

## Plastic strain in metals by shear banding.

### I. Constitutive description for simulation of metal shaping operations

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*Dedicated to Professor Piotr Perzyna  
on the occasion of his 70<sup>th</sup> birthday*

THE AIM of the paper is to study the description of plastic strain in metals produced by a hierarchy of plastic slip and shear banding processes: from slip lamellae and slip bands to coarse slip bands, which may further transform into transgranular micro-shear bands and form clusters of micro-shear bands. Constitutive description accounting for the contribution of shear banding was proposed and possible simplifications are discussed from the point of view of applications for numerical simulation of metal shaping operations.

## 1. Introduction

MULTISCALE MODELLING of large deformations of metals requires the identification of physical mechanisms of plastic strain, careful analysis of averaging procedures and proper setting of the resulting description within continuum theory of materials. The analysis and interpretation of available experimental results obtained with the application of different techniques at different scales reveal a hierarchy of plastic slip processes: from slip lamellae and slip bands to coarse slip bands, which may further transform into transgranular micro-shear bands and form clusters of micro-shear bands. This shows that crystalline solid subjected to plastic deformation is a multi-scale hierarchically organised system. It is a difficult task to describe and predict theoretically the behaviour of such a complex structure. Only partial solutions are available up to now. The mechanism of shear banding was studied in [1], where also the derivation of related macroscopic measure of the rate of deformation was presented. The continuum mechanics description of the kinematics due to shear banding made a basis for constitutive

description proposed in [2-5]. The important question, which remains unsolved, is formulation of a condition of the onset of micro-shear banding. One could expect, that such a criterion should specify at which instant of a considered loading path micro-shear bands start to contribute to plastic deformation. In [3, 4] a theoretical description of small elastic and large plastic deformations within the framework of a two-surface plasticity model, with the internal yield surface connected to back stress anisotropy and the external surface related to micro-shear band formation, was proposed.

On the other hand, the attempts to identify the proposed model for simple case of symmetric shear banding occurring in the case of channel die test made possible to specify the contribution of shear banding as a logistic function of equivalent plastic strain [5, 6]. Such a function accounts for smooth increase of a contribution of micro-shear bands, from zero for very small values of equivalent plastic strain to rapid growth within certain narrow strain interval up to the ultimate value, which is always lower than one. This result led us to the conclusion that it is possible to formulate a simpler form of flow law with a single Huber-Mises yield surface without the necessity of defining the second limit surface related with the onset of micro-shear banding. The aim of the paper is to study afresh the description of plastic strain in metals and to propose such a constitutive description of plastic strain accounting for the contribution of shear banding. In the companion paper [6], the identification and verification of the proposed model are discussed. The results presented herein show the predictive power of the model and the possibilities of applications for numerical simulation of metal shaping operations.

## 2. Physical motivation

The available results of metallographic observations reveal that in heavily deformed metals, or even at small strains if they are preceded by a properly controlled change of deformation path, a multiscale hierarchy of shear localization modes replaces the crystallographic multiple slip or twinning. Different terminology is used depending on the level of observation. In our study, the term micro-shear band is understood as a long and very thin (of order of  $0.1 \mu\text{m}$ ) sheet-like region of concentrated plastic shear crossing grain boundaries without deviation and forming a definite pattern in relation to the principal directions of strain. It bears very large shear strains and lies in a "non-crystallographic position". The term "non-crystallographic" means that micro-shear bands are usually not parallel to a particular densely packed crystallographic plane, of conventionally possible active slip system, in the crystallites they intersect. In such a case, a polycrystalline sample deforms as a "pseudocrystal" subjected to a single or double shear. This change of deformation mode modifies remarkably the ma-



terial properties and makes a basis for the development of new technologies of metal shaping operations [7]. The detailed experimental information about mechanical behaviour and related structural features is reviewed in [2-9], where also comprehensive lists of references are given. The experimental observations reveal the time and spatial organisation of dislocations. This results in the hierarchy of plastic slip processes: from coplanar dislocation groups moving collectively along active slip systems, through slip lamellae and slip bands to coarse slip bands, which may further transform into transgranular micro-shear bands and form clusters (packets) of micro-shear bands of the thickness of order  $(10 \div 100) \mu\text{m}$ . At this level of observation, the clusters become elementary carriers of plastic strain.

New information on this phenomenon, showing that it appears even more complex, provide the recent observations of the correlation of temporal instabilities and spatial localization during propagation of Portevin-Le Chatelier deformation bands with use of a novel multizone laser scanning extensometer [10, 11]. The analysis of the extensometer data reveal three types of the PLC bands: type A with continuous propagation of single band along the specimen, type B characterizing with discontinuous band propagation and type C of stochastic nucleation of single bands along the specimen. The clusters of micro-shear bands, produced for instance in rolling, form the planar structures, which are usually inclined by about  $\pm 35^\circ$  to the rolling plane and are orthogonal to the specimen lateral face. There can be, however, considerable deviations from this value within the range of  $15^\circ$  to  $50^\circ$ . As it was already stressed in [3, 5], it is typical of the clusters of active micro-shear bands that their planes are rotated relative to the respective planes of maximum shear stress by a certain angle  $\beta$ , which is usually of the order  $(5 \div 15)^\circ$ . It is worthy to stress that the problem of specifying the angle is complicated by the difficulty of distinguishing the most recently formed micro-shear bands from those that were formed earlier and subsequently rotated with material towards the rolling plane. This is related with the important observation, discussed in [2], that a particular micro-shear band operates only once and develops fully in very short time. As it was discussed in [1], the head of micro-shear band can propagate with the velocity, which is close to the elastic shear wave speed (velocity of sound) in the considered metal or alloy. The micro-shear bands, once formed, do not contribute further to the increase in plastic shear strain. They leave characteristic traces in the structure of the material but it is irrelevant for the constitutive description of plastic flow. We assume that the successive generations of active micro-shear bands competing with the mechanism of multiple crystallographic slips are responsible for the process of advanced plastic flow.



### 3. Physical model of shear strain rate produced by active micro-shear bands

The physical constraint on any continuum mechanics approach to metal plasticity, i.e. the physical dimension of the smallest representative volume element (RVE) of crystalline material, for which it is possible to define significant overall measures of stress and strain during plastic deformation and the assumptions of the averaging procedure, were thoroughly discussed in [1, 2]. The known in the literature averaging procedure is valid under the general assumption that the dominant mechanism of plastic deformation corresponds to multiple crystallographic slip. In such a case, the theory describing kinematics and constitutive structure of finite elastic-plastic deformation of crystalline solids is well established and the transition between the microscopic and macroscopic levels is well understood. In particular, relations between macro-measures of stress, strain and plastic work are related with the volume averages of their micro-measures. As it was stressed in [2, 4], the situation changes, when an additional mechanism of micro-shear banding is taken into consideration. To solve the problem of proper setting of the effects of micro-shear banding within the continuum mechanics, the description of shear strain rate produced by active micro-shear bands should be given and the concept of RVE should be redefined.

Consider the RVE containing the region of progressive shear banding, depicted schematically in Fig.1a. An active shear band consists of the clusters of micro-shear bands, which at this level of observation can be considered as elementary carriers of plastic strain. On the other hand, an active micro-shear band is produced as the effect of spatial and time organisation of large number of dislocations. They are generated and move collectively within a long and thin sheet-like regions, crossing grain boundaries without deviation and having the thickness of the order of  $0.1 \mu\text{m}$ . Therefore, from the point of view of kinematics, the micro-shear band can be considered as a thin region of concentrated plastic shear. During the passage of the active zone, of thickness  $h_{ms}$  and width  $l_{ms}$ , the local perturbation  $B_{ms}$  of the microscopic displacement field is produced which travels at the head of the micro-shear band with the speed  $v_{ms}$  as a distortion wave, cf. Fig 1c. In Fig. 1, two successive "magnifications" of the shear-banding zone are "zoomed in" and the related fundamental mechanisms of plastic shear are illustrated. The first one, depicted in Fig. 1b, corresponds to the cluster of micro-shear bands, in which the passage of large number of active micro-shear bands results in the local perturbation  $\Delta_{MS}$  of the mesoscopic displacement field  $\mathbf{u}_M = \mathbf{x}_M - \mathbf{X}_M$ , which moves with the speed  $V_S$ . The second "magnification", shown in Fig.1c, represents the aforementioned active zone of a single micro-shear band.

Consider an elementary dislocation model of plastic shear produced in the active zone at the head of a single micro-shear band, as it is depicted in Fig. 1c. According to the known approach, the shear strain  $\gamma_{ms}$  results from the generation and movement of large number of dislocations within the active zone [1]

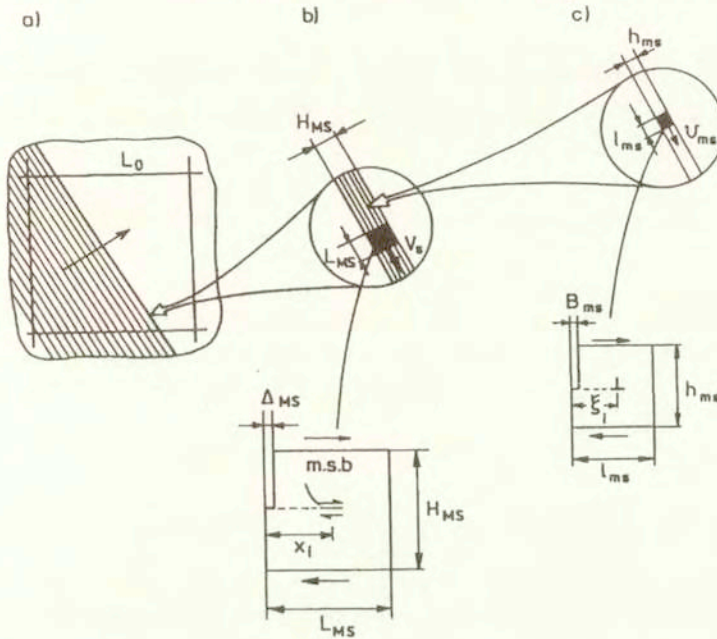


FIG. 1. Schematic view of the multiscale, hierarchically organized system of shear banding: (a) The RVE of the dimension of  $L_0 \approx 1$  mm traversed by the region of shear banding progressing in the direction pointed by the arrow. (b) The cluster of active micro-shear bands with the active zone of the thickness  $H_{MS} \approx (10 \div 100) \mu\text{m}$  and the width  $L_{MS}$  being of the same order. Beneath, the fundamental mechanism of plastic shear strain generated by the active micro-shear bands operating within the active zone, moving along distances  $x_i$ ,  $i = 1 \dots N_{MS}$  during their "lifetime", and producing the total displacement  $\Delta_{MS}$  is depicted. (c) The active zone of a single micro-shear band of the thickness  $h_{ms} \approx 0.1 \mu\text{m}$  and the width  $l_{ms}$  of the same order. Below the picture of an elementary dislocation model of plastic shear in the active zone is shown. The displacement  $B_{ms}$  is produced by  $n$  dislocations moving at the distances  $\xi_i, i = 1 \dots n$ .

$$(3.1) \quad \gamma_{ms} = \frac{B_{ms}}{h_{ms}} = \frac{bn}{l_{ms}h_{ms}} \bar{\xi}, \quad B_{ms} = \frac{b}{l_{ms}} \sum_i^n \xi_i, \quad \bar{\xi} = \frac{\sum_i^n \xi_i}{n},$$

where  $b$  is the length of Burgers vector and  $\bar{\xi}$  is the average distance that dislocations have moved. If the distance  $\bar{\xi}$  and the number of dislocations  $n$  can change



with the variable  $\tau$ , corresponding to the duration of the microscopic process of plastic shear, we have

$$(3.2) \quad \frac{d\gamma_{ms}}{d\tau} = \frac{b}{l_{ms}h_{ms}} \left( nv_d + \bar{\xi} \frac{dn}{d\tau} \right)$$

where  $v_d = \frac{d\bar{\xi}}{d\tau}$  is the average dislocation velocity. Finally, the shear strain rate, produced in a single micro-shear band, is expressed in terms of the head of micro-shear band speed  $v_{ms}$

$$(3.3) \quad \frac{d\gamma_{ms}}{d\tau} = \frac{v_{ms}}{h_{ms}}, \quad v_{ms} = \frac{b}{l_{ms}} \left( nv_d + \bar{\xi} \frac{dn}{d\tau} \right).$$

According to (3.3), generation and movement of new dislocations contribute to the plastic strain rate. If we assume that the movement of a constant number of mobile dislocations plays the prevalent role, (3.4) transforms into the well known form of the Orowan relation

$$(3.4) \quad \frac{d\gamma_{ms}}{d\tau} = b\rho v_d, \quad \rho = \frac{n}{l_{ms}h_{ms}},$$

where  $\rho$  denotes the dislocation density. In the case of micro-shear bands propagation, the systems, which are not necessarily parallel to densely packed crystallographic planes, are activated. The critical stress in such planes is very high and therefore, the generation of new dislocations may contribute remarkably to plastic shear strain rate. Therefore, it is possible that (3.3) supplemented with proper form of evolution equation for  $n$  should be taken into considerations.

Consider a number of active micro-shear bands  $N_{MS}$  of similar orientation and produced within certain time interval,  $\Delta\tau = \tau_f - \tau_i$ , which can be considered as a "time-like" variable or rescaled length of deformation path in the macroscopic description of plastic flow. The interval  $\delta\tau$  corresponds to the Representative Time Increment of the process considered earlier in [12]. Such a cluster of "simultaneously" activated (averaged over  $\Delta\tau$ ) micro-shear bands, as it is depicted in Fig. 1b, produces the shear strain on the mezosopic level

$$(3.5) \quad \gamma_{MS} = \frac{\Delta_{MS}}{H_{MS}}, \quad \Delta_{MS} = \frac{\bar{B}_{ms}N_{MS}}{L_{MS}}\bar{x}_{MS}, \quad \bar{B}_{ms} = \int_{\tau_i}^{\tau_f} v_{ms}d\tau$$

where  $\bar{B}_{ms}$  is the total displacement produced by a single micro-shear band and  $\bar{x}_{MS}$  denotes the average distance that  $N_{MS}$  micro-shear bands have moved, during their "lifetime" in the active zone. Assuming that the distance and the

number of micro-shear bands can change during propagation of the active zone of the cluster, we have from (5)

$$(3.6) \quad \dot{\gamma}_{MS} = \frac{V_S}{H_{MS}}, \quad V_S = \frac{\bar{B}_{ms}}{L_{MS}} \left( N_{MS} v_{ms} + \bar{x}_{MS} \dot{N}_{MS} \right),$$

where the dot denotes differentiation with respect to the "time-like" variable  $t$ . Let us observe that the rate  $\dot{\bar{x}}_{MS}$  can be identified with the speed  $v_{ms}$  of the head of a single micro-shear band under the simplifying assumption that  $v_{ms}$  is approximately the same for each micro-shear band in the active zone of the cluster.  $V_S$  corresponds to the speed of propagation of the disturbance of the microscopic displacement field, produced in the active zone of the cluster of  $N_{MS}$  active micro-shear bands. If the number of active micro-shear bands in the active zone can be assumed constant, the formula similar to the aforementioned Orowan relation is obtained

$$(3.7) \quad \dot{\gamma}_{MS} = \bar{B}_{ms} \rho_{MS} v_{ms}, \quad \rho_{MS} = \frac{N_{MS}}{L_{MS} H_{MS}},$$

where  $\rho_{MS}$  denotes the active micro-shear bands density in the cluster. It is a matter of further investigations upon the evolution of clusters of micro-shear bands to confirm the usefulness of this hypothesis. If we assume that  $N_{MS}$  is of the order 100, and the width and thickness of the active zone is of about 100  $\mu\text{m}$ , then the density  $\rho_{MS}$  can be estimated for about  $10^{10}$  ( $\text{m}^{-2}$ ).

#### 4. System of active micro-shear bands as a hierarchy of discontinuity surfaces

The discussed process of shear banding can be idealized mathematically as a hierarchy of singular surfaces. The necessary mathematical formalism of the theory of propagating singular surfaces is given in [13-15]. The singular surface of order zero corresponds to the local perturbation of the microscopic displacement field produced by the passage of a single micro-shear band. The passage of large number of micro-shear bands within the active zone of the cluster smoothes out the discontinuity on the micro-level and results in the perturbation of the mesoscopic displacement field traveling with the speed, which produces a discontinuity of the velocity field in the RVE it traverses. This corresponds to the singular surface of order one, called also the surface of strong discontinuity. The discussion of physical nature of the micro-shear banding process, as well as the results of the microscopic observations *in situ*, presented in [16] support the following hypothesis:

*The passage of micro-shear bands within the active zone of the cluster results in perturbation of the mesoscopic displacement field traveling with the speed  $V_S$ ,*



which produces a discontinuity of the mesoscopic velocity field  $v_M$  in the RVE it traverses. The progression of the sequences of clusters can be idealized mathematically by means of a singular surface of order one propagating through the macro-element (RVE) of the continuum.

The theory of singular surfaces allows identifying the postulated discontinuity surface of the mesoscopic velocity field  $v_M$  in RVE as a singular surface  $\Sigma(t)$  moving in the reference configuration of the RVE and its dual counterpart  $S(t)$  moving in the spatial configuration of the RVE. There exists the jump discontinuity of derivatives of the function of motion  $\chi_M$ , i.e. of the mesoscopic velocity field  $\dot{\chi}_M$  and gradient of deformation  $\mathbf{f} = \text{Grad}\chi_M$

$$(4.1) \quad [\dot{\chi}_M] = \dot{\chi}_M^+ - \dot{\chi}_M^- \neq 0, \quad [\mathbf{f}] = \mathbf{f}^+ - \mathbf{f}^- \neq 0.$$

According to the study in [1, 4] the considered surface of strong discontinuity of mesoscopic velocity field fulfills the properties of a vortex sheet with the jump discontinuity of the first derivatives of  $\chi_M$  given in the spatial configuration by

$$(4.2) \quad [v_M] = V_S \mathbf{s}, \quad [\mathbf{f}] = -\frac{V_S}{U} \mathbf{s} \otimes \mathbf{n}\mathbf{f}, \quad \text{for } U \neq 0,$$

where  $\mathbf{s}$  and  $\mathbf{n}$  are, respectively, the unit tangent and the unit normal vectors to the discontinuity surface  $S(t)$ , while  $U$  corresponds to the local intrinsic speed of propagation of  $S(t)$ , (cf. [13], p. 508). Similarly, for the material counterpart of a singular surface, the compatibility relations take the form

$$(4.3) \quad [\dot{\chi}_M] = V_S \mathbf{s}, \quad [\mathbf{f}] = -\frac{V_S}{U_N} \mathbf{s} \otimes \mathbf{N} \quad \text{for } U_N \neq 0,$$

where  $\mathbf{N}$  is the unit normal to the discontinuity surface  $\Sigma(t)$  in the reference configuration of the body and  $U_N$  is the normal component of the surface velocity (cf. [4], Fig.2). The progression of large number of clusters of micro-shear bands extending the region of shear banding can be idealized mathematically by means of the singular surface of order two propagating through the macro-element of the continuum as an acceleration wave. The application of the theory of stationary acceleration waves opens the possibility of the analysis of plastic flow instabilities, e.g. strain localization or flutter [17], in relation with shear banding. Application of the results presented in the study [18] upon the mathematical justification of the extension of the concepts of divergence and flutter instabilities to elastic-plastic materials described by incrementally nonlinear constitutive law can appear to be helpful by that.



## 5. Macroscopic measure of the rate of deformation by micro-shear banding

According to the analysis in [1,4], application of the generalized form of Gauss' theorem for the gradient of the mesoscopic velocity field, which is sufficiently smooth in each point of RVE except the singular surface, where the discussed discontinuity jump appears, results in the following relation for the rate of deformation gradient  $\dot{\mathbf{F}}$  in the reference configuration:

$$(5.1) \quad \dot{\mathbf{F}} = \frac{1}{V_0} \int_{V_0} \text{Grad} \dot{\chi}_M dV_0 + \frac{1}{V_0} \int_{\Sigma(t)} V_S \mathbf{s} \otimes \mathbf{n} dA_0.$$

If we choose the current configuration of RVE at time  $t$  as the reference one, the rate of deformation gradient  $\dot{\mathbf{F}}$  becomes then the rate of the relative deformation gradient (cf. [19], p. 54), and the averaging formula (5.1) takes the following spatial form, [1]:

$$(5.2) \quad \mathbf{L} = \frac{1}{V} \int_V \text{grad} v_M dV + \frac{1}{V} \int_{S(t)} V_S \mathbf{s} \otimes \mathbf{n} dA,$$

where  $\mathbf{L}$  denotes the macroscopic measure of velocity gradient averaged over the macro-element  $V$  traversed by the discontinuity surface  $S(t)$ . For  $V_S = 0$  the known relation is retrieved

$$(5.3) \quad \mathbf{L} \equiv \mathbf{L} = \frac{1}{V} \int_V \text{grad} v_M dV$$

The averaging formula (5.2) enables us to account for the contribution of micro-shear banding in the macroscopic measure of velocity gradient produced at finite elastic-plastic deformations:

$$(5.4) \quad \mathbf{L} = \mathbf{L} + \mathbf{L}_{SB}, \quad \mathbf{L}_{SB} = \frac{1}{V} \int_{S(t)} V_S \mathbf{s} \otimes \mathbf{n} dA.$$

Assuming that the singular surface  $S(t)$  forms a plane traversing volume  $V$  with the unit vectors  $\mathbf{s}$  and  $\mathbf{n}$  held constant, we have  $\mathbf{L}_{SB} = \dot{\gamma}_{SB} \mathbf{s} \otimes \mathbf{n}$ , where  $\dot{\gamma}_{SB}$  is the averaged macroscopic shear strain rate produced by micro-shear bands

$$(5.5) \quad \dot{\gamma}_{SB} = \frac{1}{V} \int_{S(t)} H_{MS} \rho_{MS} \bar{B}_{ms} v_{ms} dA.$$

Considering the idealized situation, in which the total displacement produced by a single micro-shear band  $\bar{B}_{ms}$  and the speed  $v_{ms}$  of the head of a single micro-shear band are held constant, we arrive at the following Orowan-type relation for the level of macro-element (RVE) traversed by shear banding zone

$$(5.6) \quad \dot{\gamma}_{SB} = \rho_{SB} \bar{B}_{ms} v_{ms}, \quad \rho_{SB} = \frac{1}{V} \int_{S(t)} H_{MS} \rho_{MS} dA,$$

where  $\rho_{SB}$  is the macroscopic measure of the density of micro-shear bands operating in the sequences of clusters contributing to the progression of shear banding zone within the RVE. If we take into account that the density  $\rho_{MS}$  can change from cluster to cluster in the course of shear banding, then the shear strain rate can be also expressed in terms of the mean rate  $\dot{\rho}_{SB}$

$$(5.7) \quad \dot{\gamma}_{SB} = \dot{\rho}_{SB} \bar{B}_{MS} L_{MS}.$$

The derived relation (5.7) is valid for a single system of micro-shear bands. This can be generalized for the case of a double shear, where

$$(5.8) \quad \mathbf{L}_{SB} = \sum_{i=1}^2 \dot{\gamma}_{SB}^{(i)} \mathbf{s}^{(i)} \otimes \mathbf{n}^{(i)}.$$

The generalization is possible if we assume that the period of time (or the deformation path length), within which the active micro-shear bands operate in both systems, is sufficiently long. Such a period can be considered, on the other hand, as an infinitesimal increment of "time-like" variable, corresponding to the Representative Time Increment, in the macroscopic description. In such a case, the "simultaneous" operation of both systems (clusters) i.e. the double shear is considered. Otherwise, the sequence of single shear systems should be taken into considerations. The derivation of kinematical relations for the rate of deformation and material spin, composed of elastic and plastic parts and accounting for the contribution of micro-shear bands in the double shear (5.8) are given in [1, 4]. The mathematical foundations, which are necessary to describe finite plastic deformation due to the sequence of single shear systems, are given in [20, 21].

## 6. Constitutive description

According to R. HILL [22], the macroscopic constitutive equations describing elastic-plastic deformations of polycrystalline aggregates are either thoroughly or partially incrementally non linear. Depending on the contribution of the mechanisms involved in plastic flow, a region of fully active loading, called also a



fully active range, separated from the total unloading (elastic) range by a truly nonlinear zone corresponding to the partially active range, may exist. The connection of the fully active range and partially active range with the geometric pattern of micro-shear bands is necessary to specify the relation for the rate of plastic deformations for different loading paths. Because multiple sources of plasticity are dealt with, the theory of multimechanisms with multiple plastic potentials can be considered. The concept of multiple potential surfaces forming a vertex on the smooth limit surface was studied earlier by Z. MRÓZ [23] within the framework of non-associated flow laws. In our case, the existence of the following plastic potentials related to the mechanisms responsible for plastic flow can be postulated [2, 4].

- The plastic potential  $g_0$  that reproduces at the macroscopic level the crystallographic multiple slips and is associated with the limit surface approximated by means of the Huber-Mises locus  $F = g_0$ .
- The non-associated plastic potentials  $g_1$  and  $g_2$  that approximate at the macroscopic level the multiplicity of plastic potential functions related with the clusters of active micro-shear bands.

The plastic potential functions  $g_1$  and  $g_2$  display the geometry of the micro-shear bands systems considered and result in two separate planes that form in the space of principal stresses  $\tau_k$  ( $k = 1, 2, 3$ ) a vertex at the loading point on the smooth Huber-Mises cylinder  $F$ . The planes are defined by normals  $\mathbf{N}_i$ , which can be expressed in terms of the unit vectors  $\mathbf{s}^{(i)}$ ,  $\mathbf{n}^{(i)}$  ( $i = 1, 2$ ) defining the “ $i$ ” th system (cluster) of micro-shear bands

$$(6.1) \quad \mathbf{N}_i = \frac{\sqrt{2}}{2} \left( \mathbf{s}^{(i)} \otimes \mathbf{n}^{(i)} + \mathbf{n}^{(i)} \otimes \mathbf{s}^{(i)} \right).$$

The normals  $\mathbf{N}_i$  can be expressed in terms of the unit normal  $\boldsymbol{\mu}_F$  to the Huber-Mises yield surface  $\frac{1}{2}\boldsymbol{\tau}' : \boldsymbol{\tau}' = k^2$ , expressed by deviators of the Kirchhoff stress tensor  $\boldsymbol{\tau}'$ , and the unit tangent  $\mathbf{T}$  to the limit surface at the loading point

$$(6.2) \quad \mathbf{N}_1 = \cos 2\beta \boldsymbol{\mu}_F + \sin 2\beta \mathbf{T}, \quad \mathbf{N}_2 = \cos 2\beta \boldsymbol{\mu}_F - \sin 2\beta \mathbf{T}, \quad \boldsymbol{\mu}_F = \frac{1}{\sqrt{2}k} \boldsymbol{\tau}'.$$

The tensor  $\mathbf{T}$  is coaxial with the tangent to the Huber-Mises locus in the deviatoric plane at the loading point

$$(6.3) \quad \mathbf{T} = T \left( \frac{\overset{\circ}{\boldsymbol{\tau}'}}{\bar{\tau}} \right) = T \left[ \overset{\circ}{\boldsymbol{\tau}'} - \left( \overset{\circ}{\boldsymbol{\tau}'} : \boldsymbol{\mu}_F \right) \boldsymbol{\mu}_F \right], \quad \bar{\tau} = \sqrt{\frac{1}{2} \boldsymbol{\tau}' : \boldsymbol{\tau}'}$$

For the pressure-insensitive Huber-Mises yield locus  $\overset{\circ}{\boldsymbol{\tau}}' : \boldsymbol{\mu}_F = \overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F$  holds, whereas  $T$  is a normalization factor

$$(6.4) \quad T = \left\| \overset{\circ}{\boldsymbol{\tau}}' - \left( \overset{\circ}{\boldsymbol{\tau}}' : \boldsymbol{\mu}_F \right) \boldsymbol{\mu}_F \right\|^{-1},$$

which due to

$$(6.5) \quad \left\| \overset{\circ}{\boldsymbol{\tau}}' - \left( \overset{\circ}{\boldsymbol{\tau}}' : \boldsymbol{\mu}_F \right) \boldsymbol{\mu}_F \right\|^2 = \overset{\circ}{\boldsymbol{\tau}}' : \overset{\circ}{\boldsymbol{\tau}}' - \left( \overset{\circ}{\boldsymbol{\tau}}' : \boldsymbol{\mu}_F \right)^2,$$

$$\overset{\circ}{\boldsymbol{\tau}}' : \boldsymbol{\mu}_F = \left\| \overset{\circ}{\boldsymbol{\tau}}' \right\| \cos \delta' = \left\| \overset{\circ}{\boldsymbol{\tau}} \right\| \cos \delta$$

is given by  $T = \left( \left\| \overset{\circ}{\boldsymbol{\tau}}' \right\| \sin \delta' \right)^{-1}$ . The symbol  $\overset{\circ}{\boldsymbol{\tau}}$  denotes the objective rate of stress, which reads

$$(6.6) \quad \overset{\circ}{\boldsymbol{\tau}} = \dot{\boldsymbol{\tau}} - \mathbf{W}^e \boldsymbol{\tau} + \boldsymbol{\tau} \mathbf{W}^e, \quad \mathbf{W}^e = \mathbf{W} - \mathbf{W}^p,$$

where  $\mathbf{W}$  is the material spin and  $\mathbf{W}^p$  is the plastic spin, which was derived in the following form [2]:

$$(6.7) \quad \mathbf{W}^p = \frac{1}{2\bar{\tau} \sin 2\beta} (\mathbf{D}^p \boldsymbol{\tau} - \boldsymbol{\tau} \mathbf{D}^p).$$

Due to the above derivations, the relation for the rate of plastic deformation takes the form

$$(6.8) \quad \mathbf{D}^p = \mathbf{D}^p + \frac{\sqrt{2}}{2} \sum_{i=1}^2 \dot{\gamma}_{SB}^{(i)} \mathbf{N}_i.$$

Assuming that  $\mathbf{D}^p$ , representing the rate of plastic deformation produced by the crystallographic multiple slips, can be expressed in the simplest case by means of the classical  $J_2$  plasticity theory, we have

$$(6.9) \quad \mathbf{D}^p = \frac{\sqrt{2}}{2} \dot{\gamma}_s \boldsymbol{\mu}_F, \quad \dot{\gamma}_s = \frac{\sqrt{2}}{2} \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{h}.$$

Due to equations (6.1), (6.2) and (6.8), (6.9), the relation for the rate of plastic deformation takes the form

$$(6.10) \quad \mathbf{D}^p = \frac{\sqrt{2}}{2} \dot{\gamma}^* \boldsymbol{\mu}_F + \frac{\sqrt{2}}{2} \dot{\epsilon}_{SB} \mathbf{T},$$



where

$$\begin{aligned}
 \dot{\gamma}^* &= \dot{\gamma}_s + \dot{\gamma}_{SB}, \\
 \dot{\gamma}_{SB} &= \cos 2\beta \left( \dot{\gamma}_{SB}^{(1)} + \dot{\gamma}_{SB}^{(2)} \right), \\
 \dot{\epsilon}_{SB} &= \sin 2\beta \left( \dot{\gamma}_{SB}^{(1)} - \dot{\gamma}_{SB}^{(2)} \right).
 \end{aligned}
 \tag{6.11}$$

The scalar functions:  $f_{SB}^{(1)}$ ,  $f_{SB}^{(2)}$ , representing the contributions of the shear banding system (1) and (2), respectively, in the total plastic shear strain rate  $\dot{\gamma}^*$ , are introduced

$$\dot{\gamma}_{SB}^{(1)} \cos 2\beta = f_{SB}^{(1)} \dot{\gamma}^*, \quad \dot{\gamma}_{SB}^{(2)} \cos 2\beta = f_{SB}^{(2)} \dot{\gamma}^*,
 \tag{6.12}$$

which are subjected to the following constraints:

$$\frac{\dot{\gamma}_s}{\dot{\gamma}^*} + f_{SB}^{(1)} + f_{SB}^{(2)} = 1, \quad \dot{\gamma}^* \neq 0, \quad f_{SB}^{(1)} + f_{SB}^{(2)} \in [0, 1), \quad f_{SB}^{(1)}, f_{SB}^{(2)} \in [0, 1].
 \tag{6.13}$$

Let us note, that this is the formal statement of the experimental observation that shear banding never appears without even slight contribution of crystallographic slips [2]. Basing also on the observation that micro-shear bands can be active only in the case of continued plastic flow, i.e. when the loading condition is fulfilled, it is assumed that for  $\dot{\gamma}^* = 0$ ,  $f_{SB}^{(1)} = f_{SB}^{(2)} = 0$ .

According to the foregoing discussion and due to (6.11) and (6.12), equation (6.10) is specified for two situations:

- For the case in which the loading direction described by the objective rate of stress  $\overset{\circ}{\boldsymbol{\tau}}$  is pointing at partially active range, i.e. for  $\delta \in \left( \delta_c, \frac{\pi}{2} \right]$

$$\begin{aligned}
 (6.14) \quad D^p &= \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{2h(1 - f_{SB})} \boldsymbol{\mu}_F \\
 &+ \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{2h(1 - f_{SB})} \frac{\Delta f_{SB} \tan 2\beta}{\left\| \overset{\circ}{\boldsymbol{\tau}} \right\| \sin \delta'} \left[ \overset{\circ}{\boldsymbol{\tau}}' - \left( \overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F \right) \boldsymbol{\mu}_F \right].
 \end{aligned}$$

- For the case in which the objective rate of stress  $\overset{\circ}{\boldsymbol{\tau}}$  is pointing at fully active range, i.e. for  $\delta \in [0, \delta_c]$

$$\begin{aligned}
 (6.15) \quad D^p &= \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{2h(1 - f_{SB})} \boldsymbol{\mu}_F \\
 &+ \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{2h(1 - f_{SB})} \frac{\Delta f_{SB} \tan 2\beta}{\tan \delta_c} \left[ \overset{\circ}{\boldsymbol{\tau}}' - \left( \overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F \right) \boldsymbol{\mu}_F \right],
 \end{aligned}$$

where  $f_{SB} = f_{SB}^{(1)} + f_{SB}^{(2)}$ ,  $\Delta f_{SB} = f_{SB}^{(1)} - f_{SB}^{(2)}$ ,  $\Delta f_{SB} \in [-1, 1]$  and  $h$  is the plastic hardening modulus obtained from the simple tensile or compression test. It is confirmed experimentally that in such tests the contribution of shear banding is negligible, [24].

## 7. Possible simplifications of constitutive description for numerical simulations of metal shaping operations

Among many possible realizations of shear banding processes, which are described by (6.14) and (6.15), one can single out the group of processes characterizing, at least approximately or for sufficiently long deformation paths, with the same contribution of both systems  $f_{SB}^{(1)} = f_{SB}^{(2)}$ . Then we have  $\Delta f_{SB} = 0$  and (6.14) simplifies. On the other hand, the non-symmetric activation of shear banding can be induced by the sufficiently large change of the loading direction or rotation of the principal axes of stress tensor in the inhomogeneous deformation process caused by boundary conditions. Therefore, we postulate that:

$$(7.1) \quad \Delta f_{SB} = 0 \text{ for } \delta \in [0, \delta_c] \text{ and } \Delta f_{SB} = A(\delta) \text{ for } \delta \in \left(\delta_c, \frac{\pi}{2}\right].$$

The function  $A(\delta)$  being a measure of the mentioned asymmetry of shear banding should be identified from numerical simulations of the experiment accounting for the change of loading direction.

As a result of the above assumptions, the flow laws (6.14) and (6.15) read respectively:

- For the case in which the loading direction described by the objective rate of stress  $\overset{\circ}{\boldsymbol{\tau}}$  is pointing at partially active range, i.e. for  $\delta \in \left(\delta_c, \frac{\pi}{2}\right]$

$$(7.2) \quad \mathbf{D}^p = \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{2h(1 - f_{SB})} \boldsymbol{\mu}_F + \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{2h(1 - f_{SB})} \frac{A(\delta) \tan 2\beta}{\left\| \overset{\circ}{\boldsymbol{\tau}}' \right\| \sin \delta'} \left[ \overset{\circ}{\boldsymbol{\tau}} - \left( \overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F \right) \boldsymbol{\mu}_F \right],$$

- For the case in which the objective rate of stress  $\overset{\circ}{\boldsymbol{\tau}}$  is pointing at fully active range, i.e. for  $\delta \in [0, \delta_c]$ :

$$(7.3) \quad \mathbf{D}^p = \frac{\overset{\circ}{\boldsymbol{\tau}} : \boldsymbol{\mu}_F}{2h(1 - f_{SB})} \boldsymbol{\mu}_F.$$



The above-mentioned assumptions leading to simplified formula (7.3) finds confirmation in experiment, for the experimental observations reveal that the spatial pattern of micro-shear bands does not change for loading conditions that deviate within limits from the proportional loading path, i.e. the load increments are confined to a certain cone (fully active range) the angle of which can be determined experimentally. For instance, according to [25] in polycrystalline Cu the critical angle  $\delta_c$  of this cone is of the order of  $22^\circ$ . A more drastic change of the loading scheme produces, however, the change of the spatial orientation of micro-shear bands. This is supported by the results presented in [26], where after cross-rolling two families of micro-shear bands inclined by about  $\pm 35^\circ$  to the most recent rolling direction were observed. The possible applications of the plastic flow law (7.3) for numerical simulation of metal shaping operations are discussed in the companion paper [6].

## 8. Conclusions

It is worth mentioning that the existence of the deviation angle  $\beta$ , which plays an essential rôle in the non-linear flow law (7.2), is typical for the micro-shear bands produced in the deformation processes carried out under nearly isothermal conditions. Thermal shear bands, i.e. the mode of plastic localization governed by a coupled thermoplastic mechanism, have also been studied by many authors (cf. e.g. [27–29]). In particular, the so-called “adiabatic shear bands” are often reported to coincide with the trajectories of maximum shear stress, which result in  $\beta=0$ . In our view, such a qualitative difference can be attributed to the influence of internal micro-stresses, which control the formation of micro-shear bands. The micro-stresses perturb locally the applied macroscopic state of stress deviating the principal axes of stress tensor. According to the hypothesis on a micro-shear band formation presented in [8], within a suitably oriented grain the critical coarse slip band is activated, which can further transform, under appropriate dynamical conditions, in a transgranular “non-crystallographic” micro-shear band propagating in the planes that are usually deviated from the planes of applied maximum shear stress. On the other hand, the effect of micro-stresses decreases while thermoplastic coupling becomes operative and “adiabatic shear bands” develop. The experimental investigations of the thermomechanical coupling during a simple shear test with use of the thermovision system show the temperature distribution along the shearing paths and reveal a misorientation of the shear-banding zone with respect to the plane of maximum shear stress for non-adiabatic conditions [30]. This confirms, at least qualitatively, the aforementioned interpretation of the deviation angle  $\beta$ . The constitutive description of plastic strain for the angle  $\beta=0$  reduces to the simple form of plastic flow law (7.3), which can be applied for the numerical simulations of cold as well as hot



metal forming operations. Some results of the relevant applications are reported in [31]. The extension of the proposed description of plastic strain in metals with an account of shear banding under dynamic loading conditions was recently proposed in [32]. The main idea was the modification of the viscoplasticity equation in the form proposed originally by P. PERZYNA [33] in such a way that the viscosity parameter depends on the contribution of shear banding  $f_{SB}$ .

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