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On nonlinear waves in elastic conductors under a magnetic field

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A STUDY OF THE BEHAVIOR of magneto-elastic waves in a nonlinear isotropic elastic conductor by applying the method of multiple scales and perturbation has been made for a general displacement wave, under the action of an arbitrarily directed, uniform magnetic field. While in the case of transverse magnetic field the shock the waves are formed, it has been shown here that, under an oblique magnetic field, the wave is distorted without the formation of shocks.

1. Introduction

THE STUDY OF ONE-DIMENSIONAL waves in nonlinear elastic media were investigated by many authors such as NAYFEH [1], LARDNER [2,3]. They considered longitudinal and transverse waves and examined the growth of amplitude and formation of shock waves, using the methods of perturbation and multiple scales. MAUGIN [4] considered the effect of a bias magnetic field on the problems of propagation of harmonic waves in hyperelastic dielectrics and in perfectly conducting nonlinear elastic conductor. HEFNI *et al.* [5] have studied general one-dimensional bulk waves in a non-linear magneto-elastic conductor. They considered both the linear and nonlinear waves from the general formulation of the constitutive equations. CHAKRABORTY [6] recently considered the problem of distortion of waves in a nonlinear magneto-elastic conductor wherein it has been shown that shock waves may be formed in a traveling wave signal, depending on the elastic coefficients and the magnetic field, the direction of the bias magnetic field being transverse to the direction of the wave propagation. In the present paper we consider the bias magnetic field to have an arbitrary direction. The method of perturbation and multiple scales have been used. Equations of motion of different orders have been obtained. Several particular cases have been considered. For the bias magnetic field oblique to the direction of wave propagation, it is seen here that no shocks are formed.

2. Basic equations

Maxwell's equations of the electromagnetic field in which the displacement current has been neglected are, in the usual notations, the following:

$$\begin{aligned} \operatorname{div}\mathbf{B} &= 0, \\ \operatorname{div}\mathbf{D} &= \rho_e, \\ \operatorname{curl}\mathbf{H} &= J, \\ \operatorname{curl}\mathbf{E} + \mathbf{B_t} &= 0, \end{aligned} \tag{2.1}$$

(ρ_e is the electrostatic charge density). The constitutive equations of the medium are taken as

$$\begin{aligned} \mathbf{B} &= \mu\mathbf{H}, \\ \mathbf{D} &= \varepsilon_1\mathbf{E} \end{aligned} \tag{2.2}$$

while Ohm's law in the generalized form is

$$\mathbf{J} = \sigma\left[\mathbf{E} + \mathbf{u_t} \times \mathbf{B}\right] \tag{2.3}$$

σ being the electrical conductivity, $\mathbf{u}_t \left(\equiv \dfrac{\partial \mathbf{u}}{\partial t}\right)$ being velocity of the material point of the medium.

The motion of the medium is governed by the stress equations of motion by including the Lorentz force of electromagnetic origin:

$$L_{ij}\,,\,j + (\mathbf{J}\times\mathbf{B})_i = \rho_0 \frac{\partial^2 u_i}{\partial t^2}, \qquad i = 1,2,3\,, \tag{2.4}$$

σ is the mass per unit volume in the undeformed state and L_{ij} is the PIOLA-KIRCHHOFF tensor derived from the strain energy W per unit volume [BLAND (7)], given by

$$L_{ij} = \frac{\partial W}{\partial u_{i,j}}, \qquad i,j = 1,2,3\,. \tag{2.5}$$

The expression for W for a nonlinear elastic solid is taken in the form

$$W = \frac{1}{2}\lambda I_1 + G I_2 + \alpha I_1^3 + \beta I_1 I_2 + \gamma I_3, \tag{2.6}$$

where λ, G correspond to elastic constants in the linear theory, and α, β, γ are higher order elastic coefficients, I_1, I_2, I_3 are the three independent strain–invariants given by

$$I_1 = e_{ii}, \quad I_2 = e_{ij}e_{ij}, \quad I_3 = e_{ij}e_{jk}e_{ki}, \qquad i,j,k = 1,2,3\,, \tag{2.7}$$

while the strain tensor e_{ij} in terms of displacement $u_i(x_1, x_2, x_3, t)$ is given by

(2.8) $$e_{ij} = \left(\frac{1}{2}\right)(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}), \qquad i, j, k = 1, 2, 3.$$

The term $\mathbf{J} \times \mathbf{B}$ in (2.4) is the Lorentz force per unit volume due to the magnetic field $\mathbf{B}$ and the current density $\mathbf{J}$.

2.1. Formulation

The displacement wave travels in the x_1 direction and the medium is acted upon by a uniform bias magnetic field $\mathbf{H}^0$ in an arbitrary direction. Let us choose the x_2-axis such that $\mathbf{H}^0$ lies in the x_1x_1 plane (this is always possible whatever would be the direction of $\mathbf{H}^0$). Hence we have

(2.9) $$\mathbf{H}^0 = (H_1^0, H_2^0, 0).$$

The perturhed magnetic field is

(2.10) $$\mathbf{H} = \mathbf{H}^0 + \mathbf{h}$$

where

(2.11) $$\mathbf{h} = (h_1, h_2, h_3).$$

For a spatially one-dimensional problem, we write equation (2.10) as

(2.12) $$\mathbf{H} = H_1, H_2, H_3$$

and

(2.13) $$H_i = H_i^0 + h_i(x_1, t,) \qquad i = 1, 2, 3.$$

The displacement components (u_1, u_2, u_3) are functions of x_1 and time t only. The equations of motion (2.4) are simplified to

(2.14) $$\frac{\partial}{\partial x_1} L_{i1} + (\mathbf{J} \times \mathbf{B})_i = \rho_0 \frac{\partial^2 u_i}{\partial t^2}, \qquad i = 1, 2, 3.$$

From Eqs. (2.5) and (2.6),

(2.15) $$L_{11} = \frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial u_{1,1}} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial u_{1,1}} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial u_{1,1}}$$
$$= \left(\lambda I_1 + 3\alpha I_1^2 + \beta I_2\right)(1 + u_{1,1}) + (G + \beta I)\,(2e_{11}(1 + u_{1,1}))$$
$$+ \gamma\left[\left(3e_{11}^2 + 3e_{12}^2 + 3e_{13}^2\right)(1 + u_{1,1})\right],$$

$$(2.16)\quad L_{21} = \frac{\partial W}{\partial u_{2,1}} = \left(\lambda I_1 + 3\alpha I_1^2 + \beta I_2\right)\ u_{2,1} + (G + \beta I_1)\ (2e_{11}u_{2,1} + 2e_{12}) + \gamma\left[3e_{11}^2 + 3e_{12}^2 + 3e_{13}^2 + \left(\frac{3}{2}\right)e_{11}\right]u_{2,1},$$

$$(2.17)\quad L_{31} = \left(\lambda I_1 + 3\alpha I_1^2 + \beta I_2\right)\ u_{3,1} + (G + \beta I_1)\ (1 + 2e_{11})\ u_{3,1} + \gamma\left[\left(3e_{11}^2 + 3e_{12}^2 + 3e_{13}^2\right) + \frac{3}{2}e_{11}\right]u_{3,1}.$$

The Lorentz force-components in our problem reduce to

$$(2.18)\qquad (\mathbf{J}\times\mathbf{B})_1 = -\mu h_3 h_{3,1} - \mu\ \left(H_2^0 + h_2\right) + h_{2,1},$$

$$(2.19)\qquad (\mathbf{J}\times\mathbf{B})_2 = \mu\ \left(H_1^0 h_{2,1} + h_1 h_{2,1}\right),$$

$$(2.20)\qquad (\mathbf{J}\times\mathbf{B})_3 = \mu h_{3,1}\ \left(H_1^0 + h_1\right),$$

where

$$(2.21)\qquad h_{2,1} = \frac{\partial h_2}{\partial x_1}, \qquad h_{3,1} = \frac{\partial h_3}{\partial x_1}.$$

For a perfectly conducting solid, we get from equations $(2.1)_4$ and (2.3):

$$(2.22)\qquad \frac{\partial}{\partial t}\mathbf{h} = \operatorname{curl}(\mathbf{u}_t \times \mathbf{H})$$

But curl $(\mathbf{u}_t \times \mathbf{H})$ in our problem has the components

$$(2.23)\qquad \begin{aligned} &0,\ u_{1_{tx_1}}H_2 - u_{1_t}h_{2_{x_1}} + u_{2_{tx_1}}H_1 + u_{2_t}h_{1_{x_1}} \\ &u_{3_{tx_1}}H_1 + u_{3_t}h_{1_{x_1}} - u_{1_t}h_{3_{x_1}} \end{aligned}$$

where

$$(2.24)\qquad u_{1_t} = \frac{\partial u_1}{\partial t}, u_{1_{tx_1}} = \frac{\partial^2 u_1}{\partial t\partial x_1} \quad \text{etc.}$$

From Maxwell's equation $(2.1)_1$ and the component of (2.22), we get

$$(2.25)\qquad \frac{\partial h_1}{\partial x_1} = 0 \quad \text{and} \quad \frac{\partial h_1}{\partial t} = 0$$

We conclude from (2.25) that

$$h_1 = 0. \tag{2.26}$$

For applying the method of perturbation, parameter ε is used to denote the order of magnitude of the displacement. We also use the slowness parameters ξ, η defined by

$$\xi = \varepsilon x_1, \ \eta = \varepsilon^2 x_1 . \tag{2.27}$$

The displacement components can now written as

$$u_i(x_1,t) = \varepsilon\, u_{i0}(x_1,t,\xi,\eta) + \varepsilon^2 u_{i1}(x_1,t,\xi,\eta) + \varepsilon^3 u_{i2}(x_1,t,\xi,\eta) + 0\left(\varepsilon^4\right), \quad i = 1,2,3 . \tag{2.28}$$

Since the magnetic perturbation arises from the motion, h_2, h_3 are of the order of ε. We write

$$h_\alpha(x_1,t) = \varepsilon\, h_{\alpha 0}(x_1,t,\xi,\eta) + \varepsilon^2 h_{\alpha 1}(x_1,t,\xi,\eta) + \varepsilon^3 h_{\alpha 2}(x_1,t,\xi,\eta) + 0\left(\varepsilon^4\right) \quad \alpha = 2,3 . \tag{2.29}$$

As a consequence of the introduction of the slowness parameters ξ, η as variables in the functions $u_{10,}u_{11}$ etc., the partial derivative $\dfrac{\partial}{\partial x_1}$ in the equations (2.1) to (2.21) is to be replaced by the operator

$$\frac{\partial}{\partial x_1} + \varepsilon\frac{\partial}{\partial \xi} + \varepsilon^2\frac{\partial}{\partial \eta}. \tag{2.30}$$

Substituting in (2.22) the expressions from equations (2.23), (2.28) and (2.29) and equating the terms with different powers of ε, on integrating w.r.t. time, we obtain

$$h_{20} = -u_{10}H_2^0 + u_{20_{x_1}} H_1^0 \tag{2.31}$$

$$h_{21} = -H_2^0 u_{11_{x_1}} - H_2^0 u_{10_\xi} + H_1^0 u_{20_\xi} + H_1^0 u_{21_{x_1}} + H_2^0 \int \left(u_{10_t} u_{10_{x_1}}\right)_{x_1} dt - H_1^0 \int \left(u_{10_t} u_{20_{x_1}}\right)_{x_1} dt \tag{2.32}$$

$$h_{22} = -H_1^0 u_{12_{x_1}} + H_2^0 u_{11_\xi} + H_1^0 u_{22_{x_1}} + H_1^0 u_{21_\xi} + H_1^0 u_{20_\eta} + \int F dt \tag{2.33}$$

where

$$F = -u_{10_{tx_1}} h_{21} - h_{20} u_{11_{tx_1}} - h_{20} u_{10_{\xi\xi}} + u_{20_{tx_1}} - h_{21_{x_1}} u_{10_t} - u_{11_t} h_{20_{x_1}} - h_{20_\xi} u_{10_t},$$

$$h_{30} = H_1^0 u_{30_{x_1}}, \tag{2.34}$$

$$h_{31} = -h_{30} u_{10_{x_1}} + H_1^0 u_{30_\xi} + H_1^0 u_{31_{x_1}} + H_1^0 \int u_{10_{x_1}} u_{30_{tx_1}} dt, \tag{2.35}$$

$$h_{32_t} = H_1^0 u_{30_{t\eta}} + H_1^0 u_{32_{tx_1}} + H_1^0 u_{31_{t\xi}} - (u_{11_t} h_{30})_{x_1} - (u_{10_t} h_{30})_\xi - (u_{10_t} h_{31})_{x_1}. \tag{2.36}$$

On substituting from equations (2.15) to (2.20), (2.28) to Eq. (2.35) in each of the three equations of (2.14), and then equating the coefficients of $\varepsilon, \varepsilon^2$ and ε^3 on both sides, we get the following equations:

$$0(\varepsilon) : D_1 u_{10} = \frac{P_2}{2} u_{20_{x_1 x_1}}, \tag{2.37}$$

$$0(\varepsilon^2) : D_1 u_{11} - D_3 u_{21} = P_1 u_{10_{x_1\xi}} + P_2 u_{20_{x_1\xi}} + T_1, \tag{2.38}$$

$$0(\varepsilon) : D_2 u_{20} = \frac{P_2}{2} u_{10_{x_1 x_1}}, \tag{2.39}$$

$$0(\varepsilon^2) : D_2 u_{21} - D_3 u_{11} = P_2 u_{20_{x_1\xi}} + P_3 u_{10_{x_1\xi}} + T_2, \tag{2.40}$$

$$\begin{aligned} 0(\varepsilon^3) : D_2 u_{22} = {} & P_2 u_{21_{x_1\xi}} + P_2 u_{20_{x_1\eta}} + \frac{P_3}{2} u_{11_{x_1\xi}} + \frac{P_2}{2} u_{20_{\xi\xi}} \\ & - c_3^2 (u_{20_\xi} u_{10_{x_1}} + u_{10_{x_1}} u_{21_{x_1}} + u_{10_\xi} u_{20_{x_1}} + u_{11_{x_1}} u_{20_{x_1}})_{x_1} \\ & - \frac{P_3}{2} \left(u_{10_{x_1\eta}} + u_{12_{x_1x_1}} \right) - \frac{P_3}{2} \int \left(u_{10_t} u_{10_{x_1}} \right)_{x_1} dt + a_1^2 \int \left(u_{10_t} u_{20_{x_1}} \right)_{x_1\xi} dt \\ & - \frac{\mu H_1^0}{P_0} \int F_x dt - c_4^2 \left(u_{20_{x_1}} u_{10_{x_1}}^2 + u_{20_{x_1}}^3 + u_{30_{x_1}}^2 u_{20_{x_1}} \right)_{x_1}. \end{aligned} \tag{2.41}$$

$$0(\varepsilon): D_2 u_{30} = 0, \tag{2.42}$$

$$0(\varepsilon^2): D_2 u_{31} = -P_2 u_{30_{x_1\xi}} + T_3, \tag{2.43}$$

where

$$D_1 \equiv \left(c_1^2 + a_2^2\right) \frac{\partial^2}{\partial x_1 \partial x_1} - \frac{\partial^2}{\partial t^2}, \tag{2.44}$$

$$D_2 \equiv \left(c_2^2 + a_1^2\right) \frac{\partial^2}{\partial x_1 \partial x_1} - \frac{\partial^2}{\partial t^2}, \tag{2.45}$$

$$D_3 \equiv a_1 a_2 \frac{\partial^2}{\partial x_1 \partial x_1}, \tag{2.46}$$

$$P_1 = -2\left(c_1^2 + a_2^2\right), \tag{2.47}$$

$$P_2 = -2\left(c_2^2 + a_1^2\right), \tag{2.48}$$

$$P_3 = 2a_1 a_2, \tag{2.49}$$

$$\begin{aligned} T_1 = &-\left(2c_2^2 + 2c_4^2 - a_2^2\right) u_{10_{x_1}} u_{10_{x_1x_1}} - \left(c_3^2 - a_1^2\right) \\ &\cdot \left(u_{20_{x_1x_1}} u_{20_{x_1}} + u_{30_{x_1x_1}} u_{30_{x_1}}\right) - a_1 a_2 \left(u_{10_{x_1x_1}} u_{20_{x_1}} + u_{10_{x_1}} u_{20_{x_1x_1}}\right) \\ &+ a_2^2 \int \left(u_{10_t} u_{10_{x_1}}\right)_{x_1x_1} dt - a_1 a_2 \int \left(u_{10_t} u_{20_{x_1}}\right)_{x_1x_1} dt, \end{aligned} \tag{2.50}$$

$$\begin{aligned} T_2 = &-c_3^2 \left(u_{10_{x_1}} u_{20_{x_1}}\right)_{x_1} - a_1 a_2 \int \left(u_{10_t} u_{10_{x_1}}\right)_{x_1x_1} dt \\ &+ a_1^2 \int \left(u_{10_t} u_{20_{x_1}}\right)_{x_1x_1} dt, \end{aligned} \tag{2.51}$$

$$\begin{aligned} T_3 = &\, a_1^2 \left(u_{10_{x_1x_1}} u_{30_{x_1}} + u_{30_{x_1x_1}} u_{10_{x_1}}\right) - c_3^2 \left(u_{10_{x_1}} u_{30_{x_1}}\right)_{x_1} \\ &+ a_1^2 \int \left(u_{10_t} u_{30_{x_1}}\right)_{x_1x_1} dt. \end{aligned} \tag{2.52}$$

$$c_1^2 = (\lambda + 2G)/_{\rho_0}, \quad c_2^2 = {}^{G}/_{\rho_0}, \tag{2.53}$$

$$c_3^2 = (\lambda + 2G + \beta + 3\gamma/2)/_{\rho_0}, \tag{2.54}$$

$$c_4^2 = (\lambda/2 + G + 3\alpha + 3\beta + 3\gamma)/_{\rho_0}, \tag{2.55}$$

$$a_1^2 = \mu H_1^{0^2} \Big/ _{\rho_0}, \quad a_2^2 = \mu H_2^{0^2} \Big/ _{\rho_0}. \tag{2.56}$$

Here c_1, c_2 are the P and S wave velocities in linear elasticity, while a_1, a_2 are the Alfven wave velocities. We know that $c_1 > c_2$ while a_1 and a_2 are much less than c_2. We consider the cases :

CASE 1. $a_1 \neq 0, a_2 \neq 0$. This occurs when the magnetic field H^0 is oblique i.e. it has non-zero components H_1^0 and H_2^0. From equations (2.37), (2.39) and (2.41) we notice that u_{10} and u_{20} satisfy two coupled equations while u_{30} satisfies a single equation. For linear wave solutions in the form

$$u_{10} = A_{10} \exp\left[ik\left(x - Vt\right)\right], \tag{2.57}$$

$$u_{20} = A_{20} \exp\left[ik\left(x - Vt\right)\right], \tag{2.58}$$

$$u_{30} = A_{30} \exp\left[ik\left(x - \sqrt{c_2^2 + a_1^2 t}\right)\right], \tag{2.59}$$

the velocity V is a solution of the biquadratic equation

$$V^4 - V^2\left(c_1^2 + c_2^2 + a_1^2 + a_2^2\right) + \left(c_1^2 + a_2^2\right)\left(c_2^2 + a_1^2\right) - a_1^2 a_2^2 = 0. \tag{2.60}$$

The two solutions, say V_1, V_2, are given by

$$V_{1,2} = \left[\left(c_1^2 + c_2^2 + a_1^2 + a_2^2\right) \Big/ 2 \pm \frac{1}{2}\left(\left(c_1^2 + a_2^2 - c_2^2 - a_1^2\right)^2 + 4a_1^2 a_2^2\right)^{1/2}\right]^{1/2} \tag{2.61}$$

Since $c_1^2 > c_2^2$, it is possible that $c_1^2 + a_2^2 > c_2^2 + a_1^2$. Then we get from (2.61) the bounds for V_1, V_2 :

$$c_1^2 + a_2^2 < V_1^2 < c_1^2 + a_2^2 + a_1 a_2 \tag{2.62}$$

and

$$c_2^2 + a_1^2 - a_1 a_2 < V_2^2 < c_2^2 + a_1^2. \tag{2.63}$$

One effect of the oblique magnetic field is therefore that there exist two possible waves, one travelling with velocity greater than $\sqrt{c_1^2 + a_2^2}$, while the other has the velocity less than $\sqrt{c_2^2 + a_1^2}$.

To assess the nonlinear wave components we notice that u_{11}, u_{21} here are involved in two equations (2.38) and (2.40). A comparison of these equations with the case treated in [6] shows that distortion of the wave

$$u_{10} = A_{10}\left(\xi, \eta\right) e^{ik(x - V_1 t)}, \tag{2.64}$$

$$u_{20} = A_{20}\left(\xi, \eta\right) e^{ik(x - V_1 t)}, \tag{2.65}$$

with distance takes place, but no shock is formed since V_1 or V_2 are different from $\sqrt{c_1^2 + a_2^2}$ and $\sqrt{c_2^2 + a_1^2}$. We therefore conclude that the presence of an oblique magnetic field prevents the formation of shock in the propagation of a magnetoelastic wave.

Case 2. $a_1 = 0,\ a_2 \neq 0$, i.e. $H_1^0 = 0,\ H_2^0 \neq 0$. This case has been treated by Chakraborty [6] in which the possibility of a shock for a longitudinal wave was studied in detail.

Case 3. $a_2 = 0,\ a_1 \neq 0$, i.e, $H_2^0 = 0,\ H_1^0 \neq 0$. In this case the longitudinal wave u_{10} is a purely elastic one, while the transverse wave u_{20} travels with velocity $\left(c_2^2 + a_1^2\right)^{\frac{1}{2}}$ Taking $u_{10} = u_{11} = u_{12} = u_{30} = 0$, i.e. assuming that no elastic longitudinal wave propagates, equation (2.41) gives, ignoring the slowness parameter ξ as it has no effect here,

$$D_2 u_{22} = P_2 u_{20\eta x_1} - c_4^2 \left(u_{20_{x_1}}^3\right)_{x_1}. \tag{2.66}$$

On substituting in (2. 66)

$$u_{20} = G\left(\theta, \eta\right) \tag{2.67}$$

with

$$\theta = t - x_1 \Big/ \sqrt{c_2^2 + a_1^2}\,, \tag{2.68}$$

$$\phi = t + x_1 \Big/ \sqrt{c_2^2 + a_1^2} \tag{2.69}$$

being a solution of equation (2.39) for a progressive transverse wave, equation (2.66) becomes

$$u_{22\theta\phi} = -\frac{\sqrt{c_2^2+a_1^2}}{2}G_{\theta\eta} + \frac{3}{4}\frac{c_4^2}{\left(c_2^2+a_1^2\right)^2}G_\theta^2 G_{\theta\theta}. \tag{2.70}$$

Integrating (2.70) w. r. t. θ, we get

$$u_{22\phi} = -\frac{\sqrt{c_2^2+a_1^2}}{2}G_{\eta} + \frac{1}{4}\frac{c_4^2}{\left(c_2^2+a_1^2\right)^2}G_\theta^3. \tag{2.71}$$

In order to be sure that integration of (2.71) w. r. t. ϕ gives an equation that keeps u_{22} finite, we get the secular equation

$$-\frac{\sqrt{c_2^2+a_1^2}}{2}G_{\eta} + \frac{c_4^2}{4\left(c_2^2+a_1^2\right)^2}G_\theta^3 = 0. \tag{2.72}$$

Differentiating (2.72) w. r. t. θ and putting $G_\theta = g$, we get the equation

$$g_\eta - M_1 g^2 g_\theta^2 = 0 \tag{2.73}$$

where

$$M_1 = \frac{3c_4^2}{2\left(c_2^2+a_1^2\right)^{\frac{5}{2}}} . \tag{2.74}$$

The solution of (2.73) corresponding to the initial condition

$$g(\theta,0) = m(\theta) \tag{2.75}$$

$$g(\theta,\eta) = m(\theta_1) \tag{2.76}$$

where θ_1 is given by (WHITHAM [8])

$$\theta_1 = \theta + M_1 m^2(\theta_1)\,\eta. \tag{2.77}$$

Equation (2.77) shows that the magneto elastic quasi-transverse wave is distorted and shock waves are possible under suitable initial condition (LARDNER [3]).

3. Conclusion

The effect of a bias magnetic field acting obliquely to the direction of a wave propagating in a nonlinear elastic medium is to prevent the shock formation of waves with displacement in its plane.

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