Influence and Green's functions for orthotropic micropolar continua

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THE ARTICLE REPORTS on a methodology to synthesize the response of orthotropic micropolar half-space subjected to concentrated and distributed loads. The disturbance due to normal and tangential loads are investigated by employing the eigenvalue approach. The integral transforms have been inverted by using a numerical technique to obtain the normal displacement, normal force stress and tangential couple stress in the physical domain. The results concerning these quantities are given and illustrated graphically.

Key words: Micropolar, orthotropic, eigenvalue, Fourier and Laplace transforms.

1. Introduction

IN MANY ENGINEERING problems, including the response of soils, geological materials and composites, the assumptions of an isotropic behaviour may not reflect some significant features of the continuum response. The formulation and solution of anisotropic problems is much more difficult and cumbersome than its isotropic counterpart. In the last years the elastodynamic response of anisotropic continuum has received the attention of several reseachers. In particular, transversely isotropic and orthotropic materials, which may not be distinguished from each other in plane strain and plane stress cases, have been more regularly studied.

The theory of micropolar elasticity introduced and developed by ERINGEN [1] rised much interest because of its possible utility in investigating the deformation properties of solids for which the classical theory is inadequate. The micropolar theory is belived to be particularly useful in investigating materials consisting of bar-like molecules which exhibit microrotation effects and which can transmit body and surface couples. Recently, CHENG and HE [11,12], ERBAY [14], KUMAR and DESWAL [15,16] have studied different problems in micropolar isotropic medium.

A review of literature on micropolar orthotropic continua shows that IESAN [3,4,5] analyzed the static problems of plane micropolar strain of a homogeneous and orthotropic elastic solid, torsion problem of homogeneous and orthotropic

cylinders in the linear theory of micropolar elasticity and bending of orthotropic micropolar elastic beams by terminal couples. NAKAMURA *et. al.* [8] derived the finite element method for orthotropic micropolar elasticity.

Most of the problems studied so far, in micropolar elasticity involve the use of potential functions. However, the use of the eigenvalue approach has the advantage of finding the solutions of equations in the coupled form directly, in the matrix notations, whereas the potential function approach requires decoupling of equations. Yet, the eigenvalue approach has not been applied in micropolar orthotropic medium. MAHALANABIS and MANNA [10,13] applied the eigenvalue approach to linear micropolar elasticity by arranging basic equations of linear micropolar elasticity in the form of matrix differential equation. Recently, KUMAR *et. al.* [17] applied the eigenvalue approach to micropolar elastic medium due to impulsive force at the origin.

2. Problem formulation

Let us consider a homogeneous and orthotropic micropolar half-space. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the plane $y = 0$, with y axis directed vertically into the medium is introduced. A normal or tangential source is assumed to be acting at the origin of the rectangular Cartesian co-ordinates.

If we restrict our analysis parallel to the xy-plane with displacement vector $\mathbf{u} = (u_1, u_2, 0)$ and microrotation vector $\boldsymbol{\phi} = (0, 0, \phi_3)$, the basic equations in the dynamic theory of the plane strain of homogeneous and orthotropic micropolar solids, in absence of body forces and body couples given by ERINGEN [2], can be recalled as:

$$(2.1) \qquad\qquad t_{ji,j} = \rho \frac{\partial^2 u_i}{\partial t^2},$$

$$(2.2) \qquad m_{i3,i} + \epsilon_{ij3} t_{ij} = \rho j \frac{\partial^2 \phi_3}{\partial t^2}.$$

The constitutive relations, given by IESAN [3], can be written as:

$$(2.3) \quad \begin{aligned} t_{11} &= A_{11}\epsilon_{11} + A_{12}\epsilon_{22}, & t_{12} &= A_{77}\epsilon_{12} + A_{78}\epsilon_{21}, \\ t_{21} &= A_{78}\epsilon_{12} + A_{88}\epsilon_{21}, & t_{22} &= A_{12}\epsilon_{11} + A_{22}\epsilon_{22}, \\ m_{13} &= B_{66}\phi_{3,1}, & m_{23} &= B_{44}\phi_{3,2}, \end{aligned}$$

where

$$(2.4) \qquad\qquad \epsilon_{ij} = u_{j,i} + \epsilon_{ji3}\phi_3.$$

In these relations, we have used the following notations: t_{ij} — components of the force stress tensor, m_{ij} — component of the couple stress tensor, ϵ_{ij} — component of the micropolar strain tensor, u_i — components of displacement vector, ϕ_3 — component of microrotation vector, ϵ_{ijk} — permutation symbol, $A_{11}, A_{12}, A_{22}, A_{77}, A_{78}, A_{88}, B_{44}, B_{66}$ — characteristic constants of the material, ρ — the density and j — the microinertia.

We introduce the dimensionless quantities

$$
(2.5) \quad
\begin{aligned}
&x^* = \frac{\omega}{c_1}x, &&y^* = \frac{\omega}{c_1}y, &&u_i^* = \frac{\omega}{c_1}u_i, &&\phi_3^* = \frac{A_{11}}{K_1}\phi_3, \\
&t_{ij}^* = \frac{t_{ij}}{A_{11}}, &&m_{ij}^* = \frac{c_1}{B_{44}\omega}m_{ij}, &&t^* = \omega t,
\end{aligned}
$$

where $c_1^2 = A_{11}/\rho$ and $\omega^2 = \chi/\rho j$.

We suppose that initially the half-space is at rest in its undeformed state, i.e., we suppose that the following homogeneous intial conditions hold for $t \geqslant 0$:

$$
u_i(x, y, 0) = \frac{\partial u_i}{\partial t} = 0, \qquad \phi_3(x, y, 0) = \frac{\partial \phi_3}{\partial t} = 0.
$$

Introducing dimensionless quantities as defined in Eq. (2.5) as well as using homogeneous intial conditions in Eqs. (2.1)-(2.4) (dropping the asterisks for convenience) and applying the Laplace transform w.r. to $'t'$ defined by

$$
(2.6) \quad \{\overline{u}_i(x,y,p), \overline{\phi}_3(x,y,p)\} = \int_0^\infty \{u_i(x,y,t), \phi_3(x,y,t)\}e^{-pt}dt, \quad i = 1, 2
$$

and then the Fourier transform w.r. to $'x'$ defined by

$$
(2.7) \quad \{\tilde{u}_i(\xi,y,p), \tilde{\phi}_3(\xi,y,p)\} = \int_{-\infty}^\infty \{\overline{u}_i(x,y,p), \overline{\phi}_3(x,y,p)\}e^{\iota\xi x}dx, \quad i = 1, 2
$$

on the resulting expressions, we obtain

$$
(2.8) \qquad \tilde{u}''_1 = Q_{11}\tilde{u}_1 + Q_{15}\tilde{u}'_2 + Q_{16}\tilde{\phi}'_3,
$$

$$
(2.9) \qquad \tilde{u}''_2 = Q_{22}\tilde{u}_2 + Q_{23}\tilde{\phi}_3 + Q_{24}\tilde{u}'_1,
$$

$$
(2.10) \qquad \tilde{\phi}''_3 = Q_{32}\tilde{u}_2 + Q_{33}\tilde{\phi}_3 + Q_{34}\tilde{u}'_1,
$$

where primes in Eqs. (2.8)-(2.10) represent the first and second order differentiation w.r. to y, respectively and

$$Q_{11} = \frac{A_{11}(\xi^2 + p^2)}{A_{88}}, \qquad Q_{15} = \frac{\iota\xi(A_{12} + A_{78})}{A_{88}}, \qquad Q_{16} = \frac{K_1^2}{A_{11}A_{88}},$$

$$Q_{22} = \frac{(\xi^2 A_{77} + p^2 A_{11})}{A_{22}}, \quad Q_{23} = -\frac{\iota\xi K_1 K_2}{A_{11}A_{22}}, \quad Q_{24} = \frac{\iota\xi(A_{12} + A_{78})}{A_{22}},$$

(2.11)
$$Q_{32} = \frac{\iota\xi K_2 A_{11}^2}{\omega^2 \rho B_{44} K_1},$$

$$Q_{33} = \frac{(\xi^2 B_{66}\omega^2 + c_1^2\chi) + jp^2\omega^2 A_{11}}{B_{44}},$$

$$Q_{34} = -\frac{A_{11}^2}{\rho\omega^2 B_{44}},$$

$$K_1 = A_{78} - A_{88}, \quad K_2 = A_{77} - A_{78}, \quad \chi = K_2 - K_1.$$

The system of Eqs. (2.8)-(2.10) can be written as

(2.12)
$$\frac{d}{dy}W(\xi, y, p) = A(\xi, p)W(\xi, y, p),$$

where

$$W = \begin{bmatrix} U \\ U' \end{bmatrix}, \quad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}, \quad U = \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{\phi}_3 \end{bmatrix},$$

(2.13)
$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & Q_{15} & Q_{16} \\ Q_{24} & 0 & 0 \\ Q_{34} & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} Q_{11} & 0 & 0 \\ 0 & Q_{22} & Q_{23} \\ 0 & Q_{32} & Q_{33} \end{bmatrix}.$$

To solve Eq. (2.12), we take

(2.14)
$$W(\xi, y, p) = X(\xi, p)e^{qy}$$

so that

(2.15)
$$A(\xi, p)W(\xi, y, p) = qW(\xi, y, p)$$

which leads to an eigenvalue problem. The characteristic equation corresponding to the matrix A is given by

$$(2.16) \qquad \det [A - qI] = 0,$$

which on expansion leads to

$$(2.17) \qquad q^6 - \lambda_1 q^4 + \lambda_2 q^2 - \lambda_3 = 0,$$

where

$$
(2.18)
\begin{aligned}
\lambda_1 &= Q_{15}Q_{24} + Q_{16}Q_{34} + Q_{11} + Q_{22} + Q_{33}, \\
\lambda_2 &= Q_{15}(Q_{24}Q_{33} - Q_{23}Q_{34}) + Q_{16}(Q_{22}Q_{34} - Q_{24}Q_{32}) \\
&\qquad\qquad + Q_{11}Q_{22} + Q_{22}Q_{33} + Q_{11}Q_{33} - Q_{23}Q_{32}, \\
\lambda_3 &= Q_{11}(Q_{22}Q_{33} - Q_{23}Q_{32}).
\end{aligned}
$$

The roots of Eq. (2.17) are $\pm q_i, \quad i = 1, 2, 3$.

The eigenvalues of the matrix A are the roots of Eq. (2.17). We assume that real parts of q_i are positive. The eigen-vector $X(\xi)$ corresponding to the eigenvalues q_i can be determined by solving the homogeneous equation

$$(2.19) \qquad [A - qI]X(\xi, p) = 0.$$

The set of eigenvectors $X_i(\xi, p), \quad (i = 1, 2, 3, 4, 5, 6)$ may be obtained as

$$(2.20) \qquad X_i(\xi, p) = \begin{bmatrix} X_{i1}(\xi, p) \\ X_{i2}(\xi, p) \end{bmatrix},$$

where

$$(2.21) \quad X_{i1}(\xi, p) = \begin{bmatrix} a_i q_i \\ b_i \\ 1 \end{bmatrix}, \quad X_{i2}(\xi, p) = \begin{bmatrix} a_i q_i^2 \\ b_i q_i \\ q_i \end{bmatrix},$$

$$q = q_i \quad i = 1, 2, 3,$$

$$(2.22) \quad X_{j1}(\xi, p) = \begin{bmatrix} -a_i q_i \\ b_i \\ 1 \end{bmatrix}, \quad X_{j2}(\xi, p) = \begin{bmatrix} a_i q_i^2 \\ -b_i q_i \\ -q_i \end{bmatrix},$$

$$j = i + 3, q = -q_i \quad i = 1, 2, 3,$$

$$a_i = (q_i^2 Q_{15} + Q_{16}Q_{32} - Q_{15}Q_{33})/\Delta_i,$$

$$(2.23) \qquad b_i = [q_i^4 - q_i^2(Q_{16}Q_{34} + Q_{11} + Q_{33}) + Q_{11}Q_{33}]/\Delta_i,$$

$$\Delta_i = q_i^2(Q_{15}Q_{34} + Q_{32}) - Q_{32}Q_{11}, \qquad i = 1, 2, 3.$$

The solution of Eq. (12) is given by

$$(2.24) \qquad W(\xi, y, p) = \sum_{i=1}^{3} [B_i X_i(\xi, p)\exp(q_i y) + B_{i+3}X_{i+3}(\xi, p)\exp(-q_i y)],$$

where $B_i (i = 1, 2, 3, 4, 5, 6)$ are arbitrary constants.

Equation (2.24) represents the solution of the general problem in the plane strain case of micropolar orthotropic elasticity by employing the eigenvalue approach, and therefore it can be applied to a broad class of problems in the domains of Laplace and Fourier transforms.

3. Application

In this section, the general solutions for displacement and stresses presented in Eq. (2.24) will be used to yield the response of a half-space subjected to a uniform traction distribution and to a point load. The constants B_i will be determined by imposing the proper boundary conditions. These constants, when substituted in Eq. (2.24), enable us to obtain the displacement and stress solutions in the Fourier and Laplace transformed (ξ, y, p) domain. The final solution in the original domain (x, y, t) is obtained by a numerical inversion of both transforms.

CASE 1. Load in normal direction: In the half-space, the load $F(x)$ is applied in normal direction at the origin of the co-ordinate system. For this loading case the boundary conditions are:

$$(3.1) \qquad t_{22}(x, 0, t) = -F(x)\delta(t), \qquad t_{21}(x, 0, t) = 0, \qquad m_{23}(x, 0, t) = 0.$$

. CASE 2. Load in tangential direction: In the half-space, the load $F(x)$ is applied in tangential direction at the origin of the co-ordinate system. For this loading case the boundary conditions are:

$$(3.2) \qquad t_{22}(x, 0, t) = 0, \qquad t_{21}(x, 0, t) = -F(x)\delta(t), \qquad m_{23}(x, 0, t) = 0.$$

It can be seen that six unknowns are to be determined in Eq. (2.24) and only three boundary conditions appear in each case. For the half-space the radition conditions implies outgoing waves with decreasing amplitudes in the positive y-direction. Therefore the radiation condition requires that $B_1 = B_2 = B_3 = 0$.

3.1. Influence functions

The method to obtain the half-space influence function, i.e., the solutions due to distributed loads applied at the half-space surface, is to set directly the distributed loads $F(x)$ in Eqs. (3.1) and (3.2). The Fourier transform w.r. to the pair (x, ξ) for the case of uniform strip load of amplitude F_o and width $2a$, applied at the origin of the coordinate system is :

$$(3.3) \qquad \tilde{F}(\xi) = F_o \frac{2\sin(\xi a)}{\xi}$$

SUBCASE 1 (a). Load acting in normal direction. The solutions for this case due to the uniformly distributed load are obtained as

$$(3.4) \qquad \tilde{u}_2(\xi, y, p) = b_1 B_4 e^{-q_1 y} + b_2 B_5 e^{-q_2 y} + b_3 B_6 e^{-q_3 y},$$

$$(3.5) \qquad \tilde{m}_{23}(\xi, y, p) = -\frac{K_1}{A_{11}}[q_1 B_4 e^{-q_1 y} + q_2 B_5 e^{-q_2 y} + q_3 B_6 e^{-q_3 y}],$$

$$(3.6) \qquad \tilde{t}_{22}(\xi, y, p) = -[N_1 B_4 e^{-q_1 y} + N_2 B_5 e^{-q_2 y} + N_3 B_6 e^{-q_3 y}],$$

where

$$(3.7) \quad B_i = 2F_o(M_j q_k - M_k q_j)\sin(\xi a)/\xi\Delta;$$
$$i = 4, \; j = 2, \; k = 3; \quad i = 5, \; j = 3, \; k = 1; \quad i = 6, \; j = 1, \; k = 2.$$

and

$$(3.8) \qquad M_i = [(-\iota\xi A_{78}b_i + A_{88}a_i q_i^2)A_{11} + K_1(A_{88} - A_{78})]/A_{11}^2,$$
$$N_i = (A_{22}b_i - \iota\xi A_{12}a_i)q_i/A_{11}; \quad i = 1, 2, 3,$$

$$(3.9) \qquad \Delta = M_1(q_2 N_3 - q_3 N_2) + M_2(q_3 N_1 - q_1 N_3) + M_3(q_1 N_2 - q_2 N_1).$$

SUBCASE 2 (a). Load acting in tangential direction. The solutions for this case are obtained as in Eqs. (3.4)-(3.6) by changing the values of the constant

$$(3.10) \quad B_i = 2F_o(N_j q_k - N_k q_j)\sin(\xi a)/\xi\Delta;$$
$$i = 4, \; j = 2, \; k = 3; \quad i = 5, \; j = 3, \; k = 1; \quad i = 6, \; j = 1, \; k = 2.$$

3.2. Green's functions

To synthesize the Green functions, *i.e.* the displacement and stress solutions due to a point load described as the Dirac Delta $F(x) = F_o\delta(x)$, its Fourier transform with respect to the pair (x, ξ)

$$(3.11) \qquad \tilde{F}(\xi) = F_o$$

must be used. The expressions for displacement and stresses may be obtained in the same way as Eqs. (3.4)-(3.6) by using the constants for the corresponding case.

SUBCASE 1 (b). Load acting in normal direction.

(3.12) $B_i = F_o(M_j q_k - M_k q_j)/\Delta;$

$$i = 4, \ j = 2, \ k = 3; \quad i = 5, \ j = 3, \ k = 1; \quad i = 6, \ j = 1, \ k = 2.$$

SUBCASE 2 (b). Load acting in tangential direction.

(3.13) $B_i = F_o(N_j q_k - N_k q_j)/\Delta;$

$$i = 4, \ j = 2, \ k = 3; \quad i = 5, \ j = 3, \ k = 1; \quad i = 6, j = 1, k = 2,$$

PARTICULAR CASES: Taking

$$A_{11} = A_{22} = \lambda + 2\mu + K, \quad A_{77} = A_{88} = \mu + K, \quad A_{12} = \lambda,$$

$$A_{78} = \mu, \quad B_{44} = B_{66} = \gamma, \quad -K_1 = K_2 = \chi/2 = K,$$

we obtain the corresponding expressions for the micropolar isotropic elastic medium.

4. Inversion of transforms

The transformed displacements and stresses (3.4)-(3.6) are functions of y, the parameters of Laplace and Fourier transforms p and ξ respectively, and hence they are of the form $\tilde{f}(\xi, y, p)$. To get the function $f(x, y, t)$ in the physical domain, first we invert the Fourier transform using

(4.1)
$$\bar{f}(x, y, p) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} e^{-\iota \xi x} \tilde{f}(\xi, y, p) d\xi,$$

$$= \frac{1}{\pi} \int\limits_{0}^{\infty} \{\cos(\xi x) f_e - \iota \sin(\xi x) f_o\} d\xi,$$

where f_e and f_o are even and odd parts of the function $\tilde{f}(\xi, y, p)$, respectively.

Thus, expression (4.1) gives us the Laplace transform $\bar{f}(x, y, p)$ of the function $f(x, y, t)$. Following HONIG and HIRDES [7], the Laplace transform function $\bar{f}(x, y, p)$ can be inverted to $f(x, y, t)$.

The last step is to evaluate the integral in Eq. (4.1). The method of evaluating this integral by PRESS *et. al.* [9], which involves the use of Romberg's integration with adaptive step size. This also uses the results of successive refinement of the extended trapezoidal rule, followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical results and discussion

For numerical computations, we take the following values of the relevant parameters for the orthotropic micropolar solid:

$$A_{11} = 13.97 \times 10^{10} \text{dyne/cm}^2, \qquad A_{22} = 13.75 \times 10^{10} \text{dyne/cm}^2,$$

$$A_{77} = 3.0 \times 10^{10} \text{dyne/cm}^2, \qquad A_{88} = 3.2 \times 10^{10} \text{dyne/cm}^2,$$

$$A_{12} = 8.13 \times 10^{10} \text{dyne/cm}^2, \qquad A_{78} = 2.2 \times 10^{10} \text{dyne/cm}^2,$$

$$B_{44} = 0.056 \times 10^{10} \text{dyne}, \qquad B_{66} = 0.057 \times 10^{10} \text{dyne}.$$

For comparison with the micropolar isotropic solid, following GAUTHIER [6], we take the following values of relevent parameters for the case of aluminum epoxy composite as

$$\rho = 2.19 \,\text{gm/cm}^3, \qquad \lambda = 7.59 \times 10^{10} \text{dyne/cm}^2,$$

$$\mu = 1.89 \times 10^{10} \text{dyne/cm}^2, \qquad K = 0.0149 \times 10^{10} \text{dyne/cm}^2,$$

$$\gamma = 0.0268 \times 10^{10} \text{dyne}, \qquad j = 0.00196 \,\text{cm}^2.$$

The comparison of dimensionless normal displacement $U_2[= u_2/F_o]$, normal force stress $T_{22}[= t_{22}/F_o]$ and couple stress $M_{23}[= m_{23}/F_o]$, for a micropolar orthotropic solid (MOS) and micropolar isotropic solid (MIS) due to normal and tangential uniform strip load (USL), have been studied and shown in Figs. 1,..,6. The computations were carried out for three values of dimensionless time $t = 0.10, 0.20$ and $t = 0.50$ at $y = 1.0$ in the range $0 \le x \le 10$. The solid lines either without the center symbols or with the center symbols represent the variations for $t = 0.1$, whereas the dashed lines with or without center symbols represent the variations for $t = 0.2$ and large dashed lines with or without center symbols represent variations for $t = 0.5$. The curves without center symbol correspond to the case of MOS whereas those with center symbol correspond to the case of MIS. All results are obtained for one value of dimensionless width $a_o = \omega a/c_1 = 1$.

CASE 1. The comparison of normal displacement $U_2[= u_2/F_o]$, normal force stress $T_{22}[= t_{22}/F_o]$ and couple stress $M_{23}[= m_{23}/F_o]$, for micropolar orthotropic solid (MOS) and micropolar isotropic solid (MIS) were studied due to a normal USL and have been shown in Figs. 1, 2 and 3.

Figure 1 presents the variation of normal displacement U_2 with x due to a normal USL. The value of displacement U_2 for MOS have been magnified by multiplying with 10, for all three values of time. For the case of MIS, as the time t increases from 0.1 to 0.5, the values of U_2 decrease at initial range of

x whereas for the cases of MOS, the response of displacement with respect to time is reverse. At the point of application of the source the values for MOS are smaller than those for MIS due to USL. The behaviour of variation is oscillatory in the whole range for both the cases.
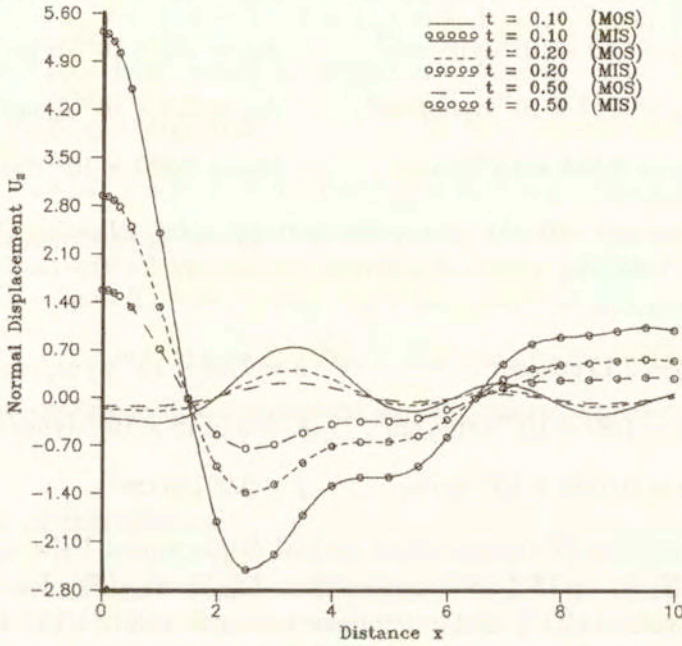

Fig. 1. Variation of normal displacement $U_2(x,1,t)(=u_2/F_0)$ with distance x due to normal USL.

Figure 2 shows the variation of normal force stress T_{22} with x due to normal USL. The values of T_{22} for MOS have been multiplied by 10 for all three times. For all three times, the values of T_{22} for MIS are greater than the corresponding values for MOS at the point of application of the source. For MOS, initially, values of T_{22} start with a small decrease and then oscillate in further range, whereas for MIS values of stress initially decrease smoothly. For both MIS and MOS at the initial stage for the maximum value of time, the value of stress is maximum.

Figure 3 shows the variations of tangential couple stress M_{23} with x due to normal USL. For all three times and for the case of MOS, the value of M_{23} starts with a sharp decrease and than start to oscillate in the range $3 \leq x \leq 10$. For the case of MIS, the behaviour of variation of couple stress in time is reverse to that for MOS. As the range of x increases, the values of couple stress tend towards zero.
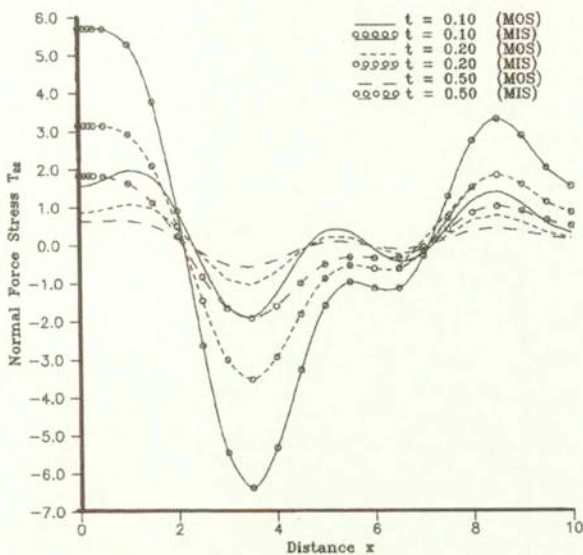
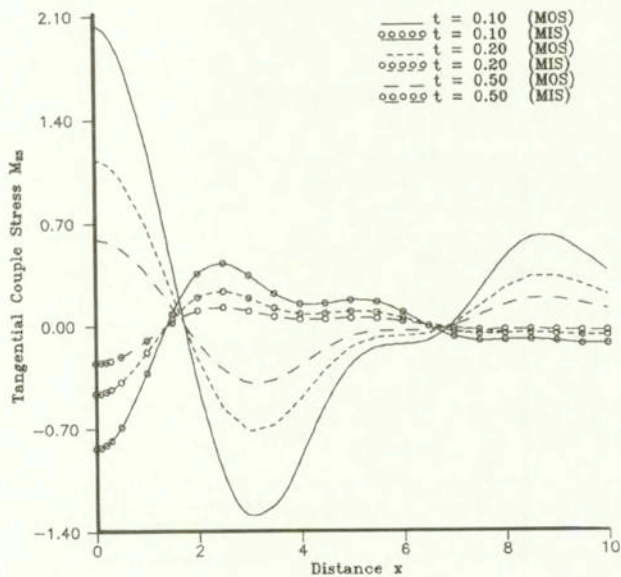FIG. 2. Variation of normal force stress $T_{22}(x, 1, t)(= t_{22}/F_o)$ with distance x due to normal USL.

FIG. 3. Variation of tangential couple stress $M_{23}(x, 1, t)(= m_{23}/F_o)$ with distance x due to normal USL.

CASE 2. Tangential Source: The comparison of normal displacement $U_2[= u_2/F_o]$, normal force stress $T_{22}[= t_{22}/F_o]$ and couple stress $M_{23}[= m_{23}/F_o]$, for

micropolar orthotropic solid (MOS) and micropolar isotopic solid (MIS) have been studied due to a tangential uniform strip load (USL), and have been shown in Figs. 4, 5 and 6. The values of U_2 and T_{22} for MOS are magnified by multiplying with 10 for all three times.
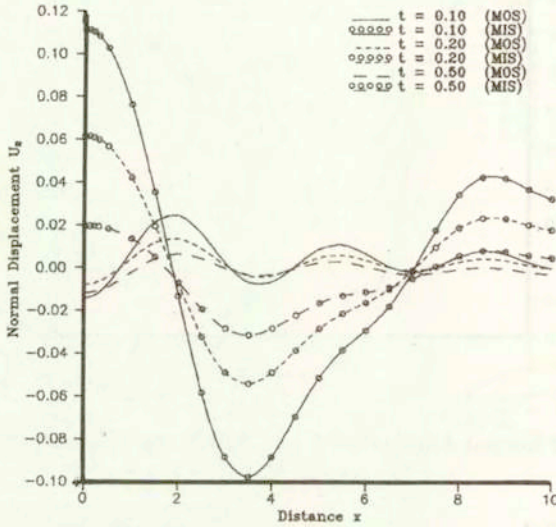

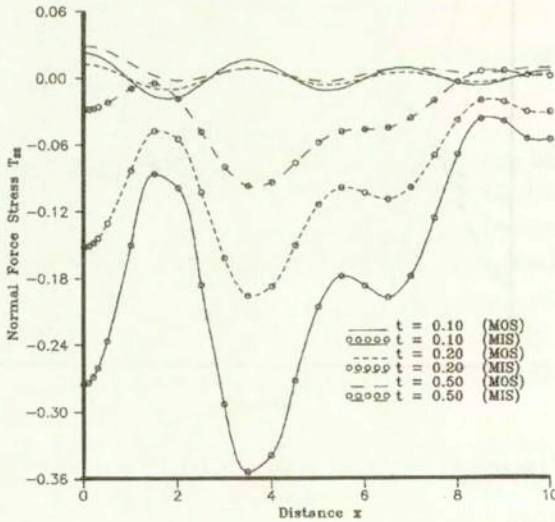FIG. 4. Variation of normal displacement $U_2(x, 1, t)(= u_2/F_o)$ with distance x due to tangential USL.

FIG. 5. Variation of normal force stress $T_{22}(x, 1, t)(= t_{22}/F_o)$ with distance x due to tangential USL.
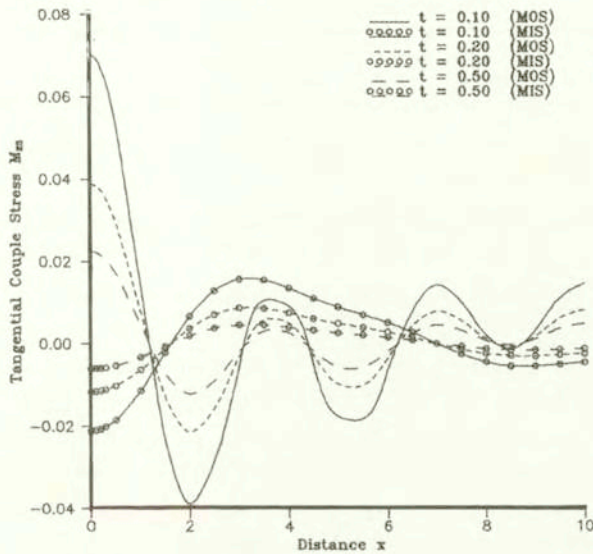
FIG. 6. Variation of tangential couple stress $M_{23}(x,1,t)(= m_{23}/F_o)$ with distance x due to tangential USL.

Figure 4 shows the variations of normal displacement U_2 with x due to tangential USL. The behaviour of variation of displacement is similar to that due to normal USL as in Fig. 1. However, their corresponding values are different.

Figure 5 shows the variation of normal force stress T_{22} with x due to tangential USL. The variation of stress for MOS is oscillating with smooth changes, whereas behaviour of variation for MIS is oscillating with greater changes. As the value of x increases the value for both the cases approaches zero.

Figure 6 shows the variation of tangential couple stress M_{23}. The variation of couple stress is similar to that due to normal USL, as in Fig. 3. However, their corresponding values are different.

6. Conclusion

A significant anisotropy effect is obtained in normal displacement, force stress and couple stress, for all values of time. Due to impulsive force, the character of solution is transient.

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