. e.g. [12], part. II, chapter 6). The calculations were made for the following

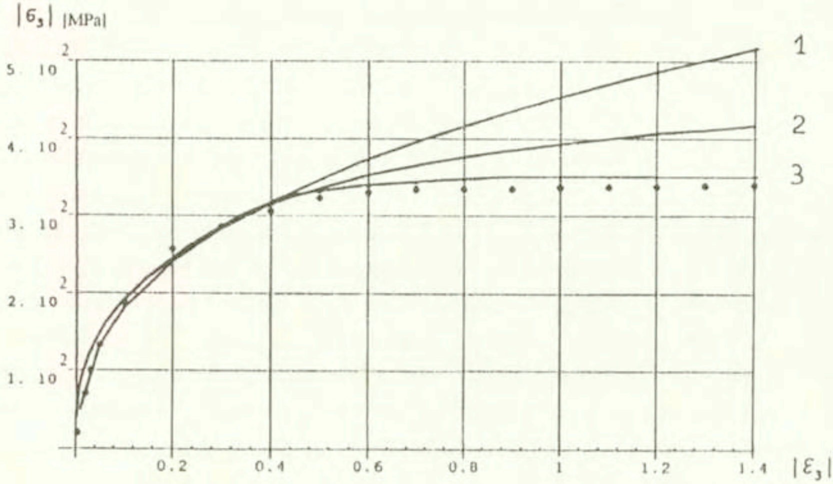

FIG. 2. Plot of the absolute value of compression stress $|\sigma_3|$ as a function of logarithmic strain $|\varepsilon_3|$ for different models of plastic flow versus the experimental results of channel-die test: points representing experimental data of C.A. BRONKHORST *et al.* [2] - •; curve 1 – results of Prandtl-Reuss model for isotropic hardening; curve 2 – numerical simulation of the aggregate of grains according to [2]; curve 3 – results of plastic flow law model expressed by Eqs. (1-4).

approximation of hardening curve taken from the free compression test of Cu polycrystalline samples presented in [2], p. 450, Fig. 2:

$$\sigma = \sigma_Y \left(\frac{E}{\sigma_Y} \varepsilon^p \right)^{\frac{1}{m}}, \tag{3.3}$$

where the yield limit $\sigma_Y = 0.02$ GPa, Young modulus $E = 126.0$ GPa, $m = 2.93$. The shape of the sought function of shear banding contribution, $F_{SB}(\varepsilon^p)$, which provides the required fitting of experimental points with the 3% error, is given in Fig. 3. The function is assumed as a logistic function of the form

$$F_{SB}(\varepsilon^p) = \frac{f_{SB_0}}{1 + \exp(a - b|\varepsilon_3|)} \tag{3.4}$$

where $f_{SB_0} = 0.95$, $a = 7.5$, $b = 13.6$.

Such a method of identification relies on an intuitive selection of the form of the sought function and fitting of relevant constants by means of repeated calculations of the process of homogeneous constrained plane strain compression and comparison of the computed values of compression stress σ_3, displayed in Fig. 3 – curve 3, with experimental points taken from the experimental data presented in [2]. To make the identification more efficient and independent of intuitive guess, an automatic procedure has been proposed by application of the iteration method. Certain starting value of the shear banding contribution function is assumed, e.g. $f_{SB}^{\text{start}} = 0,5$ and then one executes the series of calculations of the mentioned above problem of constrained plain strain compression. On each step, defined by a given strain increment $|\Delta\varepsilon_3|$, one checks if the stress values σ_3 calculated for different values of $\in$[0,1), lay sufficiently near, with an assumed error, to the values taken from the curve representing the experimental data (cf. Fig. 2). The presented in Fig. 3 results of the automatic identification procedure were obtained for the assumed error equal to 5%. It appears that an attempt of diminishing of this value leads to the rapid increase of the number of iterations and the resulting cost of computations. In the presented automatic identification procedure the new algorithms developed in [8, 9] were implemented. The algorithms are based on the solution of nonlinear regression problem with use of the method of global optimization of C. G. BOENDER *et al.* [10].

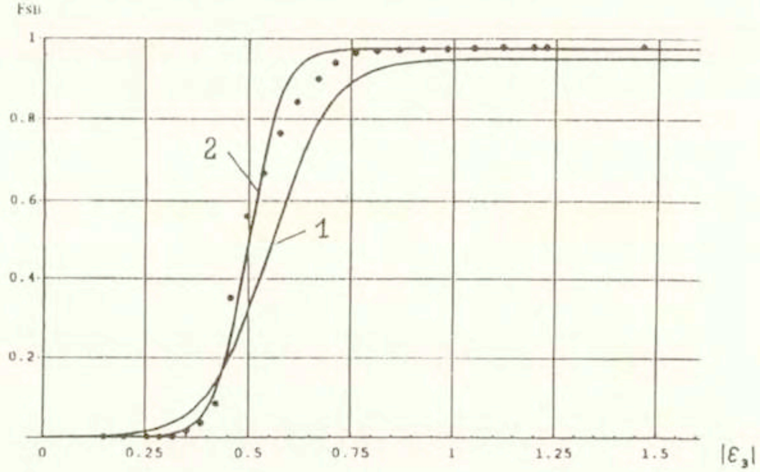

FIG. 3. Comparison of the results of simple identification with the assumed function F_{SB}, Eq. (4), and fitted constants $f_{SB_0} = 0.95$, $a = 7.5$, $b = 13.6$ – curve 1, with the results of automatic iterative identification – filled points and its approximation – curve 2 and with use of the function Eq. (4) and specified constants: $f_{SB_0} = 0.975$, $a = 12.5$, $b = 25.0$.

The details of the automatic identification procedure are discussed in [13], whereas in this paper the results of calculations are considered only. Fig. 3 displays the set of points representing the calculated values of f_{SB}, which satisfy the assumed compatibility condition with experimental data. The points can be fitted with use of a curve shown also in Fig. 3. Let us observe that the intuitively guessed logistic function $F_{SB}(\varepsilon^p)$ does not differ much from the points obtained from automatic identification.

4. Numerical verification of plastic flow law accounting for shear banding on the example of forging process

The identification of shear banding contribution function $F_{SB}(\varepsilon^p)$ makes possible to apply, as proposed in the companion paper [1], the model of elasto-plastic deformation accounting for shear banding for numerical solution of boundary value problems simulating different processes of metal shaping operations. It remains, however, an open problem of determination of the influence of the change of deformation path or loading scheme on the shape of the identified function $F_{SB}(\varepsilon^p)$. The first attempt to verify the proposed model is the numerical simulation of plane strain micro-forging process of an annealed copper polycrystalline sample, which was studied earlier experimentally and numerically in [11]. The material was chosen the same, as for the discussed above channel die test reported in [2, 3]. The scheme of matrix, the initial geometry of the sample and matrix, and finite element mesh were displayed in Fig. 4. In the upper part of the punch, which at the beginning of the test is in the contact with the sample, small indentation was made in order to prevent the displacement of the points on its upper surface in the direction x_1. The remaining surfaces of the sample and tool were carefully polished and lubricated in order to minimize the effects of friction. Due to the symmetry of the experimental setup of matrix and sample, the calculations were made only for the one half of it. In [11] the finite element program ABAQUS was used with assumption of 17 continuum plane strain eight-node elements of the type CPE8R. The contact between the upper part of the sample and the punch is modelled by assuming the constraints of the displacements in the direction of x_1, while the interaction between the remaining surfaces of the sample, punch and matrix is assumed as contact without friction. In the numerical simulation presented in [11] the authors devoted much attention to the evolution of texture in polycrystalline aggregate assuming certain modified form of the Taylor model. In our considerations we shall focus on the results presented in [11], which are related with global reaction of the system during forging operation. In Fig. 5 the plots of the force applied to the punch in dependence on the displacement of the punch with upper surface of the sample are displayed. The curve with full triangles corresponds to experimental data, whereas the plot with light triangles shows the results of numerical calculations taken from [11].

The problem discussed above was applied for verification of the plasticity model described by Eqs. (3.1)–(3.4). The similar geometry, boundary conditions and the type of finite element have been assumed. Fig. 6 shows the initial and deformed finite element mesh. For numerical calculations the program ABAQUS/ Standard [4] was implemented with the modified UMAT, as described in the previous section. The results of calculations were displayed in Fig. 5. The curve with filled circles corresponds to the classical Prandtl-Reuss flow law with isotropic hardening (called in [4] J_2-theory), while the curve drawn with heavy line pertains to the flow rule accounting for shear banding given by Eqs. (3.1)–(3.4). Comparison of the results assembled in Fig. 5 shows that the new plasticity model described by Eqs. (3.1)–(3.4) provides quite good accordance with the experimental data and the results of numerical simulation obtained in [11] for the modified Taylor model, which requires much larger computational time. It is also visible that the classical plasticity model leads to increasing discrepancies in comparison with experiment. This is due to the fact that classical model does not account for the new mechanism of plastic deformation, i.e. shear banding, which changes qualitatively material response.

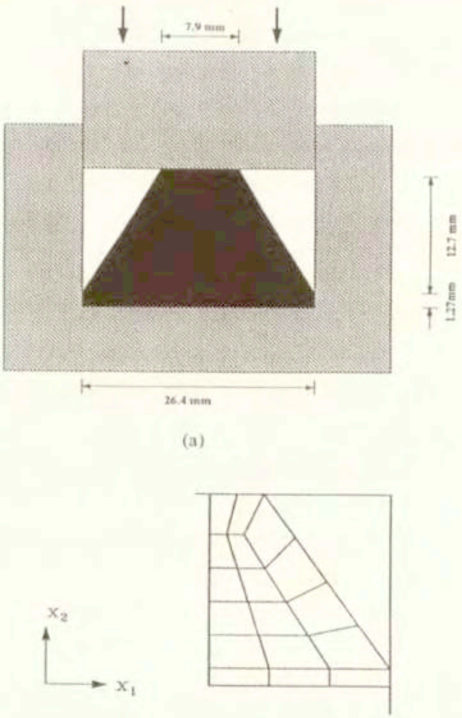

FIG. 4. Scheme of matrix, geometry of initial configuration of sample and matrix together with finite element mesh, according to [11], which were applied for numerical simulation of forging process.

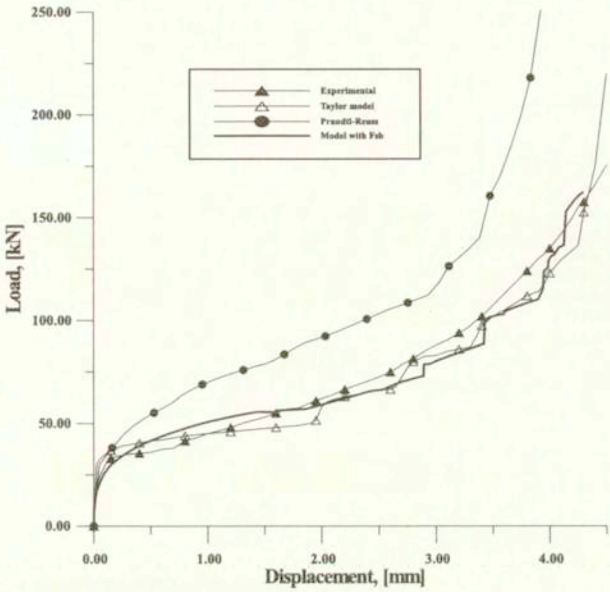

FIG. 5. Plots of load applied to the punch as a function of displacement of the punch with the upper part of the sample.

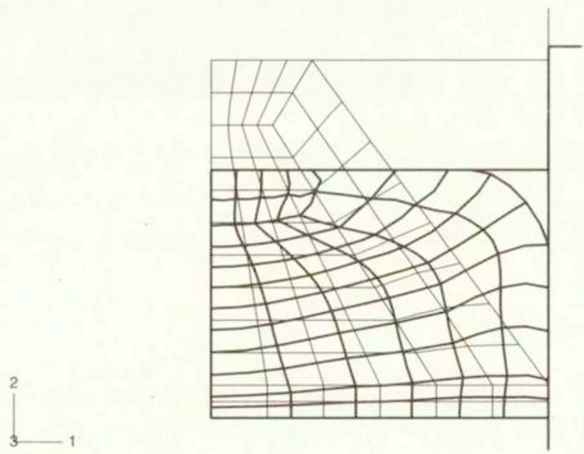

FIG. 6. Finite element mesh in initial and deformed configurations of one half of the specimen.

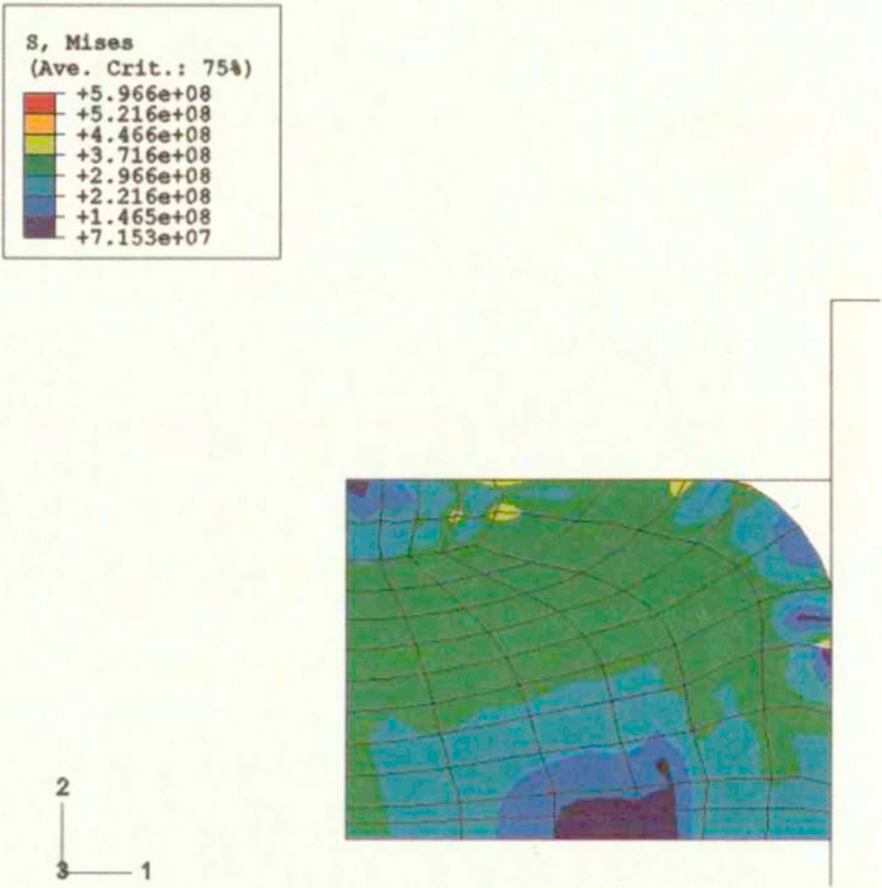

FIG. 7. Distribution of equivalent stress in the deformed configuration.

5. Conclusions

As it was stressed in [5], the motivation for the assumption of the form of function (3.4) was based on the experimental observations, which suggest that micro-shear bands contribute in shear strain rate, as a sequence of generations of active micro-shear bands. The logistic growth law for active micro-shear bands, which is well known in population dynamics, was proposed originally in [14]. The logistic function given in (3.4) may be considered as a solution of such an evolution equation. In future investigations the evolution equation for density of

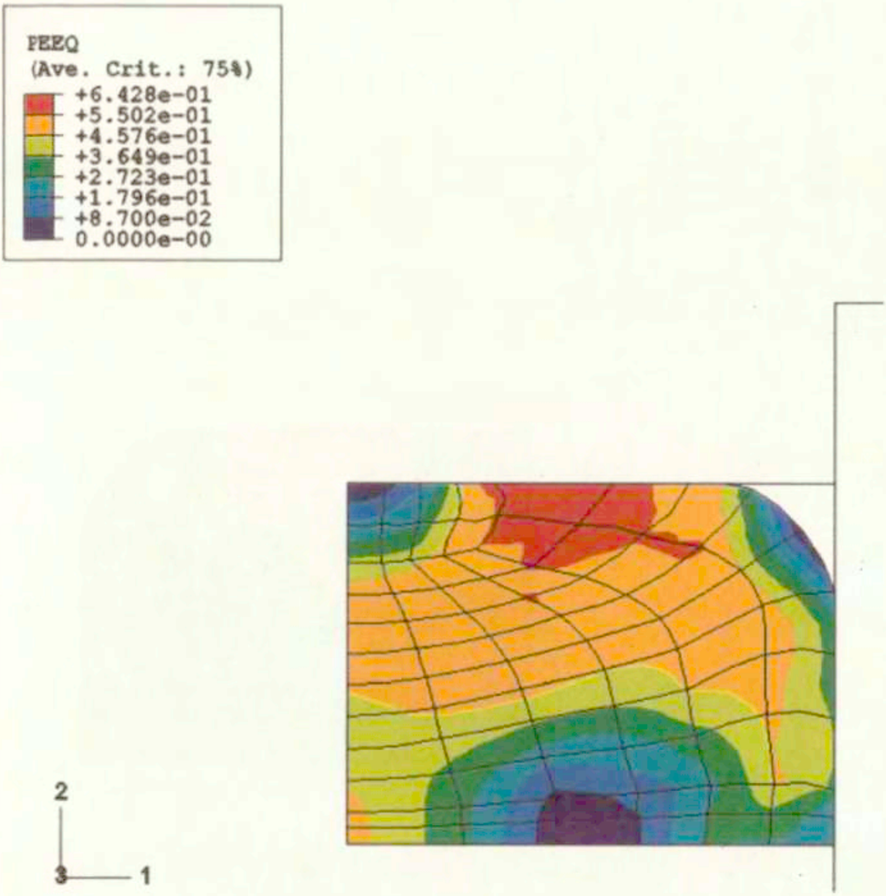

FIG. 8. Distribution of equivalent plastic strain in the deformed configuration.

active micro-shear bands should be sought. Such an evolution equation should take into account the influence of the changes of strain path and loading scheme on the contribution of shear banding f_{SB}. Identification and verification of such an evolution equation needs further studies.

The analysis of the discussed results of numerical calculations and experimental data collected in Fig. 5 leads to the observation that application of the simple model given by Eqs. (3.1)–(3.4), which is based on the flow law for symmetric system of shear banding gives reasonably good accordance with experiment, in spite of appearing inhomogeneities of stress and strain fields. The distribution of equivalent stress is illustrated in Fig. 7, while the inhomogeneous distribution

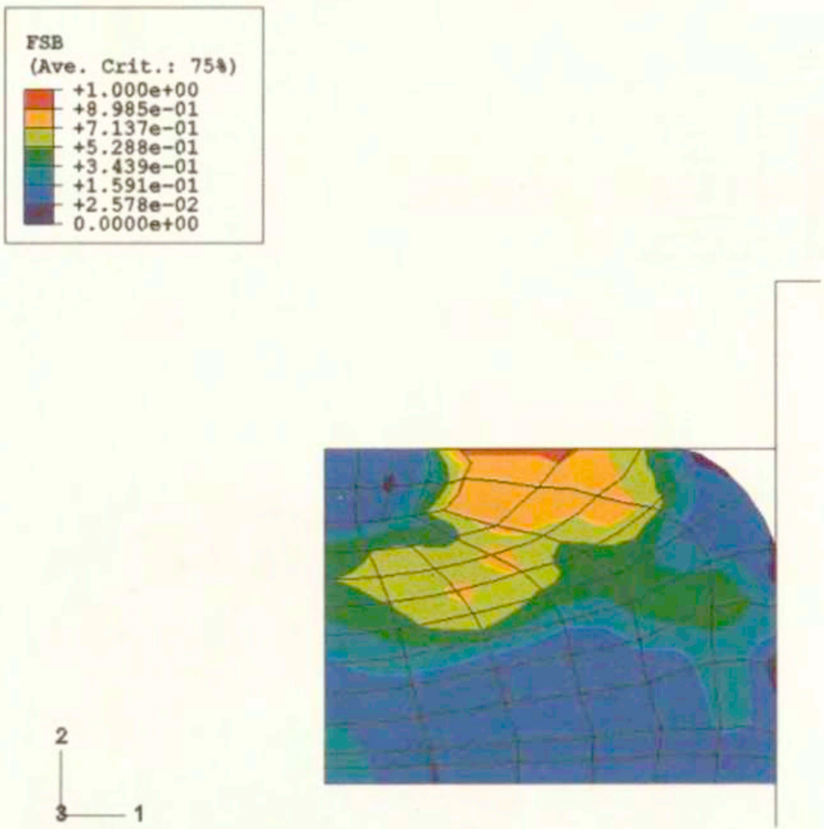

FIG. 9. Distribution of shear banding contribution function F_{SB} in the deformed configuration.

of equivalent plastic strain is displayed in Fig. 8 and the distribution of values of shear banding contribution function f_{SB} at the final stage of deformation is shown in Fig. 9. The mentioned inhomogeneous fields, produced by the assumed geometry of experimental setup and boundary conditions, are related with local rotations of principal axes of the stress tensor and rate of the deformation tensor. The results shown in Fig. 5 suggest that the simple model (3.1–3.3.4) can be also applied for certain class of problems, in which local rotations of principal axes of stress and rate of deformation are admissible. Further studies are necessary to determine the class of processes, in which the application of the simplified constitutive description could appear to be admissible. Otherwise, the more general flow law presented in [1] by Eq. (7.2) should be applied. In such a case, an additional function being a measure of the asymmetry of shear banding discussed in [1] should be specified and identified numerically with use of the experimental tests taking into account changes of the loading direction. Some preliminary results of such a tests and discussion of possible identification procedure are given in [15].

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