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Hall effect on thermosolutal instability of Walters' (model B′) fluid in porous medium

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THE THERMOSOLUTAL INSTABILITY of Walters' (model B′) fluid in porous medium is considered in the presence of uniform vertical magnetic field to include the effect of Hall currents. For the case of stationary convection, the stable solute gradient and magnetic field have stabilizing effects on the system, whereas the Hall currents have destabilizing effect on the system. The medium permeability has both stabilizing and destabilizing effects on the system depending on the Hall parameter M. The kinematic viscoelasticity has no effect for stationary convection. The kinematic viscoelasticity, stable solute gradient and magnetic field (and the corresponding Hall currents) introduce oscillatory modes in the system, which were non-existent in their absence. The sufficient conditions for the non-existence of overstability are also obtained.

Key words: Hall effect, thermosolutal instability, Walters' (model B′) fluid, porous medium.

1. Introduction

THE THERMAL CONVECTION in an electrically conducting, Newtonian fluid layer in the presence of magnetic field has been discussed in detail in the celebrated monograph by CHANDRASEKHAR [1]. BHATIA and STEINER [2] have studied the thermal instability of a Maxwellian viscoelastic fluid in the presence of magnetic field while the thermal convection in Oldroydian viscoelastic fluid in hydromagnetics has been studied by SHARMA [3]. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by VERONIS [4]. The physics is quite similar in the stellar case in which helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of single component fluid and rigid boundaries and therefore, it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The problem is of great importance because of its application to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of elastico-viscous fluids is Walters' fluid (model B′). SHARMA and KUMAR [5] have studied the steady flow and heat transfer of Walters' fluid (model B′) through a porous pipe of uniform circular cross-section with small suction.

In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in a book by PHILLIPS [6]. When the fluid permeates a porous material, the gross effect is represented by the Darcy law. As a result of this macroscopic law, the usual viscous and viscoelastic terms in the equation of Walters' fluid (model B′) motion are replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\mathbf{q}\right]$, where μ and μ' are the viscosity and viscoelasticity of the Walters' fluid, k_1 is the medium permeability and $\mathbf{q}$ is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in a porous medium is of great importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dusty 'snowball' of a mixture of frozen gases, which in the process of their journey, changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in astrophysical context (MCDONNEL [7]). The Hall effect is likely to be important in many geophysical situations as well as in flow of laboratory plasma. SHERMAN and SUTTON [8] have considered the effect of Hall currents on the efficiency of a magneto-fluid-dynamic generator. There is a growing importance of non-Newtonian fluids in chemical technology, industry and geophysical fluid dynamics. The Hall currents have relevance and importance in geophysics, MHD generators and industry. In a recent study, SHARMA *et al.* [9] studied the instability of streaming Walters' viscoelastic fluid B′ in porous medium. More recently, the effect of rotation on thermosolutal instability of Walters' fluid (model B′) in porous medium has been studied by SHARMA *et al.* [10].

Keeping in mind the importance of non-Newtonian fluids in modern technology, and various applications mentioned above, the thermosolutal instability of a electrically conducting Walters' (model B′) fluid in porous medium in the presence of uniform vertical magnetic field to include the effect of Hall currents, has been considered in the present paper.

2. Formulation of the problem and perturbation equations

Here we consider an infinite, horizontal, incompressible Walters' (model B′) fluid layer of thickness d, heated and soluted from below so that the temperatures, densities and solute concentrations at the bottom surface $z = 0$ are T_0, ρ_0 and C_0, and at the upper surface $z = d$ are T_d, ρ_d and C_d, respectively, and that a uniform temperature gradient $\beta\,(= |dT/dz|)$ and a uniform solute gradient $\beta'\,(= |dC/dz|)$ are maintained. The gravity field $\mathbf{g}\,(0,0,-g)$ and a uniform vertical magnetic field $\mathbf{H}\,(0,0,H)$ pervade the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

Let $p, \rho, T, C, \alpha, \alpha', g, \eta, \mu_e$,N,e and $\mathbf{q}\,(u,v,w)$ denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration, resistivity, magnetic permeability, electron number density, charge of an electron and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of Walters' (model B′) fluid are

$$\begin{aligned}(2.1)\quad &\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)\,\mathbf{q}\right] = \\ &\qquad -\left(\frac{1}{\rho_0}\right)\nabla p + \mathbf{g}\left(1+\frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{\mu_e}{4\pi\rho_0}(\nabla\times\mathbf{H})\times\mathbf{H},\end{aligned}$$

$$(2.2)\qquad \nabla.\mathbf{q} = 0,$$

$$(2.3)\qquad E\frac{\partial T}{\partial t} + (\mathbf{q}.\nabla)\,T = \kappa\nabla^2 T,$$

$$(2.4)\qquad E'\frac{\partial C}{\partial t} + (\mathbf{q}.\nabla)\,C = \kappa'\nabla^2 C,$$

$$(2.5)\qquad \rho = \rho_0\left[1 - \alpha\,(T - T_0) + \alpha'\,(C - C_0)\right],$$

where the suffix zero refers to values at the reference level $z = 0$ and in writing Eq. (2.1), use has been made of the Boussinesq approximation. The magnetic permeability μ_e, the kinematic viscosity ν, the kinematic viscoelasticity ν', the thermal diffusivity κ and the solute diffusivity κ' are all assumed to be constants.

The Maxwell's equations yield

$$\varepsilon \frac{d\mathbf{H}}{\mathrm{dt}} = (\mathbf{H}.\nabla)\,\mathbf{q} + \varepsilon\eta\nabla^2\mathbf{H} - \frac{c\varepsilon}{4\pi Ne}\nabla \times [(\nabla \times \mathbf{H}) \times \mathbf{H}], \tag{2.6}$$

$$\nabla.\mathbf{H} = 0, \tag{2.7}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \varepsilon^{-1}\mathbf{q}.\nabla$ stands for the convective derivative.

Here $E = \varepsilon + (1-\varepsilon)\left(\frac{\rho_s c_s}{\rho_0 c_i}\right)$ is a constant and E$'$ is a constant analogous to E but corresponding to solute rather than heat. ρ_s, c_s and ρ_0, c_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The steady state solution is

$$\mathbf{q} = (0,0,0)\,, \quad T = -\beta z + T_0,$$

$$C = -\beta' z + C_0, \quad \rho = \rho_0\left(1 + \alpha\beta z - \alpha'\beta' z\right). \tag{2.8}$$

Here we use linearized stability theory and normal mode analysis method. Consider a small perturbation on the steady state solution and let δp, $\delta\rho$, θ, γ, $\mathbf{h}\,(h_x,\, h_y,\, h_z)$ and $\mathbf{q}\,(u,\, v,\, w)$ denote, respectively, the perturbations in pressure p, density ρ, temperature T, solute concentration C, magnetic field $\mathbf{H}\,(0,0,\mathrm{H})$ and velocity $\mathbf{q}\,(0,\,0,\,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\delta\rho = -\rho_0\left(\alpha\theta - \alpha'\gamma\right). \tag{2.9}$$

Then the linearized perturbation equations become

$$\frac{1}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t} = -\frac{1}{\rho_0}\left(\nabla\delta p\right) - \mathbf{g}\left(\alpha\theta - \alpha'\gamma\right) - \frac{1}{k_1}\left(\nu - \nu'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{\mu_e}{4\pi\rho_0}\left(\nabla \times \mathbf{h}\right) \times \mathbf{H}, \tag{2.10}$$

$$\nabla.\mathbf{q} = 0, \tag{2.11}$$

$$\mathrm{E}\frac{\partial\theta}{\partial \mathrm{t}} = \beta w + \kappa\nabla^2\theta, \tag{2.12}$$

(2.13)
$$E' \frac{\partial \gamma}{\partial t} = \beta' w + \kappa' \nabla^2 \gamma,$$

(2.14)
$$\varepsilon \frac{\partial \mathbf{h}}{\partial \mathrm{t}} = (\mathbf{H}.\nabla)\, \mathbf{q} + \varepsilon\eta \nabla^2 \mathbf{h} - \frac{c\varepsilon}{4\pi N e} \nabla \times [(\nabla \times \mathbf{h}) \times \mathbf{H}],$$

(2.15)
$$\nabla.\mathbf{h} = 0.$$

3. The dispersion relation

Decomposing the disturbances into normal modes, we assume that the perturbation quantities are of the form

(3.1)
$$[w, h_z, \theta, \gamma, \zeta, \xi] = [W(z), K(z), \Theta(z), \Gamma(z), Z(z), X(z)] \exp\left(ik_x x + ik_y y + nt\right),$$

where k_x, k_y are the wave numbers along the x- and y- directions respectively, $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wave number and 'n' is the growth rate which is, in general, a complex constant. $\zeta = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}$ and $\xi = \dfrac{\partial h_y}{\partial x} - \dfrac{\partial h_x}{\partial y}$ stand for the z-components of vorticity and current density, respectively.

Expressing the coordinates x, y, z in the new unit of length d and letting $a = kd$, $\sigma = \dfrac{nd^2}{\nu}$, $p_1 = \dfrac{\nu}{\kappa}$, $p_2 = \dfrac{\nu}{\eta}$, $q = \dfrac{\nu}{\kappa'}$, $F = \dfrac{\nu'}{d^2}$, $P_\ell = \dfrac{k_1}{d^2}$ and $D = \dfrac{d}{dz}$, Eqs. (2.10)-(2.15), using (3.1), yield

(3.2)
$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_\ell}(1 - \sigma F)\right] \left(D^2 - a^2\right) W + \frac{ga^2 d^2}{\nu} \left(\alpha\Theta - \alpha'\Gamma\right) - \frac{\mu_e H d}{4\pi\rho_0 \nu} \left(D^2 - a^2\right) DK = 0,$$

(3.3)
$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_\ell}(1 - \sigma F)\right] Z = \frac{\mu_e H d}{4\pi\rho_0\nu} DX,$$

(3.4)
$$\left(D^2 - a^2 - p_2\sigma\right) K = -\left(\frac{Hd}{\eta\varepsilon}\right) DW + \frac{cHd}{4\pi N e\eta} DX,$$

(3.5)
$$\left(D^2 - a^2 - p_2\sigma\right) X = -\left(\frac{Hd}{\eta\varepsilon}\right) DZ - \frac{cH}{4\pi N e\eta d} \left(D^2 - a^2\right) DK,$$

$$(D^2 - a^2 - Ep_1\sigma)\,\Theta = -\left(\frac{\beta d^2}{\kappa}\right) W, \tag{3.6}$$

$$(D^2 - a^2 - E'q\sigma)\,\Gamma = -\left(\frac{\beta' d^2}{\kappa'}\right) W. \tag{3.7}$$

Consider now the case when both boundaries are free and perfect conductors of both heat and solute concentration, while the adjoining medium is perfectly conducting. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which Eqs. (3.2)-(3.7) must be solved, are (CHANDRASEKHAR [1])

$$W = D^2W = 0,\ DZ = 0,\ \Theta = 0,\ \Gamma = 0, \quad \text{at z } = 0 \quad \text{and } 1, \tag{3.8}$$

$DX = 0$, $K = 0$ on a perfectly conducting boundary and $X = 0$, h_x, h_y, h_z are continuous with an external vacuum field on a non-conducting boundary.

The case of two free boundaries, though a little artificial, is the most appropriate for stellar atmospheres (SPIEGEL [11]). Using the above boundary conditions, it can be shown that all the even-order derivatives of W must vanish for $z = 0$ and 1, and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi\, z, \tag{3.9}$$

where W_0 is a constant.

Eliminating Θ, X, Z, Γ and K between Eqs. (3.2)-(3.7) and substituting the proper solution $W = W_0$ sin πz, in the resultant equation, we obtain the dispersion relation

$$R_1 = \left(\frac{1+x}{x}\right)\left[\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\left(1 - i\sigma_1\pi^2 F\right)\right][1 + x + iEp_1\sigma_1] + S_1\frac{(1 + x + iEp_1\sigma_1)}{(1 + x + iE'q\sigma_1)} + Q_1\frac{\mathbf{A}}{\mathbf{B}} \tag{3.10}$$

where

$$R_1 = \frac{g\alpha\beta d^4}{\nu\kappa\pi^4}, \quad S_1 = \frac{g\alpha'\beta' d^4}{\nu\kappa'\pi^4}, \quad Q_1 = \frac{\mu_e H^2 d^2}{4\pi\rho_0\nu\eta\varepsilon\pi^2}, \quad M = \left(\frac{cH}{4\pi Ne\eta}\right)^2,$$

$$x = \frac{a^2}{\pi^2}, \quad i\sigma_1 = \frac{\sigma}{\pi^2} \quad \text{and} \quad P = \pi^2 P_\ell.$$

$$\mathbf{A} = (1 + x)\,[1 + x + iEp_1\sigma_1]\left\{\left[\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\left(1 - i\sigma_1\pi^2 F\right)\right][1 + x + ip_2\sigma_1] + Q_1\right\}$$

$$\mathbf{B} = x\Big\{\Big[\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}(1 - i\sigma_1\pi^2 F)\Big]\Big[1 + x + ip_2\sigma_1\Big]^2 + Q_1\Big[1 + x + ip_2\sigma_1\Big] + M\Big(1 + x\Big)\Big[\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\Big(1 - i\sigma_1\pi^2 F\Big)\Big]\Big\}.$$

Equation (3.10) is the required dispersion relation including the effects of magnetic field, Hall currents, medium permeability, kinematic viscoelasticity and stable solute gradient on the thermosolutal instability of Walters' (model B′) fluid in porous medium in hydromagnetics in the presence of Hall currents.

4. The stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (3.10) reduces to

$$R_1 = \left(\frac{1+x}{x}\right) . \frac{\left(\frac{1+x}{P} + Q_1\right)^2 + \frac{M\,(1+x)}{P^2}}{\frac{1+x}{P} + Q_1 + \frac{M}{P}} + S_1, \tag{4.1}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters S_1, Q_1, M and P. The parameter F accounting for the kinematic viscoelasticity effect vanishes for the stationary convection.

To study the effects of stable solute gradient, magnetic field and medium permeability, we examine the natures of $\frac{dR_1}{dS_1}$, $\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dM}$ analytically. Eq. (4.1) yields

$$\frac{dR_1}{dS_1} = +1, \tag{4.2}$$

$$\frac{dR_1}{dQ_1} = \left(\frac{1+x}{x}\right)\frac{\left(\frac{1+x}{P} + Q_1\right)}{\left(\frac{1+x+M}{P} + Q_1\right)}, \tag{4.3}$$

and

$$\frac{dR_1}{dM} = -\frac{Q_1\,(1+x)}{x} . \frac{\left(\frac{1+x}{P} + Q_1\right)}{\left(\frac{1+x}{P} + Q_1 + \frac{M}{P}\right)^2}. \tag{4.4}$$

Thus for stationary convection, the stable solute gradient and magnetic field are found to have stabilizing effects, whereas the Hall currents have a destabilizing effect on the thermosolutal instability of Walters' (model B′) fluid in porous medium in hydromagnetics in the presence of Hall currents.

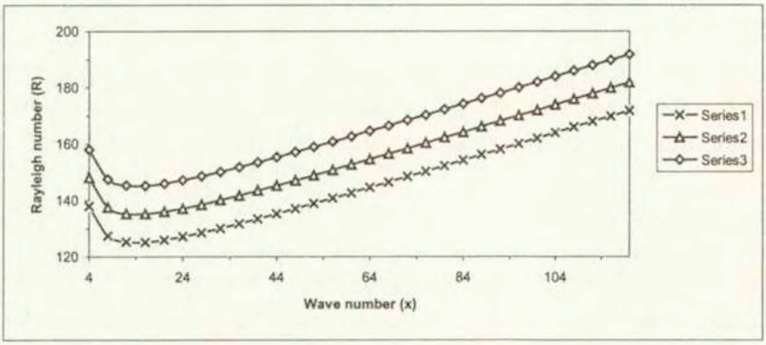

FIG. 1. The variation of Rayleigh number (R_1) with wave number (x) for $M = 0.1$, $Q_1=100$, $P = 2$; $S_1=10$ for Series 1, $S_1=20$ for Series 2 and $S_1=30$ for Series 3.

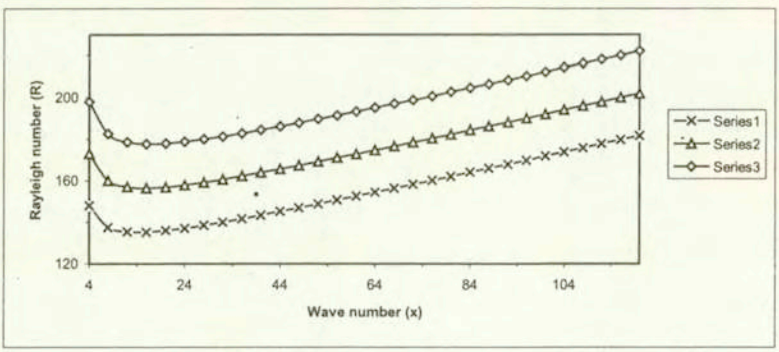

FIG. 2. The variation of Rayleigh number (R_1) with wave number (x) for $M = 0.1$, $S_1=20$, $P = 2$, $Q_1=100$ for Series 1, $Q_1=120$ for Series 2 and $Q_1=140$ for Series 3.

The dispersion relation (4.1) is analysed numerically. In Fig. 1, R_1 is plotted against x for $P = 2$, $Q_1=100$, $M = 0.1$ and $S_1=10$, 20 and 30. The stabilizing role of the stable solute gradient is clear from the increase of the Rayleigh number with increasing stable solute gradient parameter value. Figure 2 gives R_1 plotted against x for $P = 2$, $S_1=20$, $M = 0.1$ and $Q_1=100$, 120 and 140. Here we also find the stabilizing role of the magnetic field as the Rayleigh number increases with the increase in magnetic field parameter Q_1. In Fig. 3, R_1 is plotted against x for $Q_1=100$, $S_1=20$, $P = 2$ and $M = 10$, 50 and 100. Here the destabilizing role of the Hall currents is clear from the decrease of the Rayleigh number with increasing Hall currents parameter value.

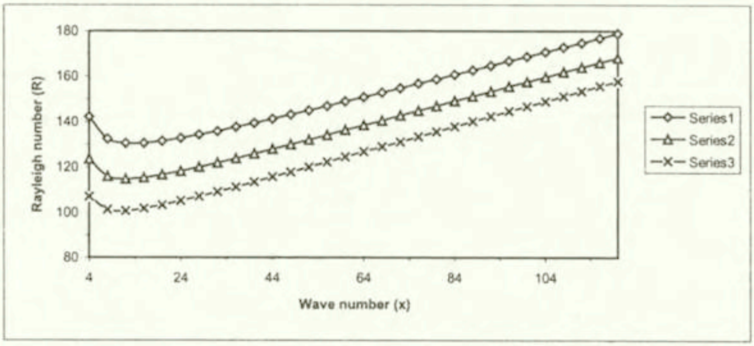

FIG. 3. The variation of Rayleigh number (R_1) with wave number (x) for Q_1=100, S_1=20, $P = 2$, $M = 10$ for Series 1, $M = 50$ for Series 2 and $M = 100$ for Series 3.

In order to investigate the effect of medium permeability, we examine the behaviour of $\frac{dR_1}{dP}$ analytically. Equation (4.1) yields

$$\frac{dR_1}{dP} = -\frac{(1+x)}{xP^2} \cdot \frac{\left(\frac{1+x}{P^2}\right)(1+x+M)^2 + \frac{2Q_1(1+x)(1+x+M)}{P} + Q_1^2(1+x-M)}{\left(\frac{1+x+M}{P} + Q_1\right)^2}. \tag{4.5}$$

which is negative. The medium permeability, therefore, has a destabilizing effect (Hall parameter M«1) on thermosolutal instability of Walters' (model B$'$) fluid in porous medium in hydromagnetics in the presence of Hall currents.

It has been shown graphically that for

- $Q_1 = 100, S_1 = 20, M = 0.1$ and $P = 1$, 3; the medium permeability has a destabilizing effect [Fig. 4].
- $Q_1 = 100, S_1 = 20, M = 10$ and $P = 1$, 3; the medium permeability has a stabilizing influence for x<7 and for x>7 they have a destabilizing effect [Fig. 5].
- $Q_1 = 100, S_1 = 20, M = 20$ and $P = 1$, 3; the medium permeability has a stabilizing influence for x<12.5 and for x>12.5 they have a destabilizing effect [Fig. 5].

It has also been shown that as Hall parameter M increases, the stabilizing range of medium permeability also increases.

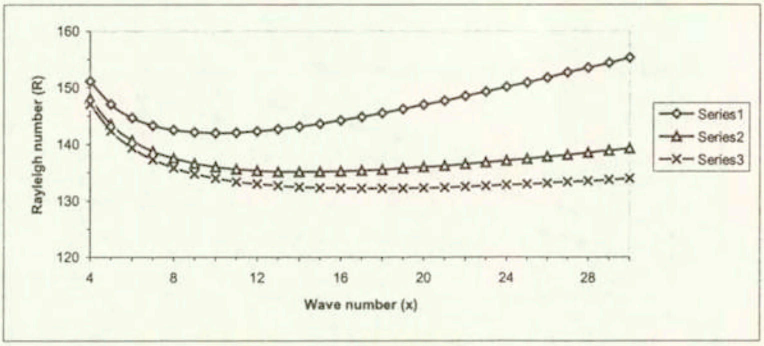

FIG. 4. The variation of Rayleigh number (R_1) with wave number (x) for Q_1=100, S_1=20, $M = 0.1$ and $P = 1$ for Series 1, $P = 2$ for Series 2 and $P = 3$ for Series 3.

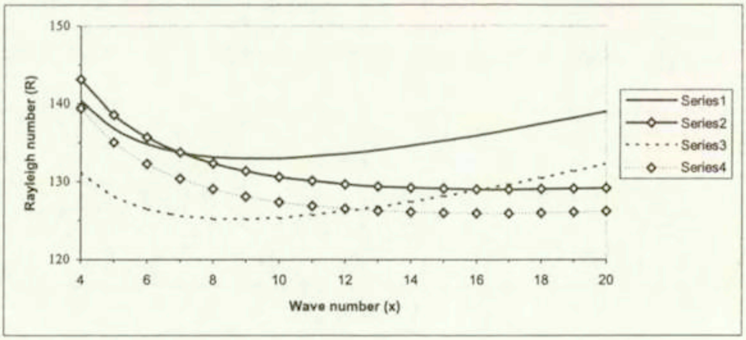

FIG. 5. The variation of Rayleigh number (R_1) with wave number (x) for $Q_1 = 100, S_1 = 20$; and $M = 10, P = 1$ for Series 1, $M = 10$, $P = 3$ for Series 2, $M = 20$, $P = 1$ for Series 3 and $M = 20, P = 3$ for Series 4.

5. Stability of the system and oscillatory modes

Here we examine the possibility of oscillatory modes, if any, on stability problem due to the presence of kinematic viscoelasticity, stable solute gradient and magnetic field. Multiplying (3.2) by $W*$, the complex conjugate of W, and using (3.3)-(3.7) together with the boundary conditions (3.8), we obtain

$$\text{(5.1)}\quad \left[\frac{\sigma}{\varepsilon}+\frac{1}{P_\ell}(1-\sigma F)\right]I_1+\left(\frac{g\alpha'\kappa' a^2}{\nu\beta'}\right)\left[I_4+E'q\sigma^* I_5\right]$$
$$+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}[I_6+p_2\sigma^* I_7]+\left(\frac{\mu_e\eta\varepsilon d^2}{4\pi\rho_0\nu}\right)[I_8+p_2\sigma I_9]$$
$$+d^2\left[\frac{\sigma^*}{\varepsilon}+\frac{1}{P_\ell}(1-\sigma^* F)\right]I_{10}-\left(\frac{g\alpha\kappa a^2}{\nu\beta}\right)[I_2+Ep_1\sigma^* I_3]=0,$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,\qquad I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,$$

$$I_3=\int_0^1\left(|\Theta|^2\right)dz,\qquad I_4=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz,$$

$$\text{(5.2)}\quad I_5=\int_0^1\left(|\Gamma|^2\right)dz,\quad I_6=\int_0^1\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz,$$

$$I_7=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,\qquad I_8=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz,$$

$$I_9=\int_0^1\left(|X|^2\right)dz,\qquad I_{10}=\int_0^1\left(|Z|^2\right)dz.$$

The integrals $I_1,...,I_{10}$ are all positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ and equating the real and imaginary parts of Eq. (5.1), we obtain

$$\text{(5.3)}\quad \left[\left(\frac{1}{\varepsilon}-\frac{F}{P_\ell}\right)I_1+\frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_5+\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu p_2}\left(I_7+d^2I_9\right)\right.$$
$$\left.+d^2\left[\frac{1}{\varepsilon}-\frac{F}{P_\ell}\right]I_{10}-\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_3\right]\sigma_r$$
$$=-\left[\frac{I_1}{P_\ell}+\frac{g\alpha'\kappa' a^2}{\nu\beta'}I_4+\frac{\mu_e\eta\varepsilon}{4}\pi\rho_0\nu\left(I_6+d^2I_8\right)+d^2\frac{1}{P_\ell}I_{10}-\frac{g\alpha\kappa a^2}{\nu\beta}I_2\right],$$

$$\left[\left(\frac{1}{\varepsilon}-\frac{F}{P_\ell}\right)I_1-\frac{g\alpha'\kappa' a^2}{\nu\beta'}E'qI_5-\frac{\mu_e\eta\varepsilon}{4\pi\rho_0\nu}p_2\left(I_7-d^2I_9\right)\right.$$
$$\left.-d^2\left(\frac{1}{\varepsilon}-\frac{F}{P_\ell}\right)I_{10}+\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_3\right]\sigma_i=0. \tag{5.4}$$

It is evident from (5.3) that σ_r is positive or negative. The system is, therefore, stable or unstable. It is clear from (5.4) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. In the absence of kinematic viscoelasticity, stable solute gradient and magnetic field (and hence Hall currents), equation (5.4) reduces to

$$\left[\frac{I_1}{\varepsilon}+\frac{g\alpha\kappa a^2}{\nu\beta}Ep_1I_3\right]\sigma_i=0, \tag{5.5}$$

and the terms in brackets are positive definite. Thus $\sigma_i=0$, which means that oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for a porous medium, in the absence of kinematic viscoelasticity, stable solute gradient and magnetic field (and hence Hall currents). This result is true for the porous as well as non-porous (CHANDRASEKHAR [1]) medium. The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, stable solute gradient and magnetic field (and hence Hall currents), which were non-existent in their absence.

6. The case of overstability

Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the critical Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (3.10) will admit the solutions with σ_1 real.

If we equate real and imaginary parts of (3.10) and eliminate R_1 between them, we obtain

$$A_4c_1^4+A_3c_1^3+A_2c_1^2+A_1c_1+A_0=0, \tag{6.1}$$

where we have put $c_1=\sigma_1^2$, $b=1+\text{x}$ and

$$A_4=E'^2q^2p_2^2\left(\frac{1}{\varepsilon}-\frac{\pi^2F}{P}\right)^2\left[\left(\frac{1}{\varepsilon}-\frac{\pi^2F}{P}\right)b+\frac{Ep_1}{P}\right], \tag{6.2}$$

$$A_3 = \left[\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^3 \left(2E'^2 q^2 + p_2^2\right)\right] b^3 \tag{6.3}$$

$$+ \left[\frac{Ep_1 p_2^2}{P}\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^2 + \frac{6E'^2 q^2 M \pi^2 F}{\varepsilon P}\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)\right.$$

$$\left. + 2E'^2 q^2 \left\{\frac{Ep_1}{P}\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^2 - \frac{M}{\varepsilon^3} + M\left(\frac{\pi^2 F}{P}\right)^3\right\}\right] b^2$$

$$+ \left[E'^2 q^2 p_2 \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)\left\{\frac{p_2}{P^2} - \frac{3Q_1}{\varepsilon} + \frac{3Q_1 \pi^2 F}{P}\right\}\right.$$

$$\left. + Ep_1 E'^2 q^2 \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^2 \left(Q_1 - \frac{2M}{P}\right)\right] b$$

$$+ p_2 \left[\frac{Ep_1 E'^2 q^2}{P}\left\{\frac{p_2}{P^2} - \frac{2Q_1}{\varepsilon} + \frac{2Q_1 \pi^2 F}{P}\right\} + S_1(b-1) p_2 \left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^2 \left(Ep_1 - E'q\right)\right].$$

Since σ_1 is real for overstability, the four values of $c_1 \left(= \sigma_1^2\right)$ are positive. The sum of roots of (6.1) is $-\dfrac{A_3}{A_4}$, and if this is to be negative, then $A_3 > 0$, $A_4 > 0$.

It is clear from (6.2) and (6.3) that A_3 and A_4 are always positive if

$$\frac{\pi^2 F}{P} < \frac{1}{\varepsilon}, \quad Ep_1 > E'q, \quad \frac{Ep_1}{P}\left(\frac{1}{\varepsilon} - \frac{\pi^2 F}{P}\right)^2 > \frac{M}{\varepsilon^3}, \quad p_2 > \frac{3Q_1 P^2}{\varepsilon} \tag{6.4}$$

$$\text{and} \quad Q_1 > \frac{2M}{P}$$

which imply that

$$\begin{gathered} \nu' < \frac{k_1}{\varepsilon}, \ E\frac{\nu}{\kappa} > E'\frac{\nu}{\kappa'}, \ E\frac{\nu}{\kappa} > \frac{k_1}{\varepsilon}\left[\frac{cHk_1}{4d\eta Ne\,(k_1 - \varepsilon\nu')}\right]^2, \\ \nu > \frac{k_1^2 \pi}{\varepsilon d^2}\left(\frac{3\mu_e H^2}{4\nu\rho_0}\right) \ \text{and} \ \nu < \frac{k_1}{\varepsilon}\left(\frac{Ne}{c}\right)^2\left(\frac{2\mu_e \pi\eta}{\rho_0}\right) \end{gathered} \tag{6.5}$$

i.e.

$$\nu' < \frac{k_1}{\varepsilon}, E\frac{\nu}{\kappa} > \max\left[E'\frac{\nu}{\kappa'}, \frac{k_1}{\varepsilon}\left\{\frac{cHk_1}{4d\eta Ne\,(k_1 - \varepsilon\nu')}\right\}^2\right] \tag{6.6}$$

and

$$\frac{k_1^2\mu_e}{\varepsilon\rho_o}\left(\frac{3H^2}{4d^2\nu}\right) < \nu < \frac{k_1\mu_e}{\varepsilon\rho_0}\left(\frac{Ne}{c}\right)^2 2\pi\eta.$$

Thus

$$\nu' < \frac{k_1}{\varepsilon}, E\frac{\nu}{\kappa} > \max\left[E'\frac{\nu}{\kappa'}, \frac{k_1}{\varepsilon}\left\{\frac{cHk_1}{4d\eta Ne\,(k_1 - \varepsilon\nu')}\right\}^2\right]$$

and

$$\frac{k_1^2\mu_e}{\varepsilon\rho_o}\left(\frac{3H^2}{4d^2\nu}\right) < \nu < \frac{k_1\mu_e}{\varepsilon\rho_0}\left(\frac{Ne}{c}\right)^2 2\pi\eta$$

are the sufficient conditions for the non-existence of overstability.

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