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Suction through point-like opening and stability of boundary layer

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A MODEL OF BOUNDARY layer suction through small opening is suggested. A procedure of neutral curve construction is described. The dependence of the shift of neutral curves on the intensity of suction and on the position of the opening is studied.

Key words: boundary layer, hydrodynamic stability, laminar flow, Orr-Sommerfeld equation, Tollmien-Schlichting wave.

1. Introduction

THE LAMINAR-TURBULENT transition in wall boundary layers is important for many practical applications. To study the problem it is necessary to treat the downstream development of the disturbances. The transition is strongly influenced by nonlinear effects, which become important when, by growth, the unstable disturbance has reached a certain level [20]. But at the first stage of the process when disturbance is sufficiently small, one can consider the linearized problem [3, 8]. Linear boundary layer stability problem reduces to investigation of Tollmien-Schlichting wave evolution [19]. In the framework of this approach stability corresponds to the downstream decrease of the Tollmien-Schlichting wave amplitude, and instability to its increase.

Suction of boundary layer is a widely used method of stream laminarization [7, 19]. We shall study the linearized problem of boundary layer. But even linear problem faces great difficulties [13, 17]. That is why it is interesting to construct rough and sufficiently simple models allowing one to estimate the influence of this small perturbation on the stability of boundary layer. It is conventional to use an approximation of uniformly distributed suction [7], but in the framework of this approach we have no possibility to investigate the influence of aperture position. There is another way- to replace a small opening (strip) by a point-like one [5, 14]. A goal of the present paper is further development of this idea. Namely, we analyze a shift of neutral curves and, correspondingly, the critical value of the Reynolds number under the influence of additional perturbation of the velocity of main flow by a point source (sink) at the wall. The result is compared with [11, 17]. The model allows one to vary the choice of the model function - disturbance of the main flow due to aperture in the boundary (in the model it is replaced by a point-like window). In [10 - 12] the excitation of the Tollmien-Schlichting waves and the receptivity of boundary layer to the Dirac line source (sink) at the wall was studied. One can use this function which corresponds to the point-like source as a disturbance in our model. But comparison of our model with the results [13, 17] shows that, in order to obtain appropriate result (outside some neighbourhood of the aperture), it is sufficient to use more rough and simpler model functions, for example, a point source for the Stokes flow [6, 16]. It should be mentioned that Stokes-flow solutions are widely used in multi-layer analysis of boundary layer for the flow in the inner sublayer [2, 18].

There is a so-called zero-width slit model which is rather effective in diffraction theory and creeping flow investigation [6, 15, 16]. It is based on the theory of self-adjoint extensions of symmetric operators. The model is similar to the zero-range potential method in quantum mechanics. The suggested approach allows one to apply the operator extension theory methods in a hydrodynamic problem.

Consider a two-dimensional flow of viscous incompressible fluid over the semi-axis $\Gamma, \Gamma = \{(x, y) : x \geq 0, y = 0\}, x, y$ are Cartesian coordinates on the plane. It is convenient to use a stream function Ψ instead of the velocity (u_x, u_y) of the flow: $u_x = \Psi'_y, u_y = -\Psi'_x$. Then the Navier-Stokes equations transform to the following boundary-initial value problem for the stream function Ψ:

$$\frac{\partial \Delta\Psi}{\partial t} + \frac{\partial\Psi}{\partial y}\frac{\partial\Delta\Psi}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial\Delta\Psi}{\partial y} - v\Delta^2\Psi = 0,$$

(1.1)

$$\Psi\,|_\Gamma = \frac{\partial\Psi}{\partial n}\,|_\Gamma = 0, \Psi\,|_{t=0} = \Psi_0,$$

where v is the kinematic viscosity.

Let ψ be the stream function of the main stationary flow, $u = \psi'_y, v = -\psi'_x$. A solution of the problem with slightly disturbed initial condition we denote by $\psi + \varphi$. Assuming that the disturbance is small, we linearize Eq. (1.1). Consider a region in which the boundary layer has been formed. Assuming that the velocity field is parallel and its components depend only on the transversal variable y, one can seek φ in the form of a Tollmien-Schlichting wave: $\varphi(x, y) = f(y)\exp(i\alpha(x - ct))$. It is possible to take into account also the transversal component v of the velocity keeping in mind that u and its derivatives are much larger than v and its derivatives [1, 7]. Taking into account that $u''_{xx} \ll u''_{yy}, v''_{xx} \ll v''_{yy}$ in the boundary layer and considering v and v_{yy} as parameters, one obtains the following modified

Orr-Sommerfeld problem (in dimensionless form) [1, 7]:

$$(1.2)\qquad f'''' - 2\alpha^2 f'' + \alpha^4 f = R(i\alpha(u-c)(f'' - \alpha^2 f) - i\alpha u''_{yy} f + v(f''' - \alpha^2 f') - v''_{yy} f'), \qquad f(0) = f'(0) = \alpha f(1) + f'(1) = 0,$$
$$f(y) \to \text{const} \quad \text{for} \quad y \to \infty,$$

where R is the Reynolds number for the boundary layer. The system of units is such that the width of the boundary layer is equal to 1. The first two conditions mean that the disturbance of the velocity field must vanish at the boundary. As for the remaining two conditions, they show that the perturbation of the velocity is concentrated in the boundary layer. It is possible to use other conditions, for example, $f(y) \to 0$ and $f'(y) \to 0$ for $y \to \infty$, but for our purposes it is more convenient to use the above mentioned condition in which the scale is fixed: the width of the boundary layer is 1.

Let α be real. It is convenient to use a neutral curve, i.e. a set of points on the plane α, R, for which $\Im c = 0$, to describe the stability. The domain of instability consists of points for which $\Im c > 0$ (because the amplitude of the corresponding Tollmien-Schlichting wave increases). Minimal value R_* of R on the neutral curve is named the critical value of the Reynolds number. For $R < R_*$ the flow is stable for any value of α. To determine R_* it is an important problem of linear hydrodynamic stability. It is essential for various applications to clarify how the critical Reynolds number is influenced by different variations of the system.

For the main flow we have $u = U(y), v = 0$ in the boundary layer. We shall estimate the influence of small aperture on the stability by means of replacement of coefficients u, v in Eq. (1.2) by the components of the flow velocity in the boundary layer in the case when the aperture is present [8]. Unfortunately, explicit construction of the velocity field in this case is very complicated. That is why it if useful to find simpler model functions εg for the perturbation of equation coefficients: $u = U(y) + \varepsilon g'_x, v = -\varepsilon g'_y$, where ε is a small parameter. Outside some neighbourhood of the opening, stationary velocity field for small aperture differs slightly from that for the model with point source at the boundary. The simplest example of such source is a potential source: $g = \arctan(y(x-a)^{-1})$, where $(a, 0)$ is a point of the opening. It should be mentioned that one can use the operator extension theory method [6, 15, 16] to construct a stream function for the potential or creeping flow with point source.

2. Construction of asymptotic expansion

Let us construct main terms of an asymptotic expansion of the solution in small parameter $(\sqrt{\alpha R})^{-1}$. We shall follow the scheme of [1], which is a modification of that from [9]. Let f_1, f_2 be the solutions of the Orr-Sommerfeld equation without viscosity (for details, see the Appendix):

$$f_1(y) = (u-c)\sum_{n=0}^{\infty} q_n(y)\alpha^{2n},$$

$$q_0 = 1, q_n(y) = \int_0^y (u-c)^{-2}dy \int_0^y (u-c)^2 q_{n-1}(y)dy,$$

$$f_2(y) = (u-c)\sum_{n=0}^{\infty} t_n(y)\alpha^{2n},$$

$$t_0 = \int_0^y (u-c)^{-2}dy, t_n(y) = \int_0^y (u-c)^{-2}dy \int_0^y (u-c)^2 t_{n-1}(y)dy.$$

Let us make a substitution

$$f(y) = \exp(\int_0^y p(y)dy)$$

in Eq. (1.2) to determine two additional solutions. Then one obtains for $p(y)$:

$$(3.1)\qquad (u-c)(p'+p^2-\alpha^2) - u'' + (iR\alpha)^{-1}vR(p'+3pp' + p^3 - \alpha^2 p) - v''_{yy}Rp(i\alpha R)^{-1} = (i\alpha R)^{-1}(p^4+6p^2p' + 3(p')^2 + 4pp'' + p''' - 2\alpha^2(p'+p^2)+\alpha^4).$$

We search a solution of (2.1) in the form of a series in powers of $(\sqrt{\alpha R})^{-1}$:

$$(3.2)\qquad p(y) = \sum_{n=0}^{\infty}(\alpha R)^{\frac{1-n}{2}}p_n(y).$$

Substitute (2.2) into (2.1) and select the terms of order αR and $\sqrt{\alpha R}$:

$$(u-c)p_0^2 = -ip_0^4,$$

$$(u-c)p_0' + 6ip_0^2p_0' - ivRp_0^3 = (-4ip_0^3 - 2(u-c)p_0)p_1.$$

Hence,

$$p_0 = \pm\sqrt{i(u-c)}, p_1 = -5p_0'(2p_0)^{-1} + vR/2.$$

Consequently, two "viscous" solutions of (1.2) have the form:

$$(3.3)\quad \begin{aligned} f_3(y) &= (u-c)^{-5/4}\exp\left(\int_0^y \left(-\sqrt{i\alpha R(u-c)} + vR/2\right) dy\right), \\ f_4(y) &= (u-c)^{-5/4}\exp\left(\int_0^y \left(\sqrt{i\alpha R(u-c)} + vR/2\right) dy\right). \end{aligned}$$

The solution should satisfy the boundary conditions

$$f(0) = f'(0) = \alpha f(1) + f'(1) = 0, f(y) \to \text{const} \quad \text{for} \quad y \to \infty.$$

But f_4 does not satisfy the last one. That is why we take the solution in the form

$$f(y) = b_1 f_1(y) + b_2 f_2(y) + b_3 f_3(y).$$

Coefficients are determined from the system of boundary conditions. It is a linear algebraic system for the coefficients. It has non-trivial solution in the case when the system determinant is equal to zero. After some calculations, this condition takes the form:

$$(3.4)\quad -\frac{f_3(0)}{f_3'(0)} = \frac{cz}{u'(0)(1+z)},$$

where

$$z = z_r + iz_i = u'(0)c(f_2'(1) + \alpha f_2(1))(f_1'(1) + \alpha f_1(1))^{-1},$$

$$(3.5)\quad z_r = -\frac{u'(0)}{u_*'} + \frac{u'(0)cu_*''}{(u_*')^3}\ln c\frac{u'(0)c}{\alpha(1-c)^2},$$

$$u(y_*) = c,\ u_*' = u'(y_*),\ u_*'' = u''(y_*),$$

$$(3.6)\quad z_i = -\pi u'(0)cu_*''(u_*')^3.$$

Under the assumption that $c \simeq y_* u'(0)$ [9], one obtains from (2.4):

$$z + 1 = \left(1 + \frac{f_3(0)}{f_3'(0)y_*}\right)^{-1}.$$

Let $\Phi(y)$ be the conventional solution of the Orr-Sommerfeld equation

$$\Phi'''' - 2\alpha^2\Phi'' + \alpha^4\Phi = Ri\alpha((u-c)(\Phi'' - \alpha^2\Phi) - U_{yy}''\Phi).$$

Then taking into account (1.5), one gets

$$f_3(y) = \Phi(y)\exp\left(\int_0^y (vR/2)dy\right).$$

Hence, (2.4) gives us

$$(3.7) \qquad z+1 = (1+\Phi(0)(y_*(\Phi'(0)+Rv(0)\Phi(0)/2))^{-1})^{-1}$$
$$(1-F(w)(1-v(0)Ry_*F(w)/2)^{-1})^{-1} = G_1(w),$$

where $F(w)=F_r(w)+F_i(w)$ is the Thitiens function [9], $w=y_*(\alpha R u_*')^{1/3}$. We use Eq. (2.7) for the determination of α, R on the neutral curve. If the model function is such that $v(0)=0$ in a cross-section of the boundary layer which is considered, then it is necessary to take into account next terms of the expansion (1.4). One obtains p_2 by collecting the terms of order $(\alpha R)^0$ in (2.1):

$$p_2 = (2p_0^3)^{-1}(vR3(p_0p_0'+p_0^2p_1) - 5p_0^2p_1' - iu''$$
$$- 5p_0^2p_1^2 - 12p_0p_1p_1' - 3(p_0')^2 - 4p_0p_0'' + \alpha^2p_0^2).$$

Substituting this expression in (2.1), one obtains the following relation for the determination of α, R on the neutral curve in the case when $v(0)=0$ instead of (2.7), by extracting the main term:

$$(3.8) \quad z+1 = (1+\Phi(0)(y_*(\Phi'(0) - 5\sqrt{R/\alpha}(4p_0(0))^{-1}v'(0)\Phi(0)))^{-1})^{-1}$$
$$= (1-F(w)(1-5\sqrt{R/\alpha}(4p_0(0))^{-1}v'(0)y_*F(w))^{-1})^{-1} = G_2(w),$$

If the model function satisfies both boundary conditions, we should take into account the next term (p_3) of the series (2.2). In this case, one must collect all terms of order $(\sqrt{\alpha R})^{-1}$ in (2.1):

$$p_3 = -(2p_0^3)^{-1}(-vR(p_0''+3p_0p_1'+3p_0^2p_2+3p_1p_0'+3p_0p_2^2-\alpha^2p_0)$$
$$+ 5p_0^2p_2' - 2p_1p_2p_0^2 + Rp_0v'' + 4p_0p_1^3 + 6p_0p_1^2 + 12p_0'p_0p_2$$
$$+ 12p_1'p_1p_0 + 6p_0'p_1' + 4p_1p_0'' + 4p_0p_1'' + p_0''' - 2\alpha^2p_0' + 2p_0p_1).$$

Then in the case $v(0)=v'(0)=0$ one obtains, by extracting the main term in p_3, the following condition for the determination of the neutral curve:

$$(3.9) \quad z+1 = (1+\Phi(0)(y_*(\Phi'(0) - 21i(8\alpha(u(0)-c))^{-1}v''(0)\Phi(0)))^{-1})^{-1}$$
$$= (1-F(w)(1+21i(8\alpha(u(0)-c))^{-1}v''(0)y_*F(w))^{-1})^{-1} = G_3(w).$$

Expressions (2.7) – (2.9) for $G_j(w)$ have similar structures:

$$(1-F(w)(1+mw^3F(w))^{-1})^{-1} = G(w,m),$$

if we introduce parameter m which has the value m_j for the case G_j:

$$\begin{aligned} m_1 &= -v(0)(2y_*^2\alpha u_*')^{-1}, \\ m_2 &= -5\sqrt{2}v'(0)(1+i)(8\sqrt{\alpha^3 R u_*}y_*^2 u_*')^{-1}, \\ m_3 &= -21v''(0)(8u_* u_*'\alpha^2 R y_*^2)^{-1}. \end{aligned}$$

3. Procedure of neutral curve construction

Let us describe a procedure of constructing the neutral curve.

Let $G(w,m) = H(w,m) + iQ(w,m), m = r + is, \Im H = \Im Q = \Im r = \Im s$. Then

$$\begin{aligned} H(w,m) = (1 + 2rw^3F_r - 2sw^3F_i + r^2w^6F_i^2 + \ s^2w^6(F_r^2 + F_i^2) \\ + r^2w^6F_i^2 - rw^3F_r^2 - F_r - rw^3F_i^2)((1 + rw^3F_r - sw^3F_i - F_r)^2 \\ + (rw^3F_i + sw^3F_r - F_i)^2)^{-1}, \\ Q(w,m) = (F_i - sw^3(F_i^2 + F_r^2))((1 + w^3(rF_r - sF_i - F_r)^2 \\ + (w^3(rF_i + sF_r) - F_i)^2)^{-1}. \end{aligned}$$

It is possible to assume a different initial approximation for the unperturbed main flow. We use the following profile: $U(y) = 2y - 5y^4 + 6y^5 - 2y^6$. Coefficients u, v in (1.2) have the following form: $u = U(y) + \varepsilon g_y', v = -\varepsilon g_x'$, where ε is a parameter, which characterizes the intensity of suction.

Let us fix ε. Choose a set of values of function $F(w)$. For each value one solves the following system of equations obtained above:

$$\begin{aligned} Q(w,m) &= z_i(y_*), \\ H(w,m) &= 1 + f_r(y_*, \alpha), \\ R &= w^3 y_*^{-3}(\alpha u_*')^{-1}, \\ m &= m(y_*, \alpha, R). \end{aligned}$$

The solution α, R of the system gives us the point of the neutral curve. Testing a sufficient set of values of the function $F(w)$, one constructs the neutral curve.

4. Suction and critical Reynolds number

To estimate the influence of suction on the stability near the critical point of the neutral curve we can use another technique. Namely, let ε be small, and we search a solution of (1.2) in the form of a power series in this small parameter:

$$f = f_0 + \varepsilon f_1 + \varepsilon^2 f_2 + ..., \qquad c = c_0 + \varepsilon c_1 + \varepsilon^2 c_2 + \ldots. \tag{4.1}$$

Substitute now these expressions into (1.2). Note that the function g depends on x, but x is treated as a parameter, i.e. the stability analysis is carried out under the assumption that the boundary layer is treated locally as a parallel flow. Terms of order ε^0 gives us the conventional Orr-Sommerfeld equation for f_0, c_0 :

$$f_0^{''''} - 2\alpha^2 f_0^{''} + \alpha^4 f_0 = Ri\alpha((u - c_0)(f_0^{''} - \alpha^2 f_0) - u_{yy}^{''} f_0). \tag{4.2}$$

For terms of order ε one obtains the inhomogeneous Orr-Sommerfeld equation for f_0, c_0 :

$$\begin{aligned} f_0^{''''} - 2\alpha^2 f_0^{''} + \alpha^4 f_0 - Ri\alpha((u - c_0)(f_0^{''} - \alpha^2 f_0) - u_{yy}^{''} f_0) \\ = R(i\alpha(g_y' - c_1)(f_0'' - \alpha^2 f_0) - i\alpha g_{yyy}''' f_0 - g_x'(f_0''' - \alpha^2 f_0') + g_{yyx}''' f_0'). \end{aligned} \tag{4.3}$$

The condition of solvability of Eq. (4.1) is the orthogonality of the right-hand part to a solution θ of the associated Orr-Sommerfeld equation:

$$\theta^{''''} - 2\alpha^2 \theta^{''} + \alpha^4 \theta = Ri\alpha((u - c_0)(\theta^{''} - \alpha^2 \theta) + 2u_y' \theta').$$

The orthogonality condition gives us the value of c_1:

$$\begin{aligned} c_1 = \left(i\alpha \int_0^\infty (f_0^{''} - \alpha^2 f_0)\overline{\theta} dy \right)^{-1} \int_0^\infty (i\alpha g_y'(f_0'' - \alpha^2 f_0) \\ - i\alpha g_{yyy}''' f_0 - g_x'(f_0''' - \alpha^2 f_0') + g_{yyx}''' f_0')\overline{\theta} dy. \end{aligned} \tag{4.4}$$

The sign of $\varepsilon \Im c_1$ shows how the stability changes. Inequality $\varepsilon \Im c_1 > 0$ corresponds to instability, and condition $\varepsilon \Im c_1 < 0$ – to stability.

5. Discussion

It is possible to choose different functions g in the model. Of course, the best is the stream function for the case of small aperture in the boundary. But it is very difficult to construct such solution [13]. That is why it is useful to choose a simpler model functions. Formula (2.7) may be used for the description of uniform suction through the surface, for example. The simplest choice of the model function for the case of suction through small opening is a potential source

$$g = \arctan(y/(x - a)).$$

Here $(a, 0)$ is a position of the opening. This function satisfies only one boundary condition in (1.1). For this case the function G should be chosen in accordance with formula (2.8). It is possible to choose a model function which satisfies both the boundary conditions. The simplest example is the corresponding solution of biharmonic equation (Stokes flow):

$$g = y^2/((x-a)^2 + y^2).$$

Note that the Stokes-flow solutions are often used as an instrument of boundary layer investigation in multi-layer analysis for inner sublayer (see, for example, [2, 18]). For such a choice of the model function, formula (2.9) for G works. But for a creeping flow, the value of flux across a line is equal to the difference between the values of stream function on the ends of the line. That is why this source is not a source (or sink) of mass, but a source of vorticity. From this point of view it is more appropriate to use a function

$$g = y^2/((x-a)^2 + y^2) + \pi/2 - \arccos(y/\sqrt{(x-a)^2 + y^2}),$$

which gives us a flux π through the opening.

For the last type of the model function we construct a neutral curve. The dependence of its shift on ε (intensity of suction) is shown in Fig. 1. Here $\Delta x = x - a$, where $(a, 0)$ is the position of the opening, x is the coordinate of the cross-

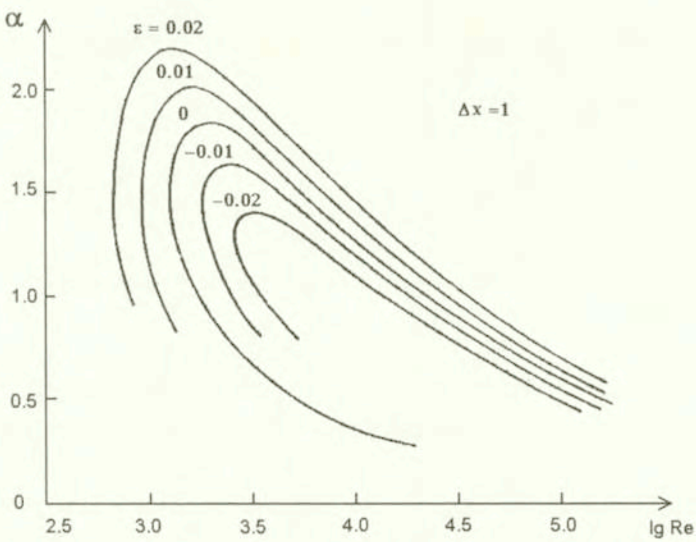

FIG. 1. Dependence of neutral curve position on the intensity of suction. Δx is the distance between the cross-section under consideration and the point of the aperture; the unit of length is the thickness of the boundary layer at the point of the opening. Curve with $\varepsilon = 0$ corresponds to absence of suction.

section for which the stability is studied. Positive ε corresponds to source. For these values of the parameter we obtain a displacement of the neutral curve to the left and extension of instability domain. For $\varepsilon < 0$ (suction) we obtain a shift to the right of the curve and, correspondingly, reduction of instability domain. The neutral curves for different Δx are shown in Fig. 2. Here the case $\Delta x = \infty$ gives us the neutral curve for the boundary layer without suction. Of course, it is possible to use more realistic model functions (for example, that obtained by approximate calculations for point-like source [12]; in this situation we have not an explicit expression for g , but it is not essential). Nevertheless, one can see that even for such a rough choice of the model function one obtains results which are in good agreement with that for more realistic, but considerably more complicated models (see, for example, [17]).

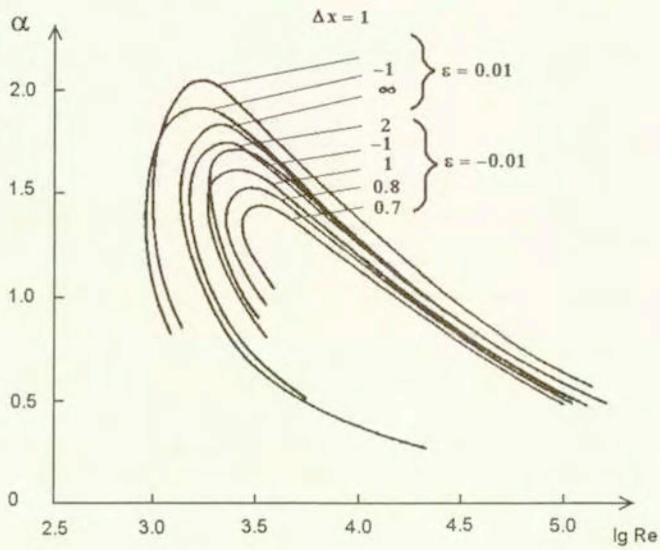

FIG. 2. Dependence of neutral curve position of the position of the aperture. Negative ε corresponds to suction (sink), positive ε- to source.

Obviously, that solution for point-like aperture is not appropriate as a model solution in the neighbourhood of the aperture of finite width (at the point of the opening the model solution has a singularity). One can estimate the size of neighbourhood outside which our model is correct. Comparison of the model function g with the stream function for the flow near aperture [13] shows that the ratio of the radius of the neighbourhood to the width of the boundary layer (at the point of the window) is of the order $10^2\varepsilon$.

Change of stability at critical point due to suction is studied by means of (4.4). The solutions f_0, θ are taken from [4] for critical values of parameters:

$\alpha = 0.304, R = 519, c_0 = 0.3967$. The dependence of $\Im c_1$ on the distance from the point of suction is shown in Fig. 3 (for the model function $g = \arctan(y/(x-a))$) and in Fig. 4 (for the model function $g = y^2/((x-a)^2 + y^2)$). Note that the second function corresponds to the situation when the total flux through the aperture is zero, for example, there is a cavity coupled with the boundary layer

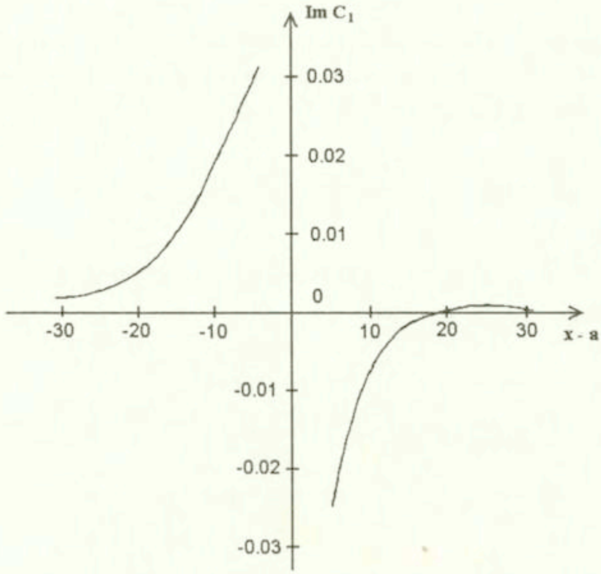

FIG. 3. The dependence of $\Im c_1$ on a distance from the point of suction for the model function $g = \arctan(y/(x-a))$.

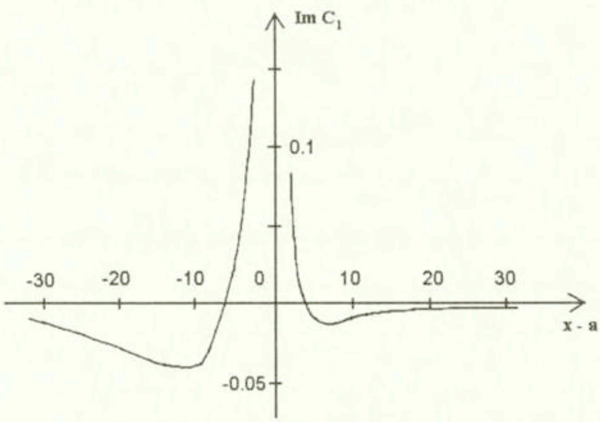

FIG. 4. The dependence of $\Im c_1$ on a distance from the point of suction for the model function $g = y^2/((x-a)^2 + y^2)$.

through small opening. For the first function, the total flux is not zero – there is a suction. It explains the qualitative difference of the pictures. We must stress that the model is correct outside a neighbourhood of the opening, i.e. the results are not valid for small $x - a$ (the unit of length is the thickness of the boundary layer).

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Appendix

Consider the form of series for $f(1)$ (and $f(2)$) in powers of α. It is a solution of the main equation without viscosity:

$$(u-c)(f'' - \alpha^2 f) - i\alpha u''_{yy} f = 0.$$

Substituting f in the form

$$f(y) = (u-c)\sum_{n=0}^{\infty} q_n(y)\alpha^{2n},$$

one gets

$$(u-c)\sum_{n=0}^{\infty}(u''q_n + 2u'q'_n + (u-c)q''_n)\alpha^{2n}$$
$$= (u-c)^2\sum_{n=1}^{\infty} q_{n-1}\alpha^{2n} + (u-c)u''\sum_{n=0}^{\infty} q_n\alpha^{2n}.$$

Comparing the terms with identical powers of α, one obtains for $n = 0$

$$2(u-c)u'q'_0 + (u-c)^2 q''_0 = 0,$$

i.e.

$$((u-c)^2 q'_0)' = 0.$$

Hence,

$$q_0 = C_1\int_0^y (u-c)^{-2}dy + C_2.$$

For $n > 0$ one gets

$$((u-c)^2 q_n')' = (u-c)^2 q_{n-1}.$$

Hence, one obtains the following recurrent relation:

$$q_n(y) = \int_0^y (u-c)^{-2} dy \int_0^y (u-c)^2 q_{n-1}(y) dy.$$

Taking two linearly independent solutions q_0, we obtain the form of series for f_1 (f_2):

$$f_1(y) = (u-c) \sum_{n=0}^{\infty} q_n(y) \alpha^{2n},$$

$$q_0 = 1, q_n(y) = \int_0^y (u-c)^{-2} dy \int_0^y (u-c)^2 q_{n-1}(y) dy,$$

$$f_2(y) = (u-c) \sum_{n=0}^{\infty} t_n(y) \alpha^{2n},$$

$$t_0 = \int_0^y (u-c)^{-2} dy, t_n(y) = \int_0^y (u-c)^{-2} dy \int_0^y (u-c)^2 t_{n-1}(y) dy.$$

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