

On importance of imperfections in plastic strain localization problems in materials under impact loading

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*Dedicated to Professor Piotr Perzyna
on the occasion of his 70th birthday*

THE WORK is the continuation of Professor Piotr Perzyna achievements in the description and analysis of the phenomenon of plastic strain localization. The ductile materials under impact loading are in focus of interest. In particular, the influence of initial imperfections on the final pattern of localization is elaborated. The computer simulations were performed in the environment of ABAQUS program.

1. Introduction

THE DESCRIPTION of plastic strain localization phenomena has been focus of research for at least the last two decades. The phenomenon is clearly observed in ductile and brittle materials as well as in soils. Localization as a precursor of failure is usually accompanied by other phenomena such like, for example, heat generation and transfer (for ductile materials loaded by impact) or fluid flow in zones of localized deformations (for soils). When trying to propose the adequate description of the phenomena, the crucial point is to choose the constitutive structure which would be the closest to the observed properties but still formulated in the frame of continuum mechanics. The careful experimental observations prove that the plastic strain localization observed on the level of continuum is a very complex phenomenon which in fact, is a kind of homogenization of changes that are observed on other scales (mezo-, macro- or nano). In many cases under consideration, the localization of plastic strains is strongly connected with softening which could be the next source of difficulties that arise in the process of solution. The crucial question that has to be answered is the well-posedness of the system of governing equations; for discussions see [7, 20, 27]. There are different approaches to the solution of the problem in the frame of plasticity and continuum formulation. All of them introduce, implicitly or explicitly, internal length scale and are viewed as regularization methods. Depending on

the process (static or dynamic), type of localization (necking or shear bands) and the main properties of materials (ductile or brittle), the different methods can successfully describe the phenomena. In computations the attention is focused on assigning the place, time and the width of localization zones. Of course, all of them strongly depend on geometry of the specimens, boundary and initial conditions including the characteristic of loading. The important feature which differs the treatment, and which follows the computations, the static cases versus dynamic, is the necessity of introducing any imperfection (geometric or constitutive) which is the source of appearing of the first plastic strain localization. For dynamic cases (impact loading) in computations, one can avoid these imperfections and the choice of the localization form depends then on waves interaction which in a natural way introduce the heterogeneity, and properly describes the merit of the phenomenon. In the work, the problem of plastic strain localization in materials under impact loading is studied. Particularly, the influence of introducing the imperfections, both geometrical in the form of additional internal boundaries and constitutive in the form of inclusions, on the pattern of localized plastic strains is elaborated and documented in the numerical examples.

2. Numerical treatment of localization phenomena

The review of the papers that approaches the description of plastic zones was done in many works viewing the problem from different standpoints; see e.g. [19]. Some classical formulations, e. g. [3] or [12] defined the fundamental criteria for creation the pattern of localized deformations (shear bands). The main point was to recognize the change of the type of governing equations, for statics, from elliptic into parabolic. In the described process this change was identified with the final state of the specimen. The analysis could not be continued. When introducing the softening behavior of the material in computations, there appeared numerical problems which were recognized as mesh dependence; see [5]. To avoid this pathological form the authors started to introduce different forms of regularization, first on the level of finite element formulation. The so-called embedded elements [5, 9, 11], which usually explicitly declared the width of localization partially allowed to avoid this loss of stability in the FE solutions. The drawback of the proposition was the necessary explicit knowledge on the width of localization which obviously is not constant for the material and depends on the boundary conditions and loadings. The other works [22] stressed the mathematical side of constitutive form. No matter if the authors accepted non-local constitutive description [6], gradient-type theories [15], plastic-damage [36], rate-dependence [7, 24, 27] or coupled fields [21, 23] they always introduced the form of regularization. After this enrichment the system of equations becomes well-posed, it means the solution is unique and stable, and the type of the system of

incremental equations that describe the process remains unchanged in the whole range of interests. It could be also proved that all the constitutive formulations introduce the dispersive character of the media [13, 27]. For metals under very fast loading (impact), it is reasonable to accept the viscous properties of the material. Rate-dependence introduces the regularization and is physically well documented in dozens of experiments; see e. g. [16] The simple constitutive form useful for practical applications was originally proposed by [4]. The modification of this form is widely used [24, 25] in industrial applications. The mathematically consistent form of constitutive visco-plastic behavior was originally proposed by PERZYNA [1, 2, 7]. This formulation which is also used in this work introduces the relaxation time of mechanical disturbances T_m which is simultaneously the constitutive parameter and a mathematical regularization parameter. In this formulation the internal scale parameter is introduced implicitly. The profits that arise from using the proposed constitutive structure were discussed by the authors in [27, 32, 35]. The important fact is that it is not necessary to use any type of imperfection to reach localization in a specimen which is, in a natural way, the result of constitutive properties and the boundary value problem characteristic. In the work the attention is focused on showing how strong the influence of imperfections can be. How the imperfection attracts the localization zones and eventually, how it could be controlled to achieve the expected behavior of the specimen.

3. Elasto-thermo-plastic formulation

The material under consideration exhibits strain softening as a result of temperature rise or/and evolution of porosity. Both of these effects for classical rate-independent plastic strain formulation with negative stress-strain constitutive relation lead to ill-posed problems and in consequence, to non-unique results in numerical applications [14].

An adiabatic flow process written in the evolution form can be presented as follows:

$$\begin{aligned}
 \dot{\phi} &= \mathbf{v}, \\
 \dot{\mathbf{v}} &= \frac{1}{\rho^0(1-\xi_0)} \left(\frac{\boldsymbol{\tau}}{\rho} \text{grad} \rho + \text{div} \boldsymbol{\tau} - \frac{\boldsymbol{\tau}}{1-\xi} \text{grad} \xi \right), \\
 \dot{\rho} &= \frac{\rho}{1-\xi} \Xi - \rho \text{div} \mathbf{v}, \\
 \dot{\boldsymbol{\tau}} &= \left[\mathcal{L}^e - \frac{1}{c_p \rho_{Ref}} \vartheta \mathcal{L}^{th} \frac{\partial \boldsymbol{\tau}}{\partial \vartheta} \right] : \text{sym} D \mathbf{v} + 2 \text{sym} \left(\boldsymbol{\tau} : \frac{\partial \mathbf{v}}{\partial \mathbf{x}} \right) \\
 &\quad - \left[\left(\frac{\chi^*}{\rho(1-\xi)c_p} \mathcal{L}^{th} \boldsymbol{\tau} + \mathcal{L}^e + \mathbf{g} \boldsymbol{\tau} + \boldsymbol{\tau} \mathbf{g} \right) : \mathbf{P} \right] \frac{1}{T_m} \left\langle \left(\frac{f}{\kappa} - 1 \right)^m \right\rangle
 \end{aligned}
 \tag{3.1}$$

$$\begin{aligned}
 & - \frac{\chi^{**} \mathcal{L}^{th}}{\rho(1-\xi)c_p} \Xi, \\
 (3.1) \quad & \dot{\xi} = \Xi, \\
 \text{[cont.]} \quad & \dot{\vartheta} = \frac{\vartheta}{c_p \rho_{Ref}} \frac{\partial \tau}{\partial \vartheta} : \text{sym} D \mathbf{v} + \frac{\chi^*}{\rho(1-\xi)c_p} \tau : \mathbf{P} \frac{1}{T_m} \left\langle \left(\frac{f}{\kappa} - 1 \right)^m \right\rangle \\
 & + \frac{\chi^{**}}{\rho(1-\xi)c_p} \Xi;
 \end{aligned}$$

where at the left-hand side consequently appear the rates of scalar values (mass density ρ , porosity ξ and temperature ϑ), vectorial (displacement ϕ and velocity \mathbf{v}) and tensorial quantities (Kirchhoff stress τ). For detailed discussion see also [27].

Let us restrict our consideration to elastic-viscoplastic associative model allowing for finite deformations. For the plastic part of the total deformation rate tensor $\boldsymbol{\varepsilon}^p$ ($\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p$) we postulate the evolution equation with the elastic-viscoplastic model of the material [20], and the tensor function \mathbf{P} is defined for associative plasticity

$$(3.2) \quad \boldsymbol{\varepsilon}^p = \lambda \cdot \mathbf{P}, \quad \lambda = \frac{1}{T_m} \langle \Phi(f - \kappa) \rangle, \quad \mathbf{P} = \frac{\partial f}{\partial \tau}.$$

In the above, T_m denotes the relaxation time for mechanical disturbances, κ is the isotropic work hardening/softening parameter, Φ denote the empirical overstress functions, $\langle \cdot \rangle$ denotes the so-called McCauly bracket and τ is the Kirchhoff stress tensor and f represents the plastic yield function [27, 35].

The different alternative hardening/softening forms of material function κ can be assumed including the effects of porosity nucleation and growth [20, 27].

Following Perzyna's achievements [1, 2, 8] we postulate the overstress viscoplastic function Φ in the form

$$(3.3) \quad \Phi(f - \kappa) = (f - \kappa)^m \quad \text{where } m = 1, 3, 5, \dots$$

We restrict our numerical tests to the initial boundary value problems for which the time scale covers only a fraction of a second and for this reason, adiabatic inelastic flow process describes sufficiently close the physical phenomena which appear.

The boundary conditions for surface traction and displacement are defined on the separate boundaries, assuming that the heat flux prescribed on the whole boundary is equal to zero and the initial conditions for all variables are given at time $t = 0$.

The whole physical and mathematical structure of the analytical and numerical model has the wave nature [13, 17].

4. Numerical Examples - Impact tension of rectangular metal specimen

As a reference (perfect) specimen, thin plate loaded by dynamic impulse is taken into computations. The constitutive relation incorporates thermal softening of the yield stress or the evolution of porosity. Twodimensional shell model is applied. The dimensions of the plate are as follows: length 25.4 mm and width 12.7 mm. The thickness of the specimen is 0.33 mm. The impact loading is defined by kinematic conditions. The bottom side of the specimen is fixed (all possible displacements) and the longitudinal velocity of 20 m/s is applied at all nodes of the top side (see Fig. 1a). Both vertical boundaries are free, without any constraints. The space discretization shown in Fig. 1b consists of finite element mesh of square elements: 80 along the length and 40 along the width. The constitutive parameters used in computations are: Young's modulus 200,000 MPa, Poisson ratio $\nu = 0.3$, strength stress 1634 MPa, initial mass density $\rho_0 = 7850 \text{ kg/m}^3$. The inelastic heat fraction 0.9, specific heat 460 J/kg $^{\circ}\text{C}$ defines the part of



FIG. 1. Mesh, plastic equivalent strain distribution and vector plot of velocity for 2D model.

thermally dissipated energy. The strength stress decreases nonlinearly to the value of 1016 MPa when temperature grows to 610°C. Density evolution describes the softening character of the material in range of void volume ratio $0.004 \div 0.3$. The fundamental relaxation time is $T_m = 2.5 \mu\text{s}$. The whole process time is equal to $t = 50 \mu\text{s}$. Process time is discretized for increments of the order $0.01 \mu\text{s}$. The stability criteria for explicit procedure and physical requirements for wave propagation are satisfied.

There are two different groups of models under the investigation. The first one contains the computations in case of no imperfections: material, geometrical nor numerical. The next one contains the analyses with considered material inclusions.

4.1. Perfect specimen

The final distribution of plastic equivalent strains and vector plots of velocities at the end of the process are shown in Fig. 1c, 5a. The achieved zones of localization are insensitive to the finite element mesh accepted for computations.

The place of localization is well recognized when looking at the plot of material point velocity.

The next two figures, Figs. 2–3, report the studies of the sensitivity and qualitative changes in the behavior of a specimen for different relaxation times T_m . If the relaxation time T_m tends to 0, the material becomes not dispersive and the energy is dissipated only due to plasticity. The plots in Fig. 2 show the differen-

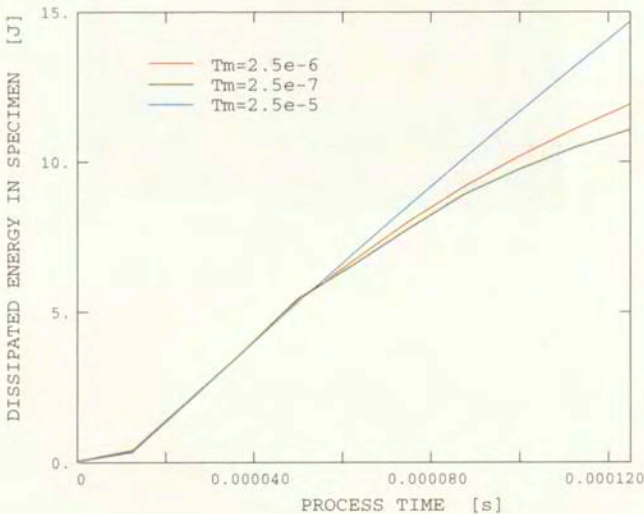


FIG. 2. History of the total structure dissipated energy for 2D model for different viscosity.

ces in the amount of energy which is related to the dispersive character of the viscoplastic medium [28,30]. Figure 3 presents the distribution of plastic strains along the axis of symmetry of the specimen for different relaxation times. If the time is relatively long, the localization remains very much diffused ($T_m = 2.5 \cdot 10^{-5}$ s). The gradient of velocity [35] splits the specimen into two parts fixing the place of localization. When using longer relaxation time, the gradient is not so sharp. The transition zone is more diffused. At the final state of deformation we can observe the clear division of the specimen into two zones where the velocities of material points vanish (left-hand side), while only the rest of the specimen moves.

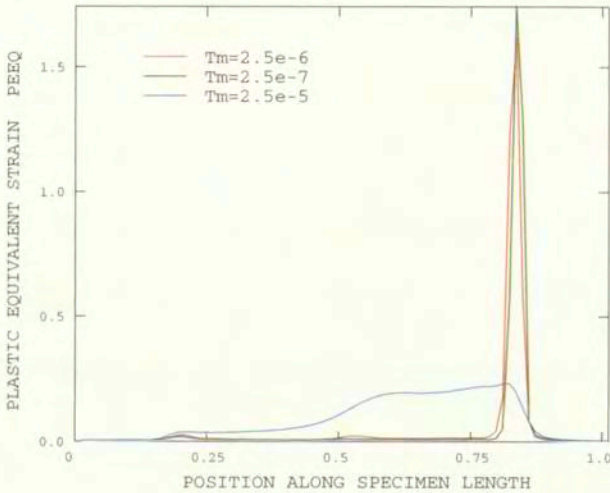


FIG. 3. Plastic strain localization for 2D model for different viscosity.

4.2. Specimen with material imperfection

In the following analyses we consider the models that contain the imperfection. The results concerning the cases with inclusions in the center of the specimen were presented in [33]. The single, material or geometrical, inclusion influenced the solution, due to the reflection and interaction of waves. The pattern and placement of localization became different than for computations with no imperfections.

We are studying three models with material inclusions. The inclusion is defined by smaller Young's modulus associated with six finite elements. All other parameters remain the same. For all cases under consideration the imperfection breaks the symmetry of the IBV problem. The assumed places of imperfections are presented in Fig. 5. For the case 1, Fig. 5b, the inclusion is located in the

middle of specimen on its left edge. For case 2, the imperfection is displaced toward the center of the specimen. In case 3, the imperfection stays at the left edge, but is moved toward the bottom constrained side.

The results, as before, are represented by plastic equivalent strain and plots of velocity vectors. Three time instants represent the history of deformations and

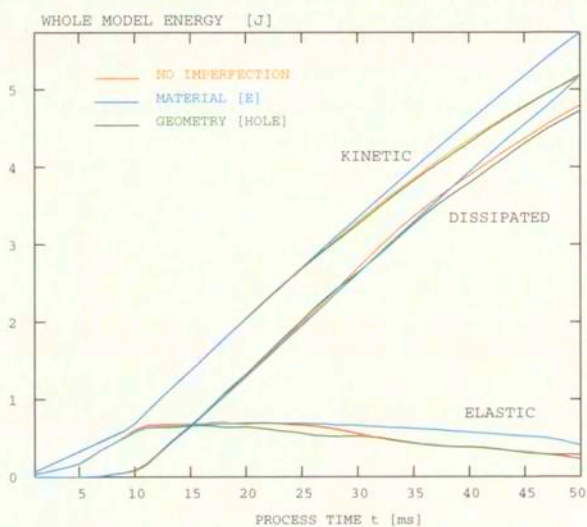


FIG. 4. Plot of energy history – division of kinetic energy into dissipated and recoverable in the whole process for three models: without imperfection, with material inclusion and with central hole [32].

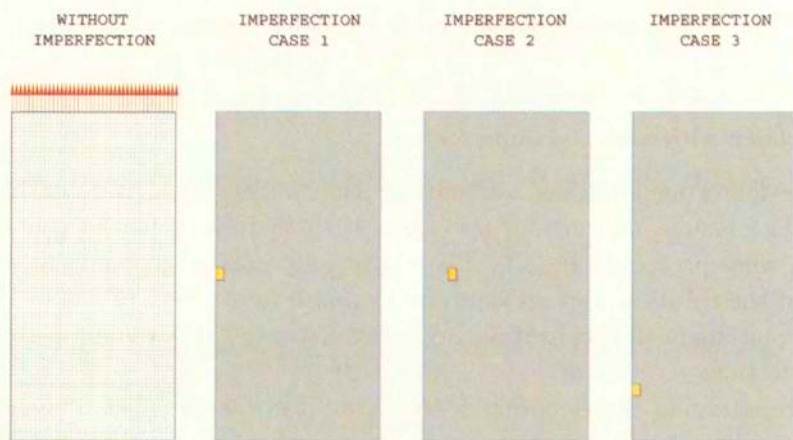
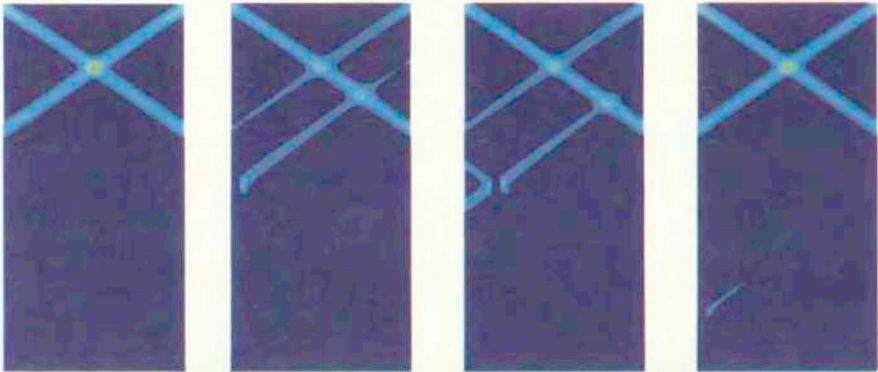


FIG. 5. Schema of initial-boundary conditions for 2D plate under tension (a), cases of material inclusions (b), (c), (d).

are taken to display the specific points of the process evolution. Figure 6 shows the distribution of equivalent plastic deformations and velocity field for the process time equal to $t = 20 \mu\text{s}$. The next Fig. 7 shows the same quantities for the process time equal to $t = 30 \mu\text{s}$. The final state for time equal to $t = 50 \mu\text{s}$ is presented in Fig. 8. For each case of imperfection, comparing with the perfect specimen, it is clearly seen that the imperfection influences both the velocity history and plastic deformation and finally the localization pattern.

PLASTIC EQUIVALENT STRAIN



t=0.000020 [s]



VELOCITY

FIG. 5. Equivalent plastic strain distribution for process time $t = 20 \mu\text{s}$ in the model without imperfection (a), and models with material inclusions (b), (c) (d).

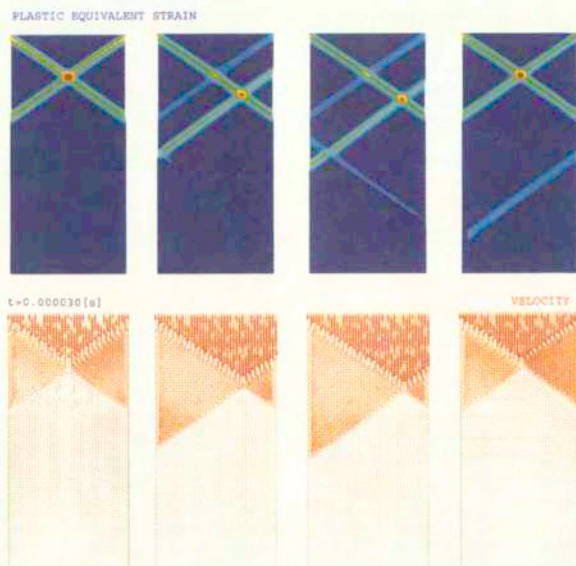


FIG. 7. Equivalent plastic strain distribution for process time $t = 20\mu\text{s}$ in the model without imperfection (a), and models with material inclusions (b), (c) (d).

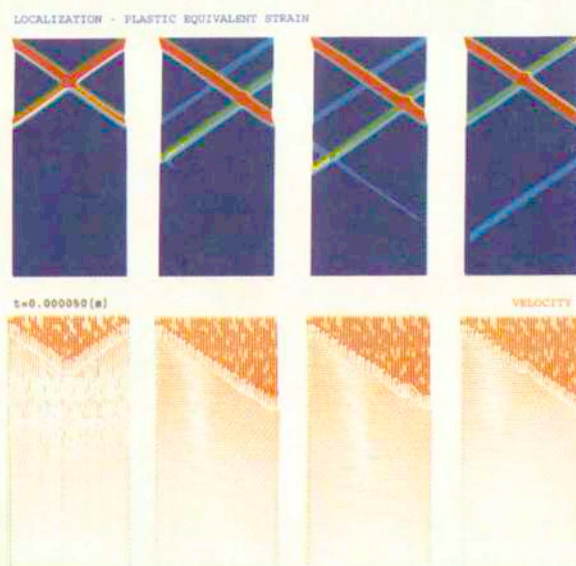


FIG. 8. Equivalent plastic strain distribution for process time $t = 20\mu\text{s}$ in the model without imperfection (a), and models with material inclusions (b), (c) (d).

5. Conclusions

The usual practice in computations, for rate-independent formulations, is to assume the initial imperfections which will guarantee the initiation of the strain localization. It is not possible to continue the computations without this introductory imperfections. The result of this assumption is always connected with the loss of symmetry which is enforced and its form strongly depends on the imperfection. One can expect a different form of localization for every single imperfection.

For dynamic processes when using rate-dependent constitutive form, even for symmetric initial-boundary-value problems the localization pattern is finally achieved as a result of waves interaction. In these cases the process of localized deformations will appear and will be growing without any artificial accelerators. In the cases studied in the paper the numerical tests were performed on perfect structure (2D plate) and we have obtained the symmetry in the behavior of the specimen and also we have compared the results with those where the inclusions were consciously introduced. The changes in the localization forms and the amount of energy that is dissipated during the whole process of deformations were in the focus of authors' interest.

It is clearly seen that the next step, in the near future, should concentrate on designing the patterns of inclusions which would generate for example the maximum amount of energy dissipated during the deformation process. The solution of this optimization problem is of crucial practical value.

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