Arch. Mech., **54**, 1, pp. 55–74, Warszawa 2002

Thermal radiation effects on free convection over a rotating axisymmetric body with application to a rotating hemisphere

M. A. HOSSAIN [1], M. ANGHEL [2] and I. POP [3]

[1] *Department of Mathematics University of Dhaka 1000 Dhaka, Bangladesh*

[2] *Faculty of Mathematics University of CLUJ R-3400 CLUJ, CP 253, Romania*

[3] *Faculty of Mathematics University of CLUJ R-3400 CLUJ, CP 253, Romania*
e-mail: popi@math.ubbcluj.ro

THIS PAPER DEALS with the interaction of thermal radiation with free convection, laminar boundary-layer flow past a heated rotating axisymmetric round-nosed body of uniform surface temperature. The fluid considered is a gray, absorbing-emitting but nonscattering medium, and Rosseland approximation is used to describe the radiative heat flux. The difficulty of having a unified mathematical treatment of this problem is due to the nonsimilarity nature of the governing equations arising from the buoyant force-field and the transverse curvature of the body. The important parameters of this problem are the Planck number, R_d, the buoyancy parameter, λ, and the wall to free stream temperature ratio, θ_w. Numerical solution of the boundary-layer equations are performed using the Keller-box method as well as the local nonsimilarity method. The theory is applied to a rotating hemisphere for a gas with Prandtl number of 0.72. The effects of the parameters λ, R_d and θ_w are shown on the velocity and temperature profiles, as well as on the local skin friction coefficient and local rate of heat transfer.

Notations

a	Rosseland mean absorption coefficient
f	dimensionless stream function
g	acceleration due to gravity
g_x	component of the acceleration due to gravity in the x direction
Gr	Grashof number
$S(x)$	function of x denotes sine of the angle between the acceleration vector and a component normal to the surface of the body
k	thermal conductivity
L	characteristic length
Nu	Nusselt number
Pr	Prandtl number
R_d	Planck number or the conduction-radiation parameter defined in Eq. (2.13)

Re	Reynolds number
R	radial distance from a surface element to the axis of symmetry
T	temperature of the fluid in the boundary-layer
T_w	surface temperature
T_∞	temperature of the ambient fluid
u	velocity component in the x direction
U_e	reference velocity
v	velocity component in the y direction
w	velocity component in the rotation direction
x	coordinate measured from the stagnation point along the surface of the body
y	coordinate normal to x
z	coordinate measured in the rotation direction

Greek letters

α	thermal diffusivity
β	thermal expansion coefficient
η	similarity variable defined in Eq. (2.15)
θ	non-dimensional temperature
θ_w	ratio of the surface temperature to the ambient temperature defined in Eq. (2.13)
ν	kinematic viscosity
λ	buoyancy parameter defined in Eq. (2.12)
ρ	density of the fluid
ξ	transformed coordinate defined in Eq. (2.15)
Ω	angular velocity
$\tau_x, \tau_z,$	skin friction coefficients in the x- and z-directions, respectively
σ	Stephan-Boltzmann constant
σ_s	scattering coefficient
ψ	stream function

1. Introduction

THE THERMAL RADIATION EFFECTS on free convection flow are important in the context of space technology and processes involving high temperatures, and very little is known about the effects of radiation on the boundary-layer flow of radiating fluid past a body of general geometry. The inclusion of thermal radiation effects in the energy equation leads to a highly nonlinear partial differential equation. In absence of the effect of radiation, investigations have been made on the laminar heat transfer from rotating axisymmetric round-nosed bodies either for forced convection or for natural convection in refs. [1-5]. The density difference arising as a result of temperature difference gives rise to a buoyancy force. The neglect of buoyancy effect on forced convection heat transfer may not be justified when the velocity is small and the temperature difference between the surface and ambient fluid is large. It may be expected that this buoyancy force will affect the momentum and the heat transfer.

Several authors [6-11] have discussed the effect of buoyancy forces on non-rotating bodies. For rotating bodies, LEE *et al.* [5] have investigated the laminar boundary-layer and heat transfer in forced flow, neglecting the buoyancy forces. They have used MERK's [10] series, modified by CHAO and FAGBENLE [11], and their results compare favorably with previous theoretical and experimental studies. SUWONO [12] considered the problem of buoyancy effects on the flow and heat transfer in rotating axisymmetric round-nosed bodies for both aiding and opposing flows. He has shown that spinning a vertical axisymmetric body in a convective flow, the fluid near the surface is forced outwards in the radial direction due to the presence of centrifugal force. Application of this idea in order to develop rotating systems for enhancing the heat transfer rate is important in the analysis of the rotary machine design. The problem posed by SUWONO [12] has later been investigated by HOSSAIN *et al.* [13] for a viscous and electrically conducting fluid, using the implicit finite difference method.

The majority of studies on interaction of thermal radiation and natural convection have been confined to the case of a vertical semi-infinite flat plate [14-20]. HOSSAIN and TAKHAR [21] have analyzed the effect of radiation on the forced and free convection flow of an optically dense viscous and incompressible fluid past a heated vertical flat plate with uniform free stream velocity and surface temperature using the Rosseland diffusion approximation, which leads to nonsimilarity solutions. The convection-radiation effects on free convection boundary-layer flow from an inclined surface with small angle of inclination to the horizontal has been investigated by HOSSAIN *et al.* [22]. In this analysis, solutions are obtained in the upstream, the downstream and the entirely mixed regimes. Very recently, HOSSAIN and ALIM [23] have studied the problem of natural convection interaction in the boundary-layer flow along a thin vertical cylinder employing two methods, namely, the implicit finite-difference method and the local non-similarity method, taking up terms to the third level of truncation.

The purpose of the present paper is to investigate the effect of the conduction-radiation interaction on the laminar free convection flow of an optically dense, viscous incompressible fluid with heated rotating axisymmetric round-nosed bodies of uniform surface temperature. The difficulty of having a unified mathematical treatment of this problem is due to the nonsimilarity nature of the governing equations arising from the buoyant force-field and the transverse curvature of the bodies. Numerical simulations of the boundary-layer equations are performed using the implicit finite-difference method, known as the KELLER-box (see CEBECI and BRADSHAW [24]) method, as well as the local nonsimilarity method, taking terms up to the second level of truncation. The results are then applied to the case of a rotating hemisphere.

2. Basic equations

Consider the steady free convection boundary-layer flow over a rotating axisymmetric round-nosed body, which rotates with the constant angular velocity Ω around its vertical axis of symmetry in an optically dense, viscous and incompressible fluid of constant ambient temperature T_∞. It is assumed that the surface of the body has the uniform temperature T_w, where $T_w > T_\infty$. Let x, y and z be a non-rotating orthogonal curvilinear coordinate system with the x-coordinate measured from the lower stagnation point along the surface of the body, y measured normal to x and z measured in the rotation direction, as shown in Fig. 1. It is also assumed that the radiative heat flux in the x-direction is negligible in comparison with that in the y-direction (see SPARROW and CESS[19]).

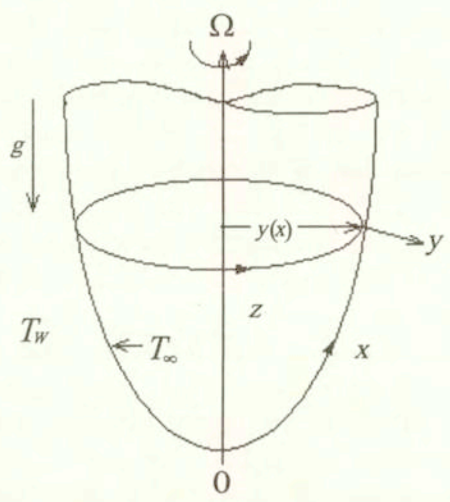


FIG. 1. Physical model and coordinate system.

(2.1) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{u}{r}\frac{dr}{dx} = 0,$$

(2.2) $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{w^2}{r}\frac{dr}{dx} = \nu\frac{\partial^2 u}{\partial y^2} + g_x\beta(T - T_\infty),$$

(2.3) $$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + \frac{uw}{r}\frac{dr}{dx} = \nu\frac{\partial^2 w}{\partial y^2},$$

(2.4)
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial x}=\frac{\nu}{Pr}\frac{\partial}{\partial y}\left[\left(1+\frac{16\sigma T^3}{3k(a+\sigma_s)}\right)\frac{\partial T}{\partial y}\right],$$

where u, and w are the velocity components along the x, y and z axes, T is the fluid temperature, g_x is the x-component of the local gravitational acceleration vector in the direction of increasing x, $r(x)$ is the radial distance from the axis of symmetry to the surface of the body, Pr is the Prandtl number; ρ, β, ν, a, σ, σ_s are, respectively, the fluid density, thermal expansion coefficient, kinematic viscosity of the fluid, Rosseland mean absorption coefficient, Stefan-Boltzmann constant and the scattering coefficient, respectively. We assume that $g_x = gS(x)$ where $S(x)$ is a non-dimensional function of x and g is the constant gravitational acceleration. Radiation effects are considered here using the Rosseland diffusion approximation (see SIEGEL and HOWEL [25]). Under this approximation, the situation is not valid where scattering is expected to be non-isotropic as well as in the immediate vicinity of the wall.

The boundary conditions to be satisfied by Eqs. (2.1) – (2.4) are

(2.5)
$$\begin{gathered} u=v=0, \quad w=r\,\Omega, \quad T=T_w \ \text{ at } y=0, \\ u\to 0, \quad w\to 0, \quad T\to T_\infty \ \text{ as } y\to\infty, \end{gathered}$$

where Ω is the angular velocity.

We now define the following non-dimensional variables:

(2.6)
$$\begin{gathered} \bar{x}=x/L, \quad \bar{y}=Re^{1/2}\,(y/L)\,, \quad \bar{r}=r/L, \\ \bar{u}=u/U, \quad \bar{v}=Re^{1/2}\,(v/U)\,, \ \bar{w}=w/U, \\ \theta=(T-T_\infty)/(T_w-T_\infty), \end{gathered}$$

with L and $U = L\Omega$ being reference length and reference velocity, respectively. Substituting these variables into Eqs. (2.1) – (2.4) and dropping the bar for brevity, we get

(2.7)
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{u}{r}\frac{dr}{dx}=0,$$

(2.8)
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{w^2}{r}\frac{dr}{dx}=\frac{\partial^2 u}{\partial y^2}+\lambda S(x)\theta,$$

(2.9)
$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+\frac{uw}{r}\frac{dr}{dx}=\frac{\partial^2 w}{\partial y^2},$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\frac{\partial}{\partial y}\left[\left\{1+\frac{4}{3}R_d(1+K\theta)^3\right\}\frac{\partial\theta}{\partial y}\right], \tag{2.10}$$

and the boundary conditions (2.5) become

$$u=v=0,\ w=r,\ \theta=1 \text{ at } y=0,\ u\to 0,\ w\to 0,\ \theta\to 0 \text{ as } y\to\infty, \tag{2.11}$$

where λ, Gr and Re are the buoyancy parameter, the Grashof number and the Reynolds number which are defined by

$$\lambda=\frac{Gr}{Re^2},\quad Gr=\frac{g\beta(T_w-T_\infty)L^3}{\nu^2},\quad Re=\frac{UL}{\nu}\,. \tag{2.12}$$

Also the parameters R_d and K in Eq. (2.10) are defined as

$$R_d=\frac{4\sigma T_\infty^3}{k(a+\sigma_s)},\qquad \mathrm{K}=\frac{T_w}{T_\infty}-1=\theta_w-1 \text{ (say)}, \tag{2.13}$$

and they are known as the Planck number and the surface temperature parameter, respectively. Further, in Eq. (2.13) θ_w is the ratio of the surface temperature to the temperature of the ambient fluid. Throughout the present investigation we assume that $K>0$. However, when the wall temperature T_w is very close to the ambient temperature T_∞ (i.e., $K=0$), the energy equation (2.10) takes the following form (see, ALI *et al.* [26]):

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\left(1+\frac{4}{3}R_d\right)\frac{\partial^2\theta}{\partial y^2}, \tag{2.14}$$

We now introduce the new coordinates (ξ,η) in place of the non-dimensional coordinates (x,y) defined as

$$\xi=\int_0^x[r(x)]^3dx,\quad \eta=r^2\frac{y}{(2\xi)^{1/2}} \tag{2.15}$$

along with the non-dimensional functions:

$$(\psi(x,y)=(2\xi)^{1/2}f(\xi,\eta),\quad w(x,y)=\frac{(2\xi)^{1/2}}{r}h(\xi,\eta), \tag{2.16}$$

where ψ is the stream function and is defined in the usual way as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial y},\quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}. \tag{2.17}$$

Using these transformations, the momentum and energy Eqs. (2.8) – (2.10) can be written in the following form:

$$f''' + ff'' - P(\xi)\left(f'^2 - h^2\right) + \lambda Q(\xi)\theta = 2\xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right), \tag{2.18}$$

$$h'' + fh' - 2P(\xi)f'h = 2\xi\left(f'\frac{\partial h}{\partial \xi} - h'\frac{\partial f}{\partial \xi}\right), \tag{2.19}$$

$$\left[\left\{1 + \frac{4}{3}R_d\left(1 + K\theta\right)^3\right\}\theta'\right]' + Pr\,f\theta' = 2Pr\xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right), \tag{2.20}$$

subject to the boundary conditions (2.11) which become

$$\begin{gathered} f(\xi,0) = f'(\xi,0) = 0,\ h(\xi,0) = 1,\ \theta(\xi,0) = 1, \\ f'(\xi,\infty) = 0,\ h(\xi,\infty) = \theta(\xi,\infty) = 0, \end{gathered} \tag{2.21}$$

where

$$P(\xi) = \frac{2\xi}{r}\frac{dr}{d\xi}, \quad Q(\xi) = \frac{2\xi K(\xi)}{r^5}. \tag{2.22}$$

Here primes denote partial differentiation with respect to η. If the wall temperature T_w is very close to the ambient temperature T_∞, the energy Eq. (2.20) takes the form:

$$\left(1 + \frac{4}{3}R_d\right)\theta'' + Pr\,f\theta' = 2Pr\,\xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right). \tag{2.23}$$

Once the solution of Eqs. (2.18) – (2.23) is known, it becomes important from the experimental point of view to determine the physical quantities like skin friction coefficients and the local heat transfer at the surface of the body. These quantities are given by

$$Re^{1/2}\tau_x = \frac{r^3}{(2\xi)^{1/2}}f''(\xi,0), \quad Re^{1/2}\tau_z = \frac{r^3}{(2\xi)^{1/2}}h'(\xi,0), \tag{2.24}$$

and

$$NuRe^{-1/2} = \frac{r^2}{(2\xi)^{1/2}}\left(1 + \frac{4}{3}R_d(1+K)^3\right)\theta'(\xi,0), \tag{2.25}$$

where τ_x and τ_z are the skin friction coefficients along the x- and z-directions, and Nu is the local Nusselt number.

3. Application to a rotating hemisphere

As an example, in this section, we discuss the application of the present analysis to the case of a rotating hemisphere of radius R with the rotating axis being parallel to the gravitational vector $\mathbf{g}$. If we select R as the reference length, i.e. $L = R$, we then have the following non-dimensional variables:

$$r(x) = \sin x, \;\; K(x) = \sin x, \;\; \xi = \frac{1}{3}\left(\cos^3 x - 3\cos x + 2\right). \tag{3.1}$$

Substituting (3.1) into Eqs. (2.18) – (2.20), we get

$$f''' + ff'' - P(x)\left(f'^2 - h^2\right) + \lambda Q(x)\theta = 2xI(x)\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right), \tag{3.2}$$

$$h'' + fh' - 2P(x)f'h = 2xI(x)\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right), \tag{3.3}$$

$$\left[\left\{1 + \frac{4}{3}R_d\left(1 + K\theta\right)^3\right\}\theta'\right]' + Pr\, f\theta' = 2Pr\, xI(x)\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right), \tag{3.4}$$

subject to the boundary conditions

$$\begin{gathered} f(\xi,0) = f'(\xi,0) = 0, \;\; h(\xi,0) = 1, \;\; \theta(\xi,0) = 1, \\ f'(\xi,\infty) = 0, \;\; h(\xi,\infty) = \theta(\xi,\infty) = 0, \end{gathered} \tag{3.5}$$

where $P(x)$, $Q(x)$ and $I(x)$ are now given by

$$\begin{gathered} P(x) = \frac{2\cos x}{3\sin^4 x}\left(\cos^3 x - 3\cos x + 2\right), \\ Q(x) = \frac{2}{3}\frac{\left(\cos^3 x - 3\cos x + 2\right)}{\sin^4 x}, \\ I(x) = \frac{\left(\cos^3 x - 3\cos x + 2\right)}{3x\sin^3 x}. \end{gathered} \tag{3.6}$$

We notice in passing that, when the conduction-radiation is absent (i.e., $R_d = 0$), Eqs. (3.2) – (3.4) reduce to those reported by Suwono [12].

4. Numerical solution

In the absence of the effect of thermal radiation (i.e. $R_d = 0$), the partial differential Eqs. (2.18) – (2.20) subject to the boundary conditions (2.21) were solved by SUWONO [12] using Görtler's series method for small values of ξ (i.e. $\xi \ll 1$), and the results were then applied to the case of a rotating hemisphere. Here we are solving the transformed Eqs. (3.2) – (3.4) subject to the boundary conditions (3.5) numerically following two distinct methods, namely: the implicit finite-difference method, known as Keller-box method (see CEBECI and BRADSHAW [24]), and the local nonsimilarity method, respectively. The latter method has been recently used very efficiently by HOSSAIN and ALIM [23]. The results obtained by employing these method are compared with those of SUWONO [12] for the case when the radiation is absent (i.e. $R_d = 0$).

4.1. Keller-box method

To employ this method, the system of partial differential Eqs. (3.2) – (3.4) is first converted to a system of seven first-order partial differential equations by introducing new unknown functions of η derivatives. This system is then put into a finite-difference form in which the nonlinear difference equations are linearized by the Newton's quasi-linearization method. The resulting linear difference equations along with the appropriate boundary conditions are finally solved by an efficient block-tridiagonal factorization technique. The details of the computational procedure have been discussed by HOSSAIN *et al.* [8, 16–19] and will not be repeated here. We note that for initiating this method, the profiles at $x = 0$ (the lower stagnation point of the hemisphere) for the functions $f(\eta)$, $g(\eta)$ and $\theta(\eta)$ and their derivatives are obtained from the exact solutions of the similarity equations:

$$f''' + ff'' + \frac{1}{2}\left(f'^2 - h^2\right) + \frac{1}{2}\lambda\theta = 0, \tag{4.1}$$

$$h''' + fh'' - f'h' = 0, \tag{4.2}$$

$$\left[\left\{1 + \frac{4}{3}R_d\left(1 + K\theta\right)^3\right\}\theta'\right]' + Prf\theta' = 0, \tag{4.3}$$

which are obtained from Eqs. (3.2) – (3.4) as $x \to 0$. The appropriate boundary conditions to be satisfied by Eqs. (4.1) – (4.3) are

$$\begin{gathered} f(0) = f'(0) = 0, \quad h'(0) = 1, \quad \theta(0) = 1, \\ f'(\infty) = 0, \quad h'(\infty) = \theta(\infty) = 0, \end{gathered} \tag{4.4}$$

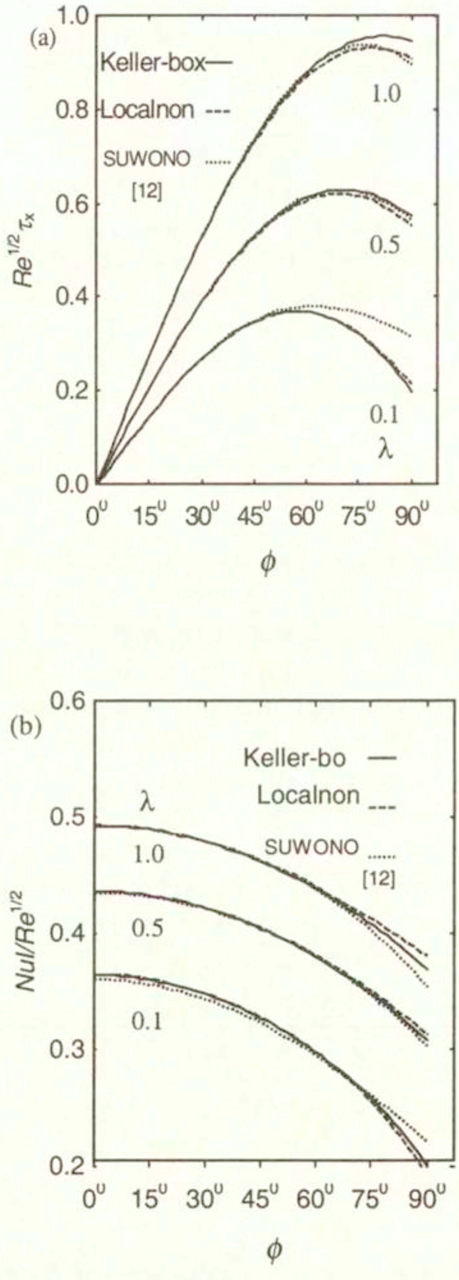


FIG. 2. Comparisonof the results without the effect of radiation ($R_d = 0$)obtained fo Pr=0.72 and $\lambda = 0.1, 0.5$ and 1.0. (a) The local skin friction coefficient along the x-direcion, and (b) the local Nusselt number.

4.2. Local nonsimilarity method

The formulation of the local nonsimilarity method for heat transfer problems has been first given by SPARROW and YU [27]. More extensive and successful use of this method has ben made recently by HOSSAIN *et al.* [23, 28]. With reference to the present problem, we derive here the equations up to the second level of truncation. To do it, we introduce the following functions:

$$F(x,\eta) = \frac{\partial f}{\partial x}, \quad G(x,\eta) = \frac{\partial h}{\partial x}, \quad \Theta(x,\eta) = \frac{\partial \theta}{\partial x}. \tag{4.5}$$

The governing Eqs. (3.2)–(3.4) can then be written as

$$f''' + ff'' + P(x)\left(f'^2 - g^2\right) + \lambda Q(x)\theta = I_1(x)\left(f'F' - f''F\right), \tag{4.6}$$

$$h'' + fh' - 2P(x)f'h = 2xI_1(x)\left(f'G - h'F\right), \tag{4.7}$$

$$\left[\left\{1 + \frac{4}{3}R_d(1+K\theta)^3\right\}\theta'\right]' + Prf\theta' = \Pr I_1(x)\left(f'\Theta - \theta'F\right), \tag{4.8}$$

subjected to the boundary conditions.

The equations for F, G and Θ can be derived by taking the derivatives of Eqs. (4.6)–(4.8) with respect to x and neglecting the terms with the derivative functions F, G and Θ with respect to x. To this end, we get

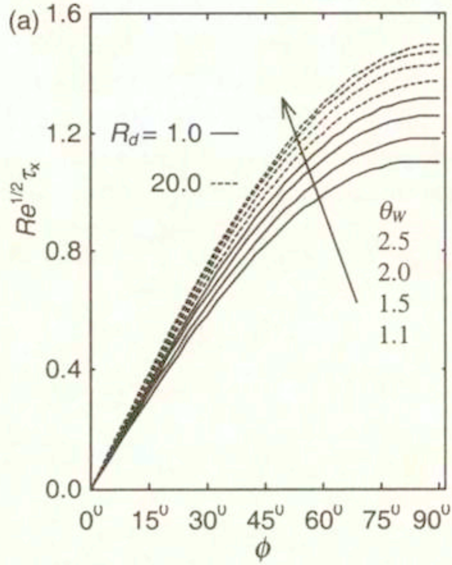
$$\begin{aligned} F''' + fF'' + (1 + 2I_2(x))\, f''F - (2P(x) + I_2(x))\, f'F' + 2P(x)gG \\ - P_1(x)\left(f'^2 - g^2\right) + \lambda\left(+Q(x)\Theta + Q_1(x)\theta\right) = I_1(x)\left(F'^2 - FF''\right), \end{aligned} \tag{4.9}$$

$$\begin{aligned} G'' + fG' + (1 + I_2(x))\, h'F - (2P(x) + I_2(x))f'G - 2P_1(x)f'h \\ = I_1(x)\left(F'G - G'F\right), \end{aligned} \tag{4.10}$$

$$\begin{aligned} \left\{1 + \frac{4}{3}R_d(1+K\theta)^3\right\}\Theta'' + 4R_d(1+K\theta)^2\theta'' \\ + 8R_dK(1+K\theta)\left[(1+K\theta)\theta'\Theta' + K\theta'^2\Theta\right] + Pr\left[f\Theta' + (1 + I_2(x))\,\theta'F\right] \\ - PrI_2(x)f'g = PrI_1(x)\left(Ff\Theta - \Theta'F\right). \end{aligned} \tag{4.11}$$

The boundary conditions to be satisfied by the equations for F, G and Θ are

$$\begin{aligned} F(x,0) = F'(x,0) = G(x,0) = \Theta(x,0) = 0, \\ F'(x,\infty) = G(x,\infty) = \Theta(x,\infty) = 0, \end{aligned} \tag{4.12}$$



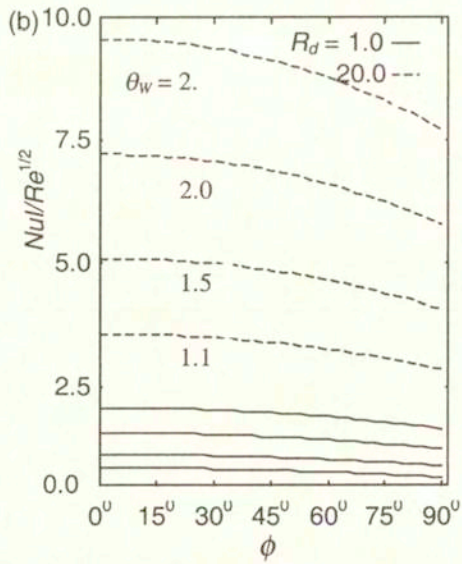


FIG. 3. Variation with ϕ of: (a) skin friction coefficient along the x-direction and (b) the local Nusselt number for some values of R_d and θ_w with $Pr = 0.7$ and $\lambda = 0.1$

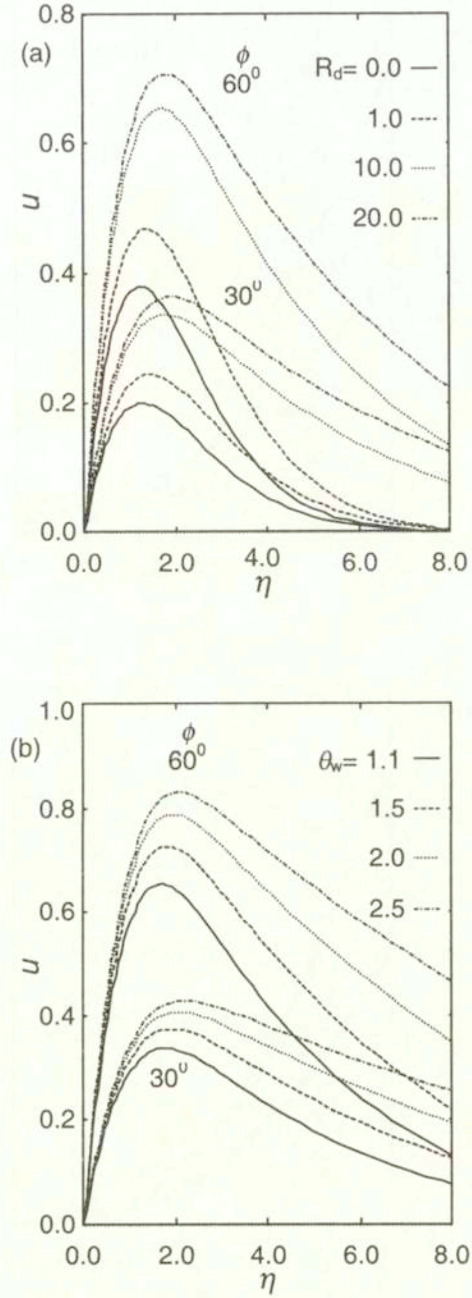


FIG. 4. Non-dimensional velocity profiles u at $\phi = 30°$ and $60°$ with $Pr = 0.72$ and $\lambda = 1.0$: (a) for some values of R_d with $\theta_w = 1.1$, and (b) for some values of θ_w with $R_d = 10.0$.

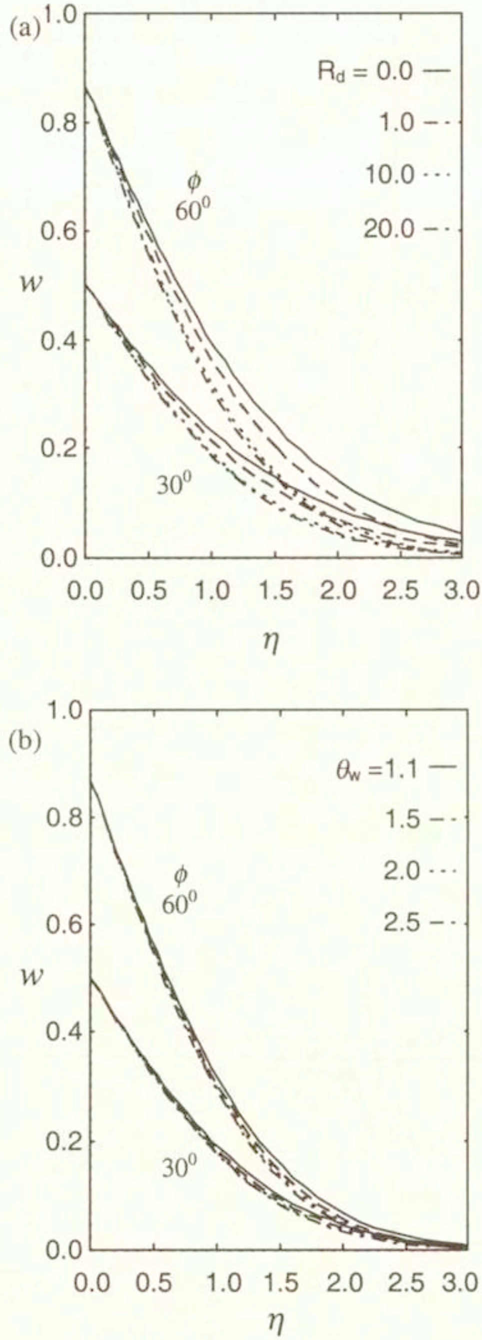


FIG. 5. Non-dimensional velocity profiles w at $\phi = 30°$ and $60°$ with $Pr = 0.72$ and $\lambda = 1.0$: (a) for some values of R_d with $\theta_w = 1.1$, and (b) for some values of θ_w with $R_d = 10.0$

The functions $P(x)$ and $Q(x)$ in Eqs. (4.6) – (4.11) were defined in the relations (3.6) and the other coefficients $P_1(x)$, $Q_1(x)$, $I_1(x)$ and $I_2(x)$ are given by

(4.13)
$$P_1(x) = \frac{4(\cos x - 1)^3}{3\sin^5 x},$$
$$Q_1(x) = 0.5926\frac{\left(\cos^4 x - 6\cos^2 x + 8\cos x - 3\right)}{2\sin^4 x},$$
$$I_1(x) = \frac{3\left(\cos^3 x - 3\cos x + 2\right)}{2\sin^3 x},$$
$$I_2(x) = \frac{dI_1}{dx} = \frac{9}{2}\frac{(\cos x - 1)^2}{2\sin^3 x}.$$

Equations (4.6) – (4.11) are coupled and highly nonlinear. The numerical solution of these equations has been obtained for some values of the involved parameters λ, R_d, θ_w and Pr using the Nachsteim-Swigert iteration technique together with the sixth order Runge-Kutta-Butcher, initial value solver, see HOSSAIN *et al.* [16-19].

5. Results and discussions

Here we discuss the effects of thermal radiation on free convection boundary-layer flow characteristics of an optically dense fluid in a rotating hemisphere by two distinct methods, namely, the Keller-box method and the local nonsimilarity scheme with second level of truncation, respectively. The numerical results for the velocity components u and w along the x- and z-directions as well as for the local skin-friction coefficient $Re^{1/2}\tau_x$ in the x-direction and the local Nusselt number $Nu/Re^{1/2}$, are obtained for some values of the involved parameters λ, R_d and θ_w for a heated surface only ($T_w > T_\infty$) with $Pr = 0.72$. It should be noted that for both CO_2 – air in the temperature range 100 ≈ 650°F (with the corresponding Prandtl number range 0.76 ≈ 0.6) and NH_3 – vapor in the temperature range 120 ≈ 400°F (with corresponding Prandtl number range 0.88 ≈ 0.84) at 1 atm, the value of the parameter R_d varies approximately from 10 to 30; whereas for water vapor in the temperature range 220 ≈ 900°F (with the corresponding Prandtl number $Pr \approx 1.0$), the value of R_d lies between 30 to 200 (see CESS [14]).

As we have mentioned before, in the absence of the effect of conduction-radiation (i.e., $R_d = 0$), the present problem has been studied by SUWONO [12] using Görtler's series expansion method. He showed the effect of the buoyancy parameter λ on the flow and heat transfer characteristics at some selected values of ϕ-positions in the interval $[0, \pi/2]$. Comparison between the present and

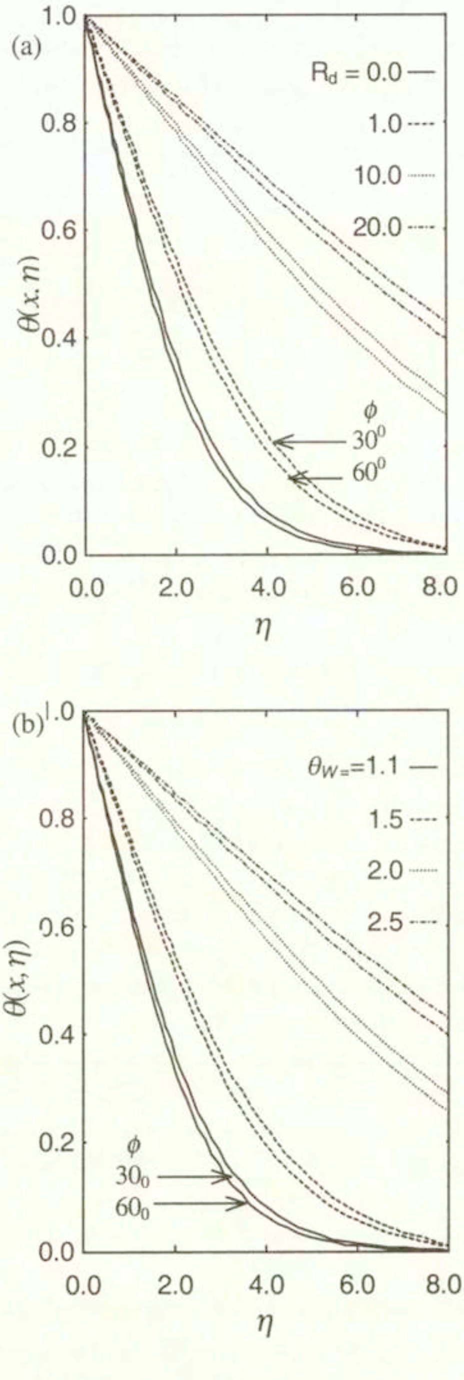


FIG. 6. Non-dimensional temperature profiles θ at $\phi = 30°$ and $60°$ with $Pr = 0.72$ and $\lambda = 1.0$: (a) for some values of R_d with $\theta_w = 1.1$ and (b) for some values of θ_w with $R_d = 10.0$.

Suwono's results, interval [0, $\pi/2$] obtained in terms of the local skin friction coefficient in the x-direction $Re^{1/2}\tau_x$ and the local Nusselt number $Nu/Re^{1/2}$ are shown in Fig. 2 (a) and 2 (b), respectively, for the values of the buoyancy parameter $\lambda = 0.1$, 0.5 and 1.0 with $Pr = 0.72$. It can be seen from these figures that the results obtained using the, Keller-box method, are very close to those of the local nonsimilarity method. They are also in good agreement with the results reported by SUWONO [12]. However, the results obtained by the Keller-box scheme are more accurate than the other methods because this method does not require any approximation.

Further results are obtained for $R_d = 1.0$ and 20.0, but for $\theta_w = 1.1$, 1.5, 2.0, 2.5 at selected axial positions ϕ in the range [0, $\pi/2$] with $\lambda = 1.0$ and $Pr = 0.7$ by the Keller-box method only. Figure 3(a) illustrates the variation of the local skin friction coefficient $\tau_x Re^{1/2}$ as a function of ϕ, and Fig. 3(b) represents that of the local Nusselt number $NuRe^{-1/2}$ for $\theta_w = 1.1$, 1.5, 2.0, 2.5. The solid lines show the values of the mentioned physical quantities for $R_d = 1.0$, while the broken lines are those for $R_d = 20.0$. It is seen from these figures that an increase in the radiation effect leads to decrease in the local skin friction coefficient $\tau_x Re^{1/2}$. On the other hand, this leads to increase in the value of the local Nusselt number $NuRe^{-1/2}$ at every station of the angular distance ϕ in the range [0, $\pi/2$]. This tendency is higher for the skin friction coefficient and is less for the local Nusselt number when the value of ϕ increases. Further observations drawn from these figures are that values of both the local skin friction and the local Nusselt number at every ϕ station increase, owing to an increase of the values of the parameter θ_w.

Representative velocity and temperature profiles are shown in Figs. 4 to 6 in which the non-dimensional velocity components u and w as well as the non-dimensional temperature profiles θ are plotted against η for some values of the conduction-radiation parameter $R_d = 0.0$, 1.0, 10.0 and 20 and the surface temperature parameter θ_w = 1.1, 1.5 2.0 and 2.5 with ϕ = 30° and 60°, and the buoyancy parameter λ=1.0. It can be seen from these figures that the transverse velocity profiles u and the temperature profiles θ increase with the increase of the parameters R_d and θ_w at all values of ϕ. On the other hand, the increase in the value of the parameters θ_w and R_d leads to decrease in the value of the circumferential velocity profile w. We also see that both the momentum and the thermal boundary layer thickness increase owing to the increase of the parameters R_d and θ_w. However, it is important to notice that the present results are available only for values of the angle $\phi < 90°$. For increasing values of the angle ϕ and, in particular, at $\phi > 90°$, the solution of the governing equations becomes unstable. Unfortunately, we are not able to compare the present results with any experimental data since we are not aware of any existing experimental results for the present problem.

6. Conclusions

Effect of the radiation-conduction interaction on free convection boundary-layer flow over rotating axisymmetric round-nosed bodies of uniform surface temperature of a gray, absorbing-emitting but nonscattering fluid medium with Rosseland approximation, has been analyzed. The nonsimilarity equations governing the flow and heat transfer have numerically been solved employing the implicit finite difference method known as the Keller-box method and the local nonsimilarity method. Solutions are obtained for different values of the pertinent parameters, such as the Planck number (radiation-conduction parameter) R_d, the surface temperature parameter θ_w and the buoyancy parameter λ. From the present investigation, the following conclusions may be drawn:

(i) Increase in the radiation parameter R_d leads to decrease of the local skin friction coefficient $\tau_x Re^{1/2}$

(ii) The rate of heat transfer $Nu/Re^{1/2}$ increases owing to the increase of the parameters R_d and θ_w

(iii) Both the tangential velocity u and the temperature θ profiles of the fluid increase, whereas the circumferential velocity w profiles decrease due to the increase of either of the value of the parameter R_d or θ_w. Furthermore, increase of the values of the parameters R_d and θ_w leads to increase in both the momentum and the thermal boundary layer thickness.

Acknowledgements

The authors are grateful to the Referee for his valuable comments.

References

1. W. H. Banks, *The laminar boundary layer on a rotating sphere*, Acta Mechanica, **24**, 273-283, 1976.

2. L. A. Dorfman and V. A. Mirnova, *Solutions of equations for the thermal boundary layer at rotating axisymmetric surface*, Int. J. Heat Mass Transfer, **13**, 81-92, 1970.

3. B. T. Chao, and R. Greif, *Laminar forced convection over rotating bodies*, Trans. ASME J. Heat Transfer, **96**, 463-466, 1974.

4. C. Cremers and D. L. Finley, *Natural convection about isothermal sphere*, In: Fourth Int. Heat Transfer Conference, Paris-Versailles , **4**, Paper No. 1.5, Elsevier, Amsterdam 1970.

5. M. H. Lee, D. R. Jeng and K. G. De Witt, *Laminar boundary layer transfer over rotating bodies in forced flow*, Trans. ASME J. Heat Transfer, **100**, 496-502, 1978.

6. A. A. KRANSE and J. SCHENK, *Thermal free convection from a solid sphere*, Appl. Sci. Res. **A15**, 397-403, 1966.

7. T. CHIANG, A. OSSIN and C. L. TIEN, *Laminar free convection from a sphere*, Trans. ASME J. Heat Transfer, **84**, 537-542, 1964.

8. T. S. CHEN and A. MUCOGLU, *Analysis of mixed, forced and free convection about a sphere*, Int. J. Heat Mass Transfer, **20**, 867-875, 1977.

9. A. MUCOGLU and T. S. CHEN, *Mixed convection about a sphere with uniform surface heat flux*, Trans. ASME J. Heat Transfer, **100**, 542-544, 1978.

10. H. J. MERCK, *Rapid calculation for boundary layer transfer for wedge solutions and asymptotic expansions*, J. Fluid Mech., **5**, 460-480, 1959.

11. B. T. CHAO and R. O. FAGBENLE, *On Merk's method of calculating boundary layer transfer*, Int. J. Heat Mass Transfer, **17**, 223-240, 1974.

12. A. SUWONO, *Buoyancy effects on flow and heat transfer on rotating axisymmetric round-nosed bodies*, Int. J. Heat Mass Transfer, **23**, 819-831, 1980.

13. M. A. HOSSAIN, S. K. DAS and I. POP, *MHD free convection flow near rotating axisymmetric round-nosed bodies*, Magnitnaya Gidrodinamika, **2**, 68-73, 1996.

14. R. D. CESS, *Interaction of thermal radiation with free convection heat transfer*, Int. J. Heat Mass Transfer, **9**, 1269-1277, 1966.

15. V. S. ARPACI, *Effect of thermal radiation on the laminar free convection from a heated vertical plate*, Int. J. Heat Mass Transfer, **11**, 871-881, 1968.

16. E. H. CHENG and M. N. OZISIK, *Radiation with free convection in an absorbing, emitting and scattering medium*, Int. J. Heat Mass Transfer, **15**, 1243-1252, 1972.

17. S. HASEGAWA, R. ECHIGO and K. FAKUD, *Analytic and experimental studies on simultaneous radiative and free convective heat transfer along a vertical plate*, Proc. Japanese Soc. Mech. Engng., **38** 2873-2883, 1972 and **39**, 250-257, 1973.

18. J. D. BANKSTON, J. R. LOYD and J. L. NOVOTNY, *Radiation convectioninteraction in an absorbing-emitting liquid in natural convection boundary layer flow*, Trans. ASME J. Heat Transfer, **99**, 125-127, 1977.

19. E. M. SPARROW and R. D. CESS, *Radiation heat transfer-augmented media*, Int. J. Heat Mass Transfer, **5**, 179-806, 1962.

20. R. D. CESS, *The interaction of thermal radiation in boundary layer heat transfer*, [in:] Proc. Third Int. Heat Transfer Conference, **5**, 154-163, 1966.

21. M. A. HOSSAIN and H. S. TAKHAR, *Radiation effect on mixed convection along a vertical plate with uniform surface temperature*, Heat and Mass Transfer, **31**, 243-248, 1966.

22. M. A. HOSSAIN, D. A. S. REES and I. POP, *Free convection-radiationinteraction from an isothermal plate inclined at a small angle to the horizontal*, Acta Mechanica, **127**, 63-73, 1998.

23. M. A. HOSSAIN and M. A. AALIM, *Natural convection-radiation interaction on boundary layer flow along a thin vertical cylinder*, Heat and Mass Transfer, **32**, 515-520, 1997.

24. T. CEBECI and P. BRADSHAW, *Physical and computational aspects of convective heat transfer*, Springer, New York 1994.

25. R. SIEGEL and J. R. HOWEL, *Thermal radiation heat transfer*, McGraw-Hill, New York 1987.

26. M. M. ALI, T. S. CHEN and B. F. ARMALY, *Natural convection-radiation interaction in boundary layer flow over horizontal surface*, AIAA J., **22**, 1797-1803, 1984.

27. E. M. SPARROW and H. S. YU, *Local non-similarity boundary layer solutions*, Trans. ASME J. Heat Transfer, **93**, 328-334, 1971.

28. M. A. HOSSAIN, N. BANU and A. NAKAYAMA, *Non-Darcy forced convection boundary layer flow over a wedge embedded in a saturated porous medium*, Num. Heat Transfer, Part A **26**, 399-414, 1994.

Received August 22, 1991; revised version January 6, 2002.