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The stored energy in metals and the concept of residual microstresses in plasticity

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DIFFERENT MEZOMECHANICAL models were analysed in an attempt to estimate how large is the portion of elastic energy stored in plastically deformed polcrystalline metals which may be attributed to the residual microstresses, its remaining part being connected with the creation of defects inside the particular crystallites. It has been indicated that durring complex plastic deformation of a metal, the portion of elastic energy stored due to the residual microstresses may be estimated with the use of the kinematical strain hardening law, in which such stresses play an important role. Such an approach is supported by results of a simple experiment.

1. Introduction

WHEN METALS UDERGO plastic deformations, a part of expended energy reappears in the form of heat, and the remaining part is retained inside the metal. The latter part is called "latent energy" or alternatively, the "stored energy". This phenonomenon was experimentally studied in the classical work by W. S. FARREN and G. I. TAYLOR in 1925 [1], and in the following papers by G. I. TAYLOR and H. QUINNEY [2, 3]. In these and in other early experiments, the latent energy in metals after plastic deformation was found to lie between 5 and 15 percent of the work done by external forces during the deformation process (cf. [4]). However, recently (see e.g. [5, 6, 7, 8]) it has been demonstrated that in some cases it can reach the amount of up to 50-60 percent in the initial stage of plastic deformation as shown for example in Fig. 1 (see [6]). The relation E_s/E_w, where E_s is the stored energy and E_w stands for the work of plastic deformation, exhibits a distinct maximum at the relatively small permanent deformation.

Usually it assumed that the stored energy is connected with the creation of crystal defects, mainly dislocations, in the crystallites of the metal structure (cf.e.g. [6, 7, 8].) However, metals subjected to complex plastic deformations display the so-called generalized Bauschinger effect connected with the residual microstresses remaining in their internal structure after the load is removed. Associated with these microstresses is the elastic energy remaining in a metal after plastic deformation. In order to assess this part of the whole stored energy, simple mezomechanical models will be used below.

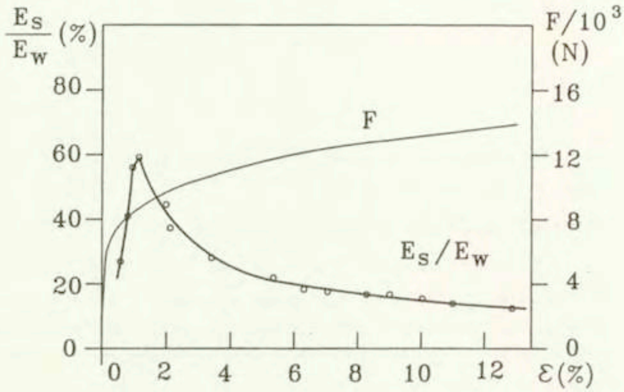

FIG. 1. The ratio E_s/E_w of the stored-to-expended energy and the pulling force F versus tensile deformation of a stainless steel specimens pulled in uniaxial tension – after [6].

2. Stored energy connected with residual microstresses

The concept of residual microstresses in plasticity has been proposed in 1958 by YU. I. KADASHEVICH and V. V. NOVOZHILOV [9] as the basic factor of the so-called kinematic strain–hardening hypothesis. If the initial non-deformed material is assumed to obey the Huber-Mises yield condition

$$s_{ij}s_{ij} = 2k^2, \tag{2.1}$$

then after plastic deformation, the yield condition may be written as

$$(s_{ij} - \alpha_{ij})(s_{ij} - \alpha_{ij}) = 2k^2, \tag{2.2}$$

where s_{ij} is the stress deviator, k is the initial yield locus in simple shear, and α_{ij} stands for an internal parameter, which is interpreted as the tensor of residual microstresses. According to such a strain–hardening law, the initial yield surface (2.1) in the six-dimensional stress space is shifted as a rigid whole. Parameter α_{ij} is represented in this space as the translation vector of the central point of the initial yield surface. The concept of residual microstresses in plasticity was later discussed in [10].

All experimental studies of the energy stored in plastically deformed metals have been done under conditions of uniaxial tension. A simple mezomechanical model shown in Fig. 2 will be used for the theoretical analysis of the process of energy storing due to residual microstresses remaining after uniaxial tension. Note, however, that the model may also be used for a more general analysis of any two-dimensional plastic deformation process. The model represents a regular array of cuboidal elements A and B, each of a different yield locus (cf.[11]). The elastic properties of elements A and B are assumed to be identical. They are defined by the elastic modulus E and by Poisson's ratio ν. Both elements may deform plastically without strain-hardening. In a more advanced variant of the model the strain-hardening effect may be accounted for.

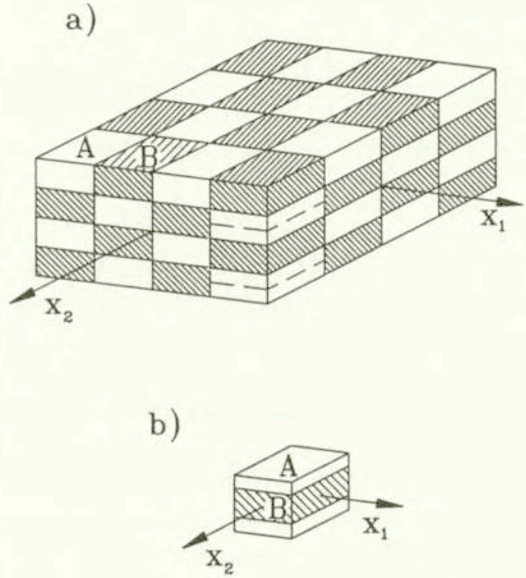

FIG. 2. Simple model of structure of polycrystalline metals with grains A and B having various yield loci: a) general layout of the model, b) its element analysed in calculations.

To simplify the analysis, plane stress states only will be taken under consideration. They will be defined by the principal stresses σ_1 and σ_2 directed along the axes x_1 and x_2 respectively. The yield loci of elements A and B are different as shown in Fig. 3. Elements A begin to deform plastically when the stresses in them satisfy the Huber-Mises yield condition

$$(\sigma_1^A)^2 - \sigma_1^A \sigma_2^A + (\sigma_2^A)^2 = (\sigma_{pl}^A)^2. \tag{2.3}$$

Similarly, the yield condition for elements B may be written as

$$(\sigma_1^B)^2 - \sigma_1^B \sigma_2^B + (\sigma_2^B)^2 = (\sigma_{pl}^B)^2. \tag{2.4}$$

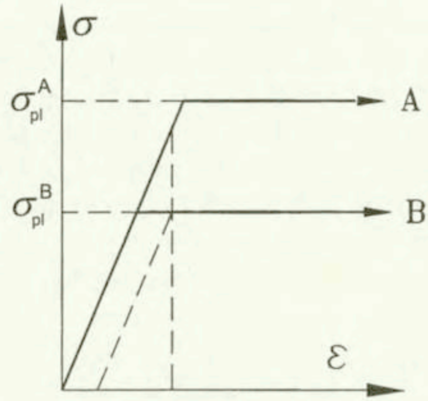

FIG. 3. Assumed theoretical stress-strain relations for elements A and B forming the model shown in Fig. 2.

The exact analysis of behavior of the model undergoing plastic deformation would be very complex, requiring application of numerical techniques with dividing each element into a number of small subelements. However, qualitative conclusions may be obtained with the use of a simplified procedure in which plastic deformation of a segment of the model shown in Fig. 2b will be separately analysed.

As an example, let us analyse the evolution of residual microstresses in the segment of the model shown in Fig. 2b undergoing complex plastic deformation. At first it is uniaxially loaded by tensile stresses σ_2 until point C (Fig. 4) beyond the initial yield stresses $\sigma^0 = \sigma_{pl}^B$, at which weaker element B begins to deform plastically. Point C has been so chosen that element A is still in the elastic state. Then after unloading until point 0, the model is reloaded by uniaxial tensile stresses σ_1 until point D. After the following total unloading until point 0 we can find the residual stresses in the model by analysing the succesive positions of the shifted yield ellipse. The initial yield ellipse shown in Fig. 4 by dashed line has been shifted after this loading programme to the position I with central point 0_D.

The coordinates of this central point 0_D represent the components of the tensor of residual microstresses σ_1^r and σ_2^r remaining in the model. Thus we can calculate the stored energy E_s

$$E_s = \frac{1}{2E}\left[\left(\sigma_1^r\right)^2 + \left(\sigma_2^r\right)^2 - 2\nu\sigma_1^r\,\sigma_2^r\right]. \tag{2.5}$$

If the model is loaded by uniaxial tension by stresses σ_2 only, then the initial yield ellipse is shifted to the position II (see Fig. 4) with centre 0_C. The stored energy due to residual microstresses is

$$E_s = \frac{1}{2E}\left(\sigma_2^r\right)^2. \tag{2.6}$$

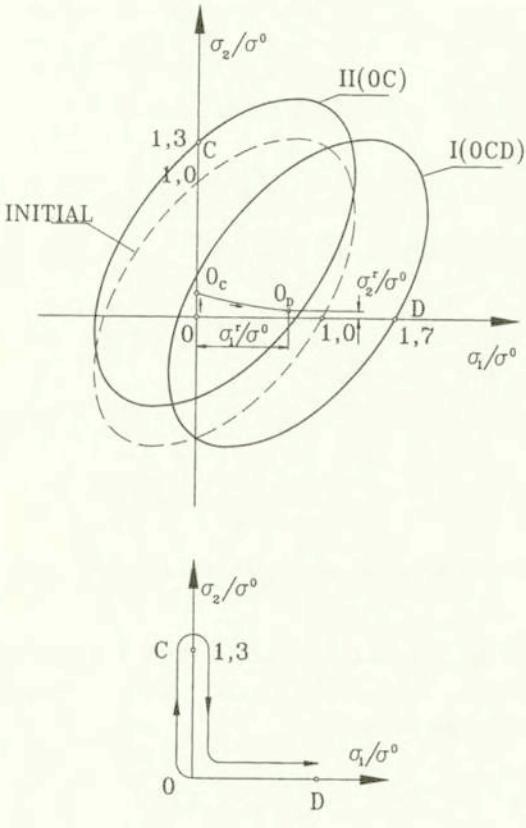

FIG. 4. An example illustrating how residual microstresses σ_1^r and σ_2^r my be calculated when a metal is biaxially deformed plastically-inset shows the loading programme.

The stress-strain relation for the initial stage of uniaxial tensile loading of the model is shown in Fig. 5. Along the segment 0-1 of the loading path, the deformation of the two elements A and B is fully elastic. When stresses in element B reach the yield locus σ_{pl}^B it begins to deform plastically, while element A remains in the elastic state and deforms along the segment 1-b of the diagram. The deformation of the entire model is then represented by the segment 1-2 until point 2 at which also element A begins to deform plastically. From this point the deformation of the model is represented by the horizontal line 2-3. Thus starting from the limiting value of strain ε^*, the model yields under constant limiting value of stress σ^*.

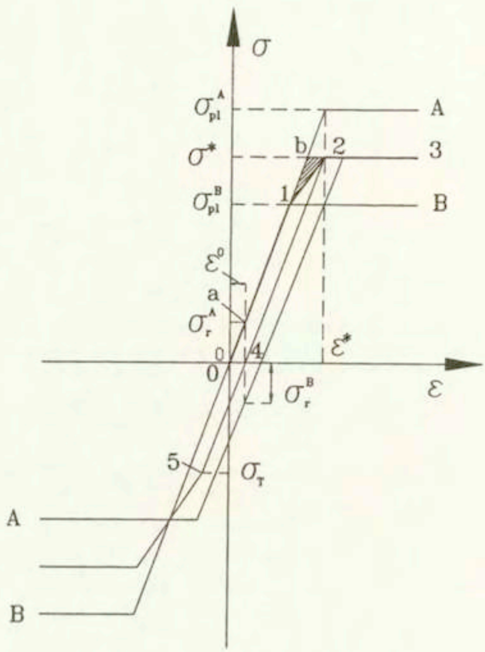

FIG. 5. Stress-strain relation for the model shown in Fig. 2b subject to uniaxial tension.

After unloading from, for example, point 2 until point 4 along straight segment 2-4, there remains in the model a certain plastic deformation ε^0. In element A remain the tensile residual stresses

$$\sigma_r^A = \frac{1}{2}\left(\sigma_{pl}^A - \sigma_{pl}^B\right). \tag{2.7}$$

The residual compressive stresses in element B are of the same absolute value.

Thus in the unit volume of the model is stored the elastic energy

$$E_s = \frac{1}{2E}\sigma_r^2, \tag{2.8}$$

where $\sigma_r = \sigma_r^A = -\sigma_r^B$. Note that this energy is represented in Fig. 5 by the area of the triangle 0-4-a or, in other words, by the dashed area of triangle 1-2-b.

Thus, having found the stress-strain diagram from the tension test we may assess the internal energy stored in the metal due to residual microstresses remaining in it, by measuring the area dashed in Fig. 6 (compare e. g. [12]). This method will be used here for the estimation of the partition of the stored energy into a part connected with the microstresses playing the key role in the strain-hardening hypotheses of the theory of plasticity, and the other part due to the creation of defects inside the crystallites of the metal structure. In Fig. 7 is presented the quotient E_s/E_w calculated for the model shown in Fig. 2b on the basis

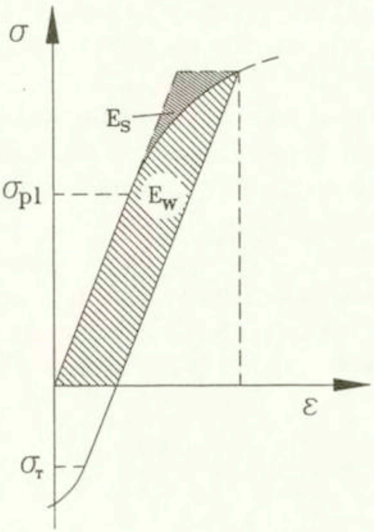

FIG. 6. Schematic diagram showing how to assess the elastic energy E_s stored due to the formation of residual microstresses in plastically deformed metals.

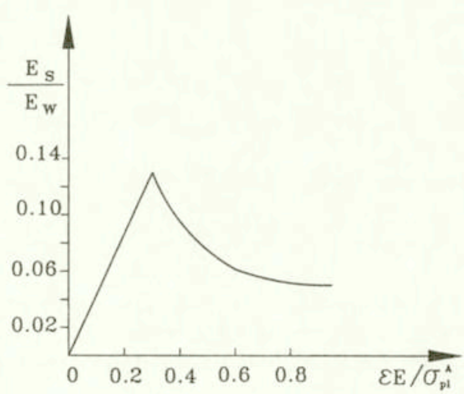

FIG. 7. Relation E_s/E_w versus plastic strain calculated for the model shown in Fig. 2b undergoing uniaxial tension as shown in Fig. 5.

of the diagram for uniaxial tension shown in Fig. 5. The general layout of the diagram is similar to the analogous diagrams found experimentally – e.g. Fig. 1 and diagrams given in [7] and [8]. However, in real metals the quotient E_s/E_w is remarkably larger that obtained for our model. It is possible to increase the maximum value of this quotient in the model up to the value say 0.2 by assuming for example that $\sigma_{pl}^{A} = 2\sigma_{pl}^{B}$. Note that such a large difference between the yield loci of crystallites may be expected in real polycrystalline metals – (see [15]) and Sec. 4 of the present paper.

Applying the method of assessing the amount of stored energy shown in Fig. 6 to the analysis of the stress-strain diagram of the stainless steel given in Fig. 1, we find the maximum value $(E_s/E_w)_{max} = 0.1$, while the real maximum value measured experimentally was found to be about 0.6. Thus that part of the stored energy which is connected with residual microstresses constitutes at the initial stage of plastic deformation a relatively small fraction (about one fifth) of the total stored energy, the remaining portion being attributed to the creation of defects (mainly dislocations) inside the crystallites of the metal structure.

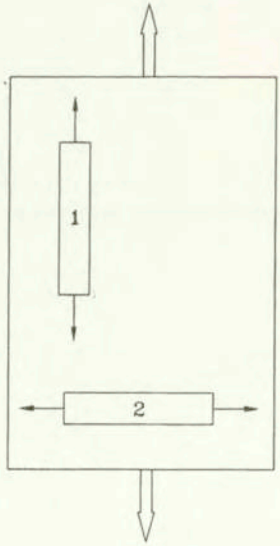

FIG. 8. Scheme of the performed experiment. A large specimen was plastically deformed by uniaxial tension. Then after unloading two small specimens 1 and 2 have been cut out from it, each of them being later plastically deformed under uniaxial tensile loading.

Let us now use this method to the analysis of results of a simple experiment performed according to the loading programme shown previously in Fig. 4. A sheet 6.5 mm thick of an AlMg aluminium alloy (4.7 percent of Mg) was used.

At first, a large specimen cut out from the sheet was loaded by uniaxial tensile stresses until the permanent deformation reached 1.92 percent. Then after unloading, two small specimens, 15 mm wide and 180 mm long, were cut out from it, one in the direction of the previous prestressing of large specimen, and the other in the perpendicular direction (Fig. 8).

Each of these small speciment was then loded by uniaxial tension. In Fig. 9 is shown the stress-strain diagram for the specimen cut out in the direction of previous deformation of the large specimen. For comparison, in the figure is also shown the stress-strain diagram for another specimen cut out in the same direction from the non-deformed sheet. It is seen that using the procedure shown in Fig. 6, we do not find any considerable increase of the stored energy during the initial stage of plastic deformation of our specimen.

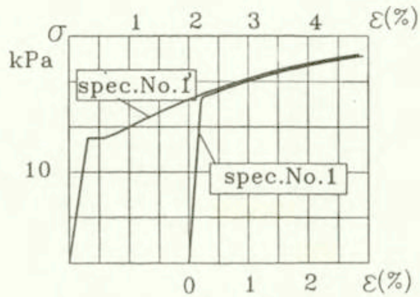

FIG. 9. Stress-strain diagram for specimen No. 1 cut out from the deformed large specimen and then loaded by uniaxial tension, compared with the diagram for the analogous specimen No. 1' cut out from the non-deformed sheet.

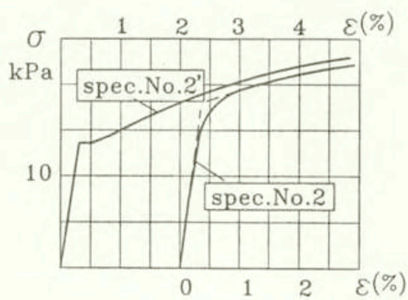

FIG. 10. Stress-strain diagram for specimen No. 2 cut out from the deformed large specimen and then loaded by uniaxial tension, compared with the diagram for the analogous specimen No. 2' cut out from the non-deformed sheet.

Figure 10 presents an analogous stress-strain diagram for the other specimen cut out in the direction perpendicular to the previous uniaxial deformation of large specimen. In the figure is also shown the diagram for a specimen cut out in the same direction from the non-deformed sheet. It is seen that now the material retains the ability to store the elastic energy due to redistribution of residual microstresses during the process of plastic deformation. Figure 11 shows how the quotient E_s/E_w changes the value during the initial stage of deformation. This diagram was obtained by analysing the diagram shown in Fig. 10 with the use of the procedure shown schematically in Fig. 6.

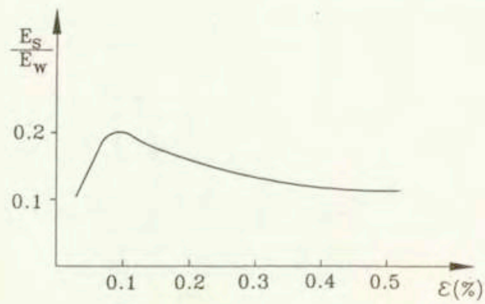

FIG. 11. Relation E_s/E_w versus plastic strain experimentally determined for specimen No. 2 on the basis of the stress-strain diagram shown in Fig. 10.

3. Stored energy and Bauschinger effect

Let use note, coming back to Fig. 5, that the amount of that part of the stored energy, which is connected with the residual microstresses, may also be estimated by measuring the Bauschinger effect. When the model after previous deformation by uniaxial tensile stresses is then loaded by uniaxial compressive stresses along the segment 4-5 of the diagram, its plastic deformation begin at stresses σ_T. In the particular case shown in Fig. 5 we can write

$$\sigma_r^A = \frac{1}{2}\Big(\sigma^* - |\sigma_T|\Big). \tag{3.1}$$

In practice the magnitude of residual microstresses may be estimated on the basis of the stress-strain diagram by measuring the ordinate of the central point of sector 2-5. The elastic energy stored due to these microstresses may be than calculated from formuła (2.8). This procedure holds valid for any value of stresses along the segment 1-2 followed by successive compression.

4. Another model for the study of residual microstresses

In the previous model (Fig. 2) of plycrystalline structure of metals it was assumed that its elements have different yield loci. Plastic deformations in metals are caused by shearing along certain allowable slip-planes identified with the appropriate planes of their crystalline structure. In the previous model it was implicitly assumed that any arbitrary plane inside its elements may be activated as a slip-plane.

However, in real polycrystalline metals there exist in each crystallite only certain admissible directions of slip-planes connected with its atomic structure. A model accounting for this fact was proposed by T. H. LIN and M. ITO [13] and used for the analysis of global plastic properties of polycrystalline aggregates. Analogous model was later used by the same authors for the study of latent energy in plastically deformed such aggregates [14]. In the latter work the model is formed by 64 cubic-shaped "crystals" having different orientations. Each "crystal" was assumed to have one slip-planes orientation only, on which there are three slip directions.

Despite the simplifications introduced in the model, the calculations were complicated and required large computation time. The latent energy remaining in the plastically deformed model after unloading was calculated when at first the residual microstresses were found. Particular calculations have been done for an aggregate of zinc crystals at the temperature $-196°$C. The calculated latent energy was found to reach the astonishing value of about 90 percent of the work done during plastic deformation of the model.

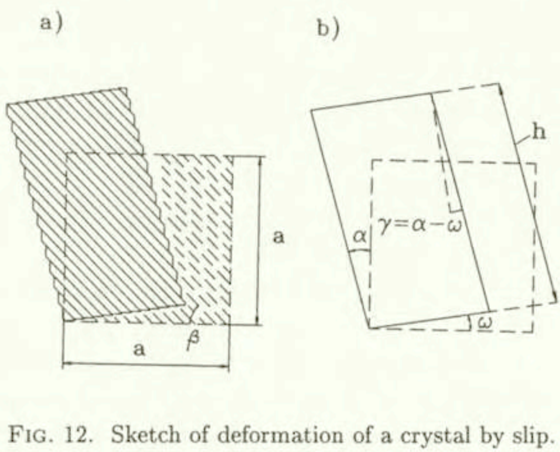

FIG. 12. Sketch of deformation of a crystal by slip.

In what follows, much simpler models of aggregates of elements with different orientations of allowable slip-planes will be analysed in order to estimate how much elastic energy may be stored in them after plastic deformations. In Fig. 12 is shown the basic mechanism of plastic deformation of a single crystal with one allowable orientation of slip-planes. The initially quadratic crystal is deformed into a rhomboid. Its angle of rotation is

$$\omega = \arctan \frac{\tan\beta \sin\beta \sin\alpha}{\cos(\alpha+\beta) - \sin\beta\sin\alpha}. \tag{4.1}$$

Dimension h characterizing the elongation of the crystal is

$$h = a\Big[\cos\alpha + \sin\alpha\tan(\alpha+\beta)\Big]\cos\gamma, \tag{4.2}$$

where $\gamma = \alpha - \omega$ is the angle of shearing distortion.

Let us now consider a simple model composed of two elements with the same but oppositely oriented inclination angles β of allowable slip-planes (Fig. 13). Let the model be loaded by tensile uniaxial stresses as shown in the figure.

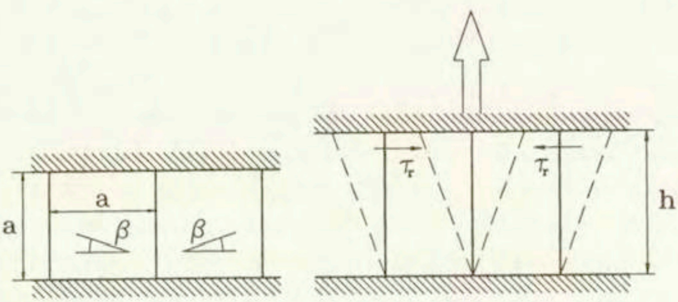

FIG. 13. Simple two-component model with oppositely oriented slip-planes.

Assume that to begin the sliding process, the shear stresses on the slip-planes must reach a certain constant limiting value k. No strain-hardening on these planes is taken into account. Thus the model begins to deform plastically when the tensile stresses in its components reach the value

$$\sigma^* = 2k/\sin 2\beta. \tag{4.3}$$

The components of the model are assumed to be attached to rigid external plates. Thus they cannot rotate and suffer shearing distortion. To remove such a distortion shown in the figure by dashed lines, the elastic residual stresses τ_r must be introduced.

Theoretically the limiting value τ_r^* of these residual shear stresses is that at which the shear stresses on the slip-plane are equal to the yield stress k assumed in the model. Thus we can write

$$\tau_r^* = k/\cos 2\beta. \tag{4.4}$$

Note, however, that for $\beta = 45°$ we get infinite value of the allowable residual shear stress remaining in the model. Such singular properties of models with elements in which only a single orientation of slip-planes is assumed may lead to unrealistic theoretical estimations of the stored elastic energy, such as that calculated in [14].

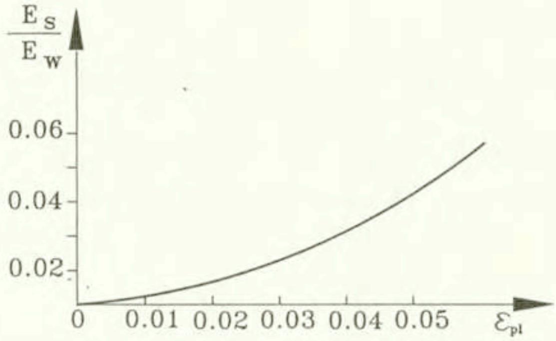

FIG. 14. Theoretical relation E_s/E_w versus plastic deformation calculated for the model shown in Fig. 13.

Nevertheless, our model may be used for rough estimation of the energy stored due to the residual shear microstresses at the initial stage of plastic deformation. In Fig. 14 is shown the diagram E_s/E_w versus plastic longitudinal deformation calculated for $\beta = 45°$ and $k = \mathrm{E}/2000$. Thus the energy stored due to residual shear microstresses is rather small. Let us notice that for $\varepsilon_{pl} = 0.023$ these microstresses reach the value $\tau_r = k$. For this value of residual microstresses we get, assuming $\beta = 30°$, the value $E_s/E_w = 0.031$.

In the model presented in Fig. 13, elongations of two its elements were of the same magnitude – only their distortions had opposie signs.

Now let us analyse a slightly more advanced model consisting of elements with different inclinations of allowable slip-planes (Fig. 15a). In the initial stage of deformation at first begin to deform plastically elements B according to the scheme shown in Fig. 12, while elements A having smaller inclination angle β_A of allowable slip-planes remain in the elastic state. The shear stresses on these planes are smaller then the yield stresses k. In Fig. 15b dashed lines represent the natural shapes of elements A and B after plastic deformation of the model followed then by total unloading. In order to assure the compatibility of the aggregate, the residual shear microstresses τ_r^B must be applied to the elements and moreover, the residual normal microstresses σ_r^A and σ_r^B must appear in the two elements. Thus, ignoring the shear microstresses τ_r^B we have arrived to the situation analogous to that in the previous model shown in Fig. 2. Thus the two models (Fig. 2 and Fig. 15) initially deform plastically almost identically. However, different are the physical mechanisms of their similar behaviour. In the first model any arbitrary plane in its elements may be taken as the slip-plane, while in the second model one orientation of slip-planes only was permissible.

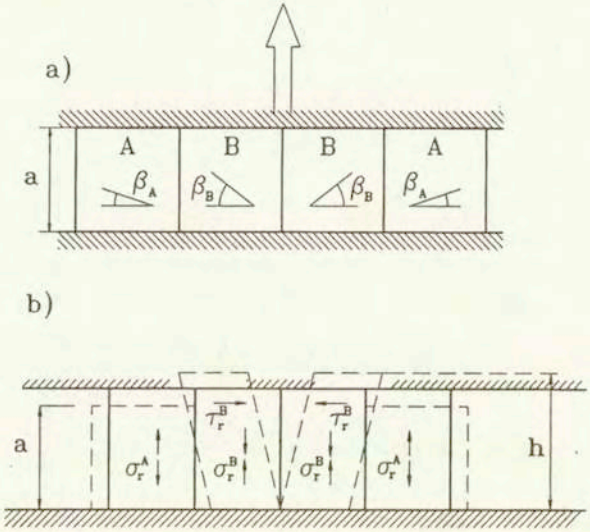

FIG. 15. Model with variously oriented slip-planes in elements.

5. Final remarks

Different simple mezomechanical models of polycrystalline aggregates were analysed in an attempt to estimate how large is the portion of elastic energy stored in plastically deformed metals which may be attributed to the residual microstresses, the remaining part of the energy being connected with the creation of defects in particular grains (mainly dislocations). It was found that the contribution of residual microstresses is rather small but not negligible. Theoretical analysis of these models confirmed by a simple experiment shows that the diagram of the quotient E_s/E_w (E_s is the stored energy and E_w is the work of plastic deformation) versus the permanent deformation has a distinc maximum at the relatively small plastic strains. Such a maximum observed in numerous experimental studies concerning the total stored energy seems to be caused, according to the present study, by the residual microstresses rather than by the creation of defects in particular grains of a polycrystal. It was shown that during complex plastic deformation, the portion of stored energy due to residual microstresses may be theoretically estimated on the basis of the kinematical strain-herdening hypothesis of the theory of plasticity. The residual microstresses are the basic parameter in this hypothesis.

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