

## Buoyancy effects in boundary layers on a continuously moving vertical surface with a parallel free stream

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AN ANALYSIS IS PERFORMED to study the effect of the buoyancy force on the flow and heat transfer characteristics in a viscous fluid over a heated vertical continuously moving surface with a parallel free stream. The buoyancy force varies with the streamwise distance  $x$  and hence introduces nonsimilarity in the flow field. Both the constant wall temperature and constant heat flux conditions are included in the analysis. The partial differential equations governing the flow are solved numerically. Closed form solutions are obtained when the wall and free stream velocities are equal and there is no buoyancy force. Also the correlation equations for the local Nusselt number are developed. It is found that for opposing flow or for an upstream moving wall, the solution does not exist beyond a certain value of the buoyancy parameter or the ratio of wall and free stream velocities.

### Notations

$C_{fx}$	local skin friction coefficient
$f, F$	reduced stream functions
$g$	gravitational acceleration
$Gr_x, Gr_x^*$	Grashof numbers
$G, H$	dimensionless temperatures
$k$	thermal conductivity
$Nu_x$	local Nusselt number
$Pr$	Prandtl number
$q_w$	local surface heat transfer rate per unit area
$Re_x$	local Reynolds number
$T$	fluid temperature
$T_w, T_\infty$	wall and free stream temperatures
$u, v$	velocity components in $x$ and $y$ directions
$u_w, u_\infty$	wall and free stream velocities
$x, y$	axial and normal coordinates

## Greek symbols

$\alpha$	thermal diffusivity
$\beta$	volumetric coefficient of thermal expansion
$\eta$	pseudo-similarity variable
$\mu, \nu$	dynamic and kinematic viscosities
$\xi, \xi_1$	buoyancy force parameters
$\psi$	stream function

## Subscripts

$w$	condition on the surface
$x, y$	derivatives with respect to $x$ and $y$ , respectively
$\infty$	condition in the free stream

## Superscript

prime	derivative with respect to $\eta$
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## 1. Introduction

THE STUDY OF FLOW AND HEAT TRANSFER in the boundary layer induced by a continuously moving surface with a parallel free stream is useful in many manufacturing processes in various fields of industry such as cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, and the boundary layer along a liquid film in condensation processes. In order to understand the main features of such a process, the flow field induced by a continuous flat surface which issues from a slot and moves with a constant velocity into a fluid, either at rest or with constant free stream, has been considered by many investigators [1 – 4]. In recent years several authors [4 – 9] have considered the effect of buoyancy force on a continuously moving surface in a fluid at rest.

Here the effect of buoyancy force on the flow and heat transfer on a heated vertical, continuous moving surface with a parallel free stream has been studied. Both the constant wall temperature and constant heat flux conditions have been considered. The partial differential equations governing the nonsimilar flow have been solved using an implicit finite difference scheme. The nonsimilarity in the flow field is due to the buoyancy force which varies with axial distance  $x$ . In absence of the buoyancy force, the governing equations reduce to the ordinary differential equations and closed-form solutions have been obtained. Also correlation equations for the local Nusselt number have been developed. Particular cases of the present results have been compared with those of MUCOGLU and CHEN [10], MOUTSOGLU and CHEN [6] and ABDELHAFEZ [2].

## 2. Governing equations

Let us consider a continuous flat surface that originates from a slot and is moving with a constant velocity  $u_w$  in vertical direction in a fluid having constant free stream velocity  $u_\infty$  and temperature  $T_\infty$ . The positive  $x$  coordinate is measured along the direction of the moving surface with the slot as the origin, and the positive  $y$  coordinate is measured normal to the surface in the outward direction toward the fluid (see Fig. 1). The surface is assumed to be either maintained at a constant wall temperature  $T_w$  or subjected to a constant surface heat flux  $q_w$ . The velocity of the moving surface  $u_w$  is considered to be either greater or smaller than the free stream velocity  $u_\infty$ . The buoyancy force is assumed to act in the same direction or in the direction opposite to the forced flow (i.e., the buoyancy force assists or opposes the forced flow). On the basis of boundary layer approximations and with the use of Boussinesq approximations, the governing equations for the problem under consideration for constant wall temperature (CWT) case based on the principles of conservation of mass, momentum and energy can be written as [10]

$$(2.1) \quad u_x + v_y = 0,$$

$$(2.2) \quad uu_x + vv_y = \nu u_{yy} + g\beta(T - T_\infty),$$

$$(2.3) \quad uT_x + vT_y = \alpha T_{yy}.$$

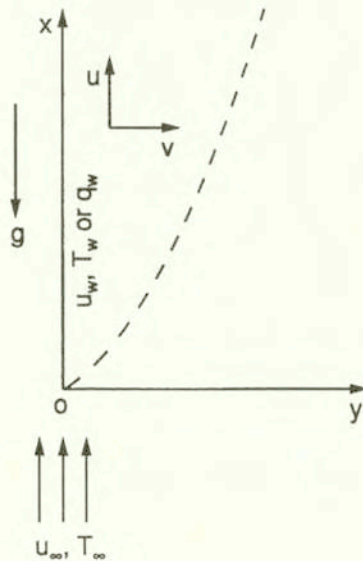


FIG. 1. Physical model and co-ordinate system.

The boundary conditions satisfying the no-slip conditions at the wall and matching with the inviscid flow solutions at the edge of the boundary layer are given by

$$(2.4) \quad \begin{aligned} u(x, 0) &= u_w, & v(x, 0) &= 0, & T(x, 0) &= T_w, \\ u(x, \infty) &= u_\infty, & T(x, \infty) &= T_\infty, \\ u(0, y) &= u_\infty, & T(0, y) &= T_\infty, & y > 0. \end{aligned}$$

In order to transform equations (2.1) – (2.3) in dimensionless form, we apply the following transformations:

$$(2.5) \quad \begin{aligned} \eta &= (u_\infty/\nu x)^{1/2} y, & \xi &= \xi(x) = Gr_x/\text{Re}_x^2, \\ f(\xi, \eta) &= \psi(x, y)/(\nu u_\infty x)^{1/2}, \\ G(\xi, \eta) &= (T(x, y) - T_\infty)/(T_w - T_\infty), \end{aligned}$$

$$Gr_x = g\beta(T_w - T_\infty)x^3/\nu^2, \quad \text{Re}_x = u_\infty x/\nu, \quad f' = \partial f/\partial \eta,$$

$$u = \psi_y, \quad v = -\psi_x, \quad \text{Pr} = \nu/\alpha, \quad f'(\xi, \eta) = u/u_\infty, \quad G' = \partial G/\partial \eta$$

to (2.1) – (2.3) and we find that (2.1) is identically satisfied, and (2.2) and (2.3) are reduced to

$$(2.6) \quad f''' + 2^{-1}ff'' + \xi G = \xi(f'\partial f'/\partial \xi - f''\partial f/\partial \xi),$$

$$(2.7) \quad \text{Pr}^{-1}G'' + 2^{-1}fG' = \xi(f'\partial G/\partial \xi - G'\partial f/\partial \xi).$$

The boundary conditions (2.4) can be expressed as

$$(2.8) \quad \begin{aligned} f(\xi, 0) &= f'(\xi, 0) - (u_w/u_\infty) = G(\xi, 0) - 1 = 0, \\ f'(\xi, \infty) - 1 &= G(\xi, \infty) = 0. \end{aligned}$$

When the surface velocity is greater than the freestream velocity (i.e.,  $(u_w/u_\infty) > 1$ ), we take the reference velocity as  $u_w$  instead of  $u_\infty$  in Eq. (2.5). The governing Eqs. (2.6) and (2.7) remain unaltered. The boundary conditions can be written as

$$(2.9) \quad \begin{aligned} f(\xi, 0) &= f'(\xi, 0) - 1 = G(\xi, 0) - 1 = 0 \\ f'(\xi, \infty) - (u_\infty/u_w) &= G(\xi, \infty) = 0 \end{aligned}$$

For the constant heat flux (CHF) case, the governing equations along with the boundary conditions for  $u_w/u_\infty \leq 1$  can be expressed in the form [10]

$$(2.10) \quad F''' + 2^{-1}FF'' + \xi_1 H = (3/2)\xi_1(F'\partial F'/\partial \xi_1 - F''\partial F/\partial \xi_1),$$

$$(2.11) \quad \text{Pr}^{-1}H'' + 2^{-1}(FH' - F'H) = (3/2)\xi_1(F'\partial H/\partial \xi_1 - H'\partial F/\partial \xi_1),$$

$$(2.12) \quad F(\xi_1, 0) = F'(\xi_1, 0) - (u_w/u_\infty) = H'(\xi_1, 0) + 1 = 0,$$

$$F'(\xi_1, \infty) - 1 = H(\xi_1, \infty) = 0.$$

When  $u_w/u_\infty > 1$ , the boundary conditions are

$$(2.13) \quad F(\xi_1, 0) = F'(\xi_1, 0) - 1 = H'(\xi_1, 0) + 1 = 0,$$

$$F'(\xi_1, \infty) - (u_\infty/u_w) = H(\xi_1, \infty) = 0.$$

For  $u_w/u_\infty \leq 1$

$$(2.14) \quad \eta = (u_\infty/\nu x)^{1/2}y, \xi_1 = \xi_1(x) = \text{Gr}_x^*/\text{Re}_x^{5/2},$$

$$F(\xi_1, \eta) = \psi(x, y)/(\nu u_\infty x)^{1/2},$$

$$H(\xi_1, \eta) = (T - T_\infty)\text{Re}_x^{1/2}/(q_w x/k),$$

$$\text{Gr}_x^* = g\beta q_w x^4/k\nu^2, F'(\xi_1, \eta) = u/u_\infty.$$

For  $u_w/u_\infty > 1$ ,  $u_\infty$  is replaced by  $u_w$  in (2.14).

From the solutions of Eqs. (2.6) and (2.7) under conditions (2.8) and (2.9) and Eqs. (2.10) and (2.11) under conditions (2.12) and (2.13), we can obtain the stream function, velocity and temperature in dimensionless form for both CWT and CHF cases ( $f, f', G, F, F', H$ ).

It may be noted that Eqs. (2.6) and (2.7) under conditions (2.8) and Eqs. (2.10) and (2.11) under conditions (2.12) for  $u_w/u_\infty = 0$  (i.e., for a stationary surface) reduce to those of MUCOGLU and CHEN [10] who studied the mixed convection flow over a stationary surface. Similarly, Eqs. (2.6) and (2.7) under conditions (2.9) and Eqs. (2.10) and (2.11) under conditions (2.13) for  $u_\infty/u_w = 0$ , (i.e., for a moving surface in a quiescent liquid) reduce to those of MOUTSOGLU and CHEN [6] who studied the effect of the buoyancy force on the moving surface. Also, Eqs. (2.6) and (2.7) under conditions (2.8) and (2.9) for  $\xi = 0$  (i.e., no buoyancy force) reduce to those of ABDELHAFEZ [2].

The local skin friction coefficient and the local Nusselt number (heat transfer coefficient) for the CWT case are expressed in the form [10].

$$(2.15) \quad C_{fx} = 2\mu(u_y)_w/\rho U^2 = 2\text{Re}_x^{-1/2}f''(\xi, 0),$$

$$\text{Nu}_x = -k(T_y)_w x/(T_w - T_\infty) = -\text{Re}_x^{1/2}G'(\xi, 0).$$

The corresponding results for the CHF case are given by [10]

$$\begin{aligned}
 (2.16) \quad C_{f_x} &= 2\text{Re}_x^{-1/2} F''(\xi_1, 0), \\
 \text{Nu}_x &= \text{Re}_x^{1/2} / H(\xi_1, 0).
 \end{aligned}$$

Here the reference velocity  $U$  is either  $u_\infty$  or  $u_w$ .

It may be remarked that for  $\xi = 0$  (CWT case), Eqs. (2.6) and (2.7) and for  $\xi_1 = 0$  (CHF case), Eqs. (2.10) and (2.11), reduce to ordinary differential equations and they, under conditions (2.8) and (2.12), admit a closed-form solution when  $u_w = u_\infty$ . The solution for the CWT case is given by

$$\begin{aligned}
 (2.17) \quad f(\eta) &= \eta, \quad G(\eta) = \text{erfc} \left[ (\text{Pr})^{1/2} \eta/2 \right], \\
 G'(0) &= -(\text{Pr}/\pi)^{1/2}.
 \end{aligned}$$

For the CHF case, the solution is expressed as

$$\begin{aligned}
 (2.18) \quad F(\eta) &= \eta, \\
 H(\eta) &= -2(B\text{Pr})^{-1} \exp \left[ -\text{Pr}(\text{Pr} + 2)\eta^2/16 \right] \\
 &\quad \times \left[ A_1 F_1 \left( \frac{2 + 3\text{Pr}}{8}, \frac{1}{2}, \frac{(\text{Pr}\eta)^2}{8} \right) \right. \\
 &\quad \left. + B(\text{Pr}\eta/2) {}_1F_1 \left( \frac{6 + 3\text{Pr}}{8}, \frac{3}{2}, \frac{(\text{Pr}\eta)^2}{8} \right) \right],
 \end{aligned}$$

where the confluent hypergeometric function  ${}_1F_1$  is given by

$$\begin{aligned}
 (2.19) \quad {}_1F_1(a, b; x) &= \sum_{i=0}^{\infty} C_i x^i, \quad C_i = \frac{a(a+1) \cdots (a+i-1)}{b(b+1) \cdots (b+i-1)} \frac{1}{i!}, \\
 A &= \Gamma(1/2)/\Gamma(6 + 3\text{Pr})/8, \quad B = 2^{1/2} [\Gamma(2 + 3\text{Pr})/8] / \Gamma(-1/2).
 \end{aligned}$$

The above results are found to be in very good agreement with those obtained numerically (they differ by less than 0.5 per cent).

The correlation equations for the local Nusselt number ( $\text{Nu}_x$ ) involving Prandtl number ( $\text{Pr}$ ), buoyancy parameter ( $\xi$  or  $\xi_1$ ) and the ratio of free stream and wall velocities ( $u_\infty/u_w$ ) have been obtained following the analysis of CHURCHILL [11] and RAMACHANDRAN *et al.* [8]. The correlation equation for the local Nusselt number ( $\text{Nu}_x$ ) for the CWT case which is valid for  $0.5 \leq \text{Pr} \leq 50$ ,  $-0.5 \leq \xi \leq 5$ ,  $-0.25 \leq (u_\infty/u_w) \leq 1$ , is given by

$$(2.20) \quad (\text{Nu}_x/\text{Nu}_{x1})^3 = 1 \pm (\text{Nu}_{xN}/\text{Nu}_{x1})^3$$

which can be expressed as

$$(2.21) \quad Y^3 = 1 \pm X^3,$$

where

$$(2.22) \quad Y = (\text{Nu}_x/\text{Nu}_{x1}) , \quad X = (\text{Nu}_{xN}/\text{Nu}_{x1}) .$$

The local Nusselt number for the pure forced convection flow ( $\text{Nu}_{x1}$ ) can be expressed as

$$(2.23) \quad \text{Nu}_{x1} = F_1(\text{Pr}, \xi, \lambda) (\text{Re}_x)^{1/2} , \quad \lambda = u_\infty/u_w \leq 1,$$

where

$$(2.24) \quad F_1(\text{Pr}, \xi, \lambda) = [1.8865(\text{Pr})^{11/12} - 1.4447(\text{Pr})^{1/4}] [1 + (0.325)^{(1+\xi^{1/2})} \lambda] .$$

The local Nusselt number for the pure free convection flow ( $\text{Nu}_{xN}$ ) is given by [12]

$$(2.25) \quad \text{Nu}_{xN} = F_2(\text{Pr})(\text{Gr}_x)^{1/4},$$

where

$$(2.26) \quad F_2(\text{Pr}) = 0.75\text{Pr}^{1/2} [2.5 (1 + 2\text{Pr}^{1/2} + 2\text{Pr})]^{-1/4} .$$

When  $\lambda_1 = u_w/u_\infty < 1$ ,  $F_1(\text{Pr}, \xi, \lambda_1)$  is given by

$$(2.27) \quad F_1(\text{Pr}, \xi, \lambda_1) = [1.7732(\text{Pr})^{11/12} - 1.4412(\text{Pr})^{1/4}] \times [1 + (0.5001)^{(1+\xi^{5/6})} \lambda] .$$

The relation (2.17) holds good for the CHF case also, but  $F_1$  and  $F_2$  are replaced by  $G_1$  and  $G_2$ , respectively, and they are expressed as

$$(2.28) \quad G_1(\text{Pr}, \xi_1, \lambda) = [2.8452(\text{Pr})^{13/32} - 2.0947(\text{Pr})^{1/4}] \times [1 + (0.150)^{(1+\xi_1)^{1/4}} \lambda] ,$$

$$(2.29) \quad G_2(\text{Pr}) = (\text{Pr})^{2/5} [4 + 9(\text{Pr})^{1/2} + 10\text{Pr}]^{-1/5} .$$

For  $\lambda_1 < 1$

$$(2.30) \quad G_1(\text{Pr}, \xi_1, \lambda_1) = [2.7347(\text{Pr})^{13/12} - 2.3342(\text{Pr})^{1/4}] \times [1 + (0.702)^{(1+\xi_1)^{6/5}} \lambda_1] .$$

These correlations give results which are accurate to within 8 percent of the predicted results.

### 3. Results and discussion

The nonlinear coupled partial differential Eqs. (2.6) and (2.7) under conditions (2.8) and (2.9) and Eqs. (2.10) and (2.11) under conditions (2.12) and (2.13) have been solved numerically using an implicit finite-difference scheme which is described in detail in [13]. In order to assess the accuracy of our method, we have compared our skin friction and heat transfer results ( $2^{-1}\text{Re}_x^{1/2} C_{fx}$ ,  $\text{Re}_x^{-1/2}\text{Nu}_x$ ) for  $u_w/u_\infty = 0$  with those of MUCOGLU and CHEN [10], and for  $u_\infty/u_w = 0$  with those of MOUTSOGLU and CHEN [6]. Also for  $\xi = 0$ , we have compared our skin friction and heat transfer results for the CWT case with those of ABDELHAFEZ [2]. In all the cases the results are found to be in very good agreement (the maximum difference is about 1.5 per cent). The comparison is shown in Tables 1 and 2 and Figs. 2 and 3.

**Table 1.** Comparison of skin friction and heat transfer parameters for constant wall temperature case (CWT) when  $u_\infty/u_w = 0$  in Eq. (2.9).

Pr	$\xi$	Present results		MOUTSOGLU and CHEN [6]	
		$f''(\xi, 0)$	$-G'(\xi, 0)$	$f''(\xi, 0)$	$-G'(\xi, 0)$
0.7	0.0	-0.44370	0.35190	-0.44375	0.34924
0.7	0.5	-0.10676	0.41307	-0.10558	0.41320
0.7	1.0	0.19090	0.44928	0.19425	0.45505
0.7	2.0	0.72831	0.49883	0.73552	0.50031
0.7	3.0	1.21948	0.53468	1.23103	0.53681
0.7	4.0	1.68038	0.56342	1.69652	0.56609
7.0	0.0	-0.44370	1.37164	-0.44375	1.38703
7.0	0.5	-0.28239	1.39670	-0.28376	1.41322
7.0	1.0	-0.12624	1.41978	-0.12876	1.43712
7.0	2.0	0.17354	1.46130	0.16880	1.48026
7.0	3.0	0.45995	1.49803	0.45318	1.51864
7.0	4.0	0.73573	1.53110	0.72697	1.55334

Figure 2 shows the local Nusselt number ( $\text{Re}_x^{-1/2}\text{Nu}_x$ ) and local skin friction coefficient ( $\text{Re}_x^{1/2}C_{fx}/4$ ) for both CWT and CHF cases as a function of velocity difference ( $1 - u_w/u_\infty$ ) when  $u_w/u_\infty \leq 1$  for three values of the buoyancy parameter  $\xi$  or  $\xi_1$ . For a given buoyancy parameter  $\xi$  or  $\xi_1$ , the local skin friction coefficient ( $\text{Re}_x^{1/2}C_{fx}/4$ ) increases as the velocity difference ( $1 - u_w/u_\infty$ ) increases, but the local Nusselt number ( $\text{Re}_x^{-1/2}\text{Nu}_x$ ) decreases. The effect is



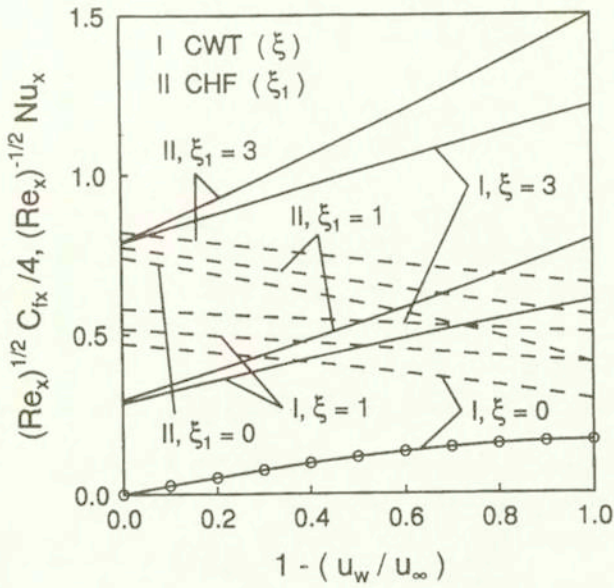


FIG. 2. Local Nusselt number  $(Re_x^{-1/2} Nu_x)$  and local skin friction coefficient  $(Re_x^{1/2} C_{fx} / 4)$  for  $Pr = 0.7$  and  $u_w / u_\infty \leq 1$ . —  $Re_x^{1/2} C_{fx} / 4$ ; - - - -  $Re_x^{-1/2} Nu_x$ ; o, Ref. [2].

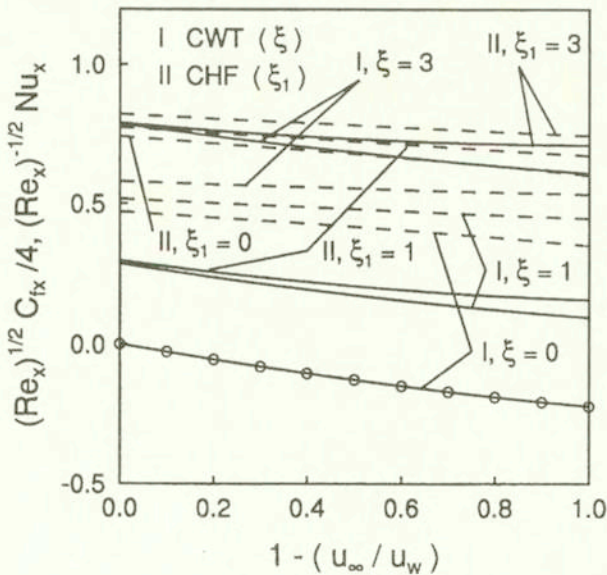


FIG. 3. Local Nusselt number  $(Re_x^{-1/2} Nu_x)$  and local skin friction coefficient  $(Re_x^{1/2} C_{fx} / 4)$  for  $Pr = 0.7$  and  $u_w / u_\infty \geq 1$ . —  $Re_x^{1/2} C_{fx} / 4$ ; - - - -  $Re_x^{-1/2} Nu_x$ ; o, Ref. [2].

more pronounced on the skin friction coefficient than on the Nusselt number, because velocity profiles and velocity gradient at or near the wall are strongly affected by the difference in the velocities at the wall and the free stream. On the other hand, this velocity difference affects the Nusselt number indirectly. Similarly, for a given velocity difference  $(1 - u_w/u_\infty)$ , both the skin friction and Nusselt number increase with the buoyancy parameter  $\xi$  or  $\xi_1$ . When  $\xi$  or  $\xi_1 > 0$ , the buoyancy parameter acts as a favourable pressure gradient which accelerates the fluid what results in thinner momentum and thermal boundary layers. This in turn increases both the skin friction and heat transfer. For  $\xi = \xi_1 = 0$ , the momentum equation is uncoupled with the energy equation. Hence, the skin frictions for both CWT and CHF cases are identical.

**Table 2.** Comparison of skin friction and heat transfer parameters for constant heat flux (CHF) when  $u_\infty/u_w = 0$  in Eq. (2.13).

Pr	$\xi_1$	Present results		MOUTSOGLOU and CHEN [6]	
		$F''(\xi_1, 0)$	$H(\xi_1, 0)$	$F''(\xi_1, 0)$	$H(\xi_1, 0)$
0.7	0.0	-0.44370	1.65905	-0.44375	1.66240
0.7	0.5	-0.02949	1.55216	-0.02740	1.55428
0.7	1.0	0.31673	1.48657	0.32209	1.48674
0.7	2.0	0.90802	1.40075	0.91813	1.39925
0.7	3.0	1.42089	1.34279	1.43543	1.34001
0.7	4.0	1.88434	1.29907	1.90290	1.29536
7.0	0.0	-0.44370	0.45287	-0.44375	0.44878
7.0	0.5	-0.38610	0.45186	-0.38688	0.44761
7.0	1.0	-0.32921	0.45087	-0.33072	0.44656
7.0	2.0	-0.21748	0.44895	-0.22051	0.44453

Figure 3 shows the skin friction and Nusselt number ( $Re_x^{1/2} C_{fx}/4$ ,  $Re_x^{-1/2} Nu_x$ ) as a function of the velocity difference  $(1 - u_\infty/u_w)$  when  $u_w/u_\infty \geq 1$  for three values of the buoyancy parameter. The results for both CWT and CHF cases have been presented in Fig. 3. It is found that the Nusselt number for  $u_w/u_\infty > 1$  is greater than that for  $u_w/u_\infty < 1$ . The maximum difference for the CWT case is about 17 percent and for the CHF case it is about 50 percent. When  $u_w/u_\infty > 1$ , the skin friction  $Re_x^{1/2} C_{fx} < 0$  for  $\xi = \xi_1 = 0$  and it becomes positive in the range  $0 \leq 1 - u_\infty/u_w \leq 1$  when the buoyancy parameter exceeds a certain critical value.

The skin friction and heat transfer ( $4^{-1} C_{fx}(Re_x)^{1/2}$ ,  $(Re_x)^{-1/2} Nu_x$ ) for both CWT and CHF cases for opposing flow ( $\xi < 0, \xi_1 < 0$ ) and assisting flow ( $\xi > 0, \xi_1 > 0$ ) are shown in Fig. 4. The skin friction and heat transfer increase for assisting flow ( $\xi > 0, \xi_1 > 0$ ) and decrease for opposing flow ( $\xi < 0, \xi_1 < 0$ ), because for assisting flow the buoyancy force acts like a favourable pressure

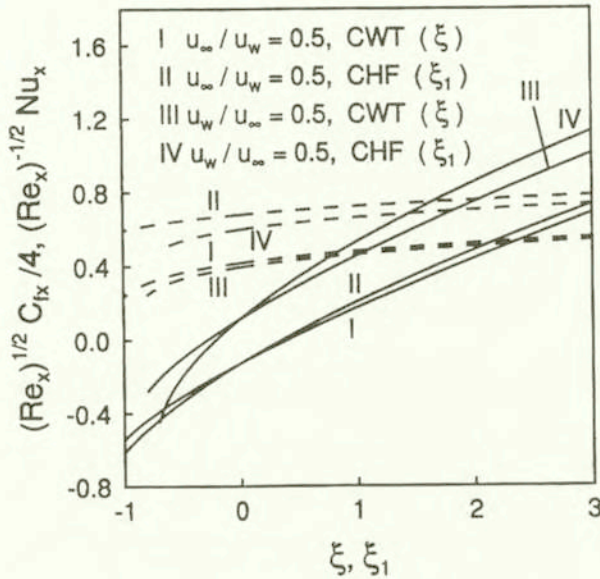


FIG. 4. Local Nusselt number  $(Re_x^{-1/2} Nu_x)$  and local skin friction coefficient  $(Re_x^{1/2} C_{fx}/4)$  for  $Pr = 0.7$  ———  $Re_x^{1/2} C_{fx}/4$ ; - - - -  $Re_x^{-1/2} Nu_x$ .

gradient, and for opposing flow it acts like an adverse pressure gradient. The skin friction ( $f''(\xi, 0)$  or  $F''(\xi_1, 0)$ ) vanishes for certain value  $\xi$  or  $\xi_1$ . Since it is a moving wall case, the vanishing of skin friction does not imply separation of flow. It is also negative in a certain range of  $\xi$  or  $\xi_1$  which implies that the fluid is being dragged by the plate. For opposing flow, the solution does not converge beyond a certain critical value of  $\xi$  or  $\xi_1$ .

The skin friction and heat transfer for the CWT case for both upstream ( $u_w/u_\infty < 0$ ) and downstream ( $u_w/u_\infty > 0$ ) moving wall are presented in Fig. 5. It is found that for the upstream moving wall ( $u_w/u_\infty < 0$ ), the solution does not exist beyond a certain critical value of  $u_w/u_\infty < 0$  (say  $(u_w/u_\infty)_0$ ). For the CHF case similar trend is also observed, but is not presented here for the sake of brevity.

Figure 6 shows the variation of the Nusselt number  $((Re)_x^{-1/2} Nu_x)$  with the buoyancy parameter  $\xi$  for the CWT case starting from the pure forced convection flow ( $\xi = 0$ ) to pure free convection flow ( $\xi \rightarrow \infty$ ). The forced and free convection asymptotes are also shown. The forced convection ( $\xi = 0$ ) asymptote for CWT case is given [14] as

$$Nu_x(Re_x)^{-1/2} = 0.293 \quad \text{when} \quad u_w/u_\infty = 0, \quad Pr = 0.7,$$

$$Nu_x(Re_x)^{-1/2} = 0.3526 \quad \text{when} \quad u_w/u_\infty = 0.5, \quad Pr = 0.7.$$

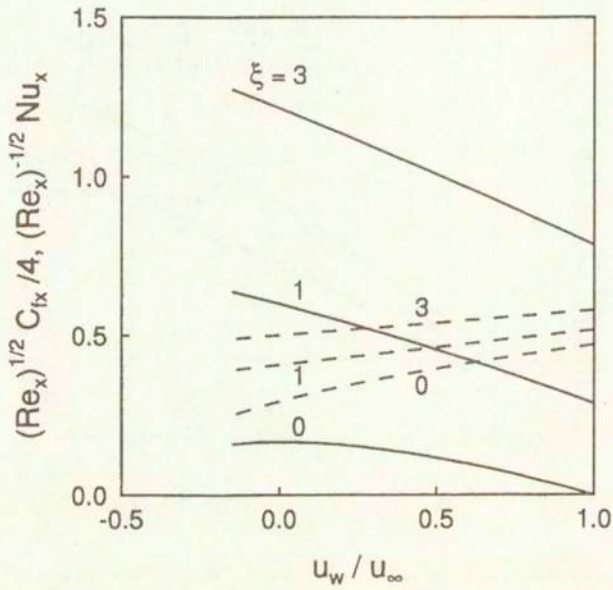


FIG. 5. Local Nusselt number  $(Re_x^{-1/2}Nu_x)$  and local skin friction coefficient  $(Re_x^{1/2}C_{fx}/4)$  for the CWT case with  $Pr = 0.7$  ———  $Re_x^{1/2}C_{fx}/4$ ; - - - -  $Re_x^{-1/2}Nu_x$ .

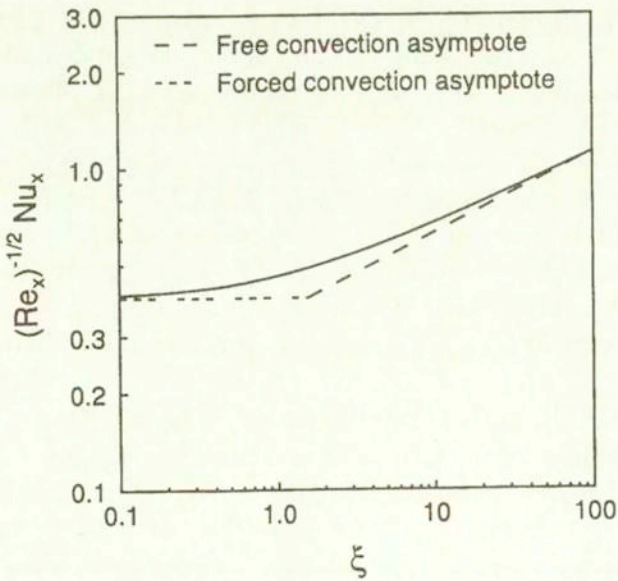


FIG. 6. Local Nusselt number  $(Re_x^{-1/2}Nu_x)$  for the CWT case when  $Pr = 0.7$  and  $u_w/u_\infty = 0.5$ .

The Nusselt number for the free convection flow over a vertical plate for the CWT case with  $Pr = 0.7$  can be expressed as [15]

$$Nu_x = 2^{-1/2} (Gr_x)^{1/4} \quad G'(\xi, 0) = 2^{-1/2} (Gr_x)^{1/4} (0.49950).$$

From the above expression, the free convection asymptote can be written as

$$Nu_x(Re_x)^{-1/2} = 0.3532(\xi)^{1/4}.$$

The value of  $G'(\xi, 0)$  for  $Pr = 0.7$  was found to be 0.49951 by HASAN and EICHHORN [16].

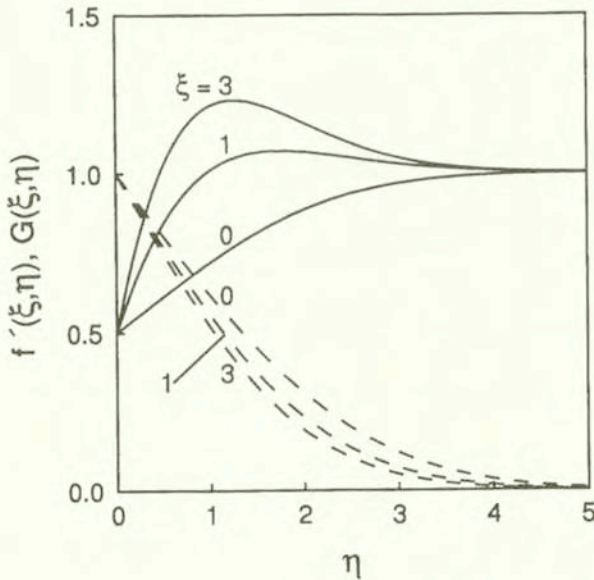


FIG. 7. Velocity and temperature profiles ( $f'(\xi, \eta), G(\xi, \eta)$ ) for the (CWT) case when  $\xi = 0, 1, 3, u_w/u_\infty = 0.5, Pr = 0.7$ . —  $f'(\xi, \eta)$ ; - - - -  $G(\xi, \eta)$ .

The effect of the buoyancy parameter  $\xi$  on the velocity and temperature profiles ( $f'(\xi, \eta), G(\xi, \eta)$ ) for the CWT case when  $u_w/u_\infty = 0.5$  and  $Pr = 0.7$  is shown in Fig. 7. The corresponding velocity and temperature profiles for the CHF case ( $F'(\xi_1, \eta), H(\xi_1, \eta)$ ) are presented in Fig. 8. The velocity profiles for the CHF case are qualitatively and to some extent quantitatively similar to those of the CWT case. In both the cases, there is a velocity overshoot (i.e., the velocity at a certain value of  $\eta$  exceeds the velocity at the edge of the boundary layer) when the buoyancy parameter  $\xi$  or  $\xi_1$  exceeds a certain value (say  $\xi = \xi_0$  or  $\xi_1 = \xi_{10}$ ). Both  $\xi_0$  and  $\xi_{10}$  are nearly equal to 1 when  $u_w/u_\infty = 0.5$  and  $Pr = 0.7$ . The reason for the velocity overshoot is that the positive buoyancy force ( $\xi > 0$  or  $\xi_1 > 0$ ) acts like a favourable pressure gradient and accelerates the fluid within

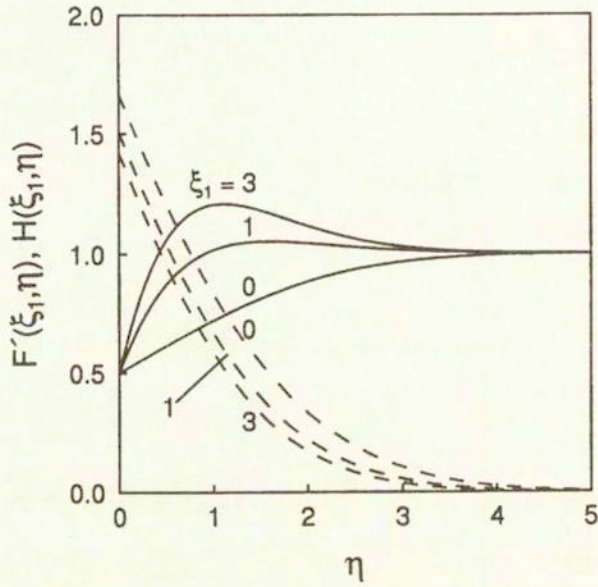


FIG. 8. Velocity and temperature profiles ( $F'(\xi_1, \eta), H(\xi_1, \eta)$ ) for the CHF case when  $\xi = 0, 1, 3, u_w/u_\infty = 0.5, Pr = 0.7$ . —  $F'(\xi_1, \eta)$ ; - - - -  $H(\xi_1, \eta)$ .

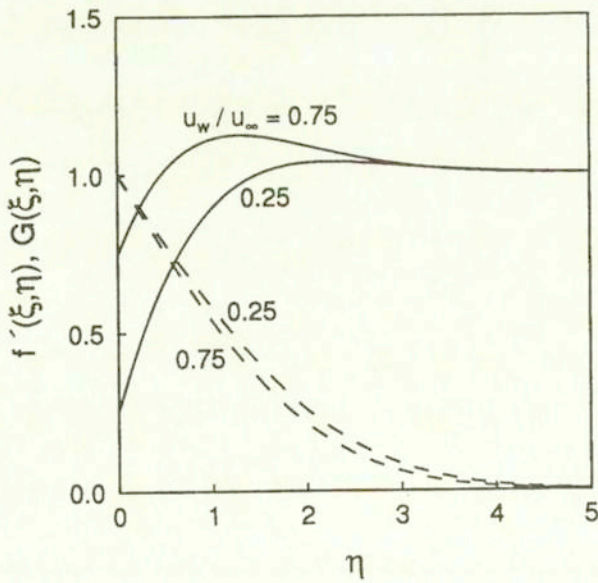


FIG. 9. Velocity and temperature profiles ( $f'(\xi, \eta), G(\xi, \eta)$ ) for the CWT case when  $u_w/u_\infty = 0.25, 0.75, \xi = 1, Pr = 0.7$ . —  $f'(\xi, \eta)$ ; - - - -  $G(\xi, \eta)$ .

the boundary layer. This in turn increases the velocity beyond its edge value in a certain region near the wall. The temperature profiles ( $G(\xi, \eta), H(\xi_1, \eta)$ ) become more steep with increasing  $\xi$  or  $\xi_1$  due to the reduction in the thermal boundary layer thickness.

The effect of the wall temperature  $u_w/u_\infty$  on the velocity and temperature profiles for the CWT and CHF cases ( $f'(\xi, \eta), G(\xi, \eta), F'(\xi_1, \eta), H(\xi_1, \eta)$ ) when  $\xi = \xi_1 = 1$ ,  $\text{Pr} = 0.7$  are displayed in Figs. 9 and 10, respectively. The magnitude of the velocity overshoot increases with increasing wall velocity because it imports additional momentum to the boundary layer which enhances the velocity. Also the temperature profiles become more steep with increasing wall velocity.

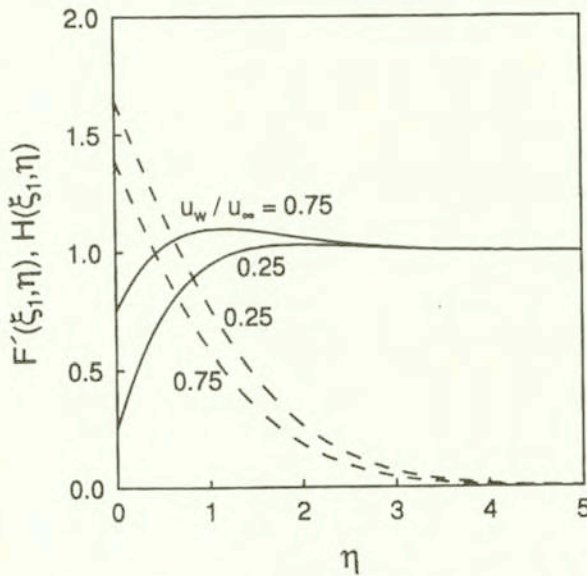


FIG. 10. Velocity and temperature profiles ( $F'(\xi_1, \eta), H(\xi_1, \eta)$ ) for the CHF case when  $u_w/u_\infty = 0.25, 0.75$ ,  $\xi = 1$ ,  $\text{Pr} = 0.7$ . —  $F'(\xi_1, \eta)$ ; - - -  $H(\xi_1, \eta)$ .

#### 4. Conclusions

For the same velocity difference and the buoyancy parameter, the Nusselt number for  $u_w/u_\infty > 1$  is found to be greater than that for  $u_w/u_\infty < 1$ . The Nusselt number and skin friction increase for assisting flow and decrease for opposing flow. For opposing flow or for an upstream moving wall, the solution does not exist beyond a certain critical value of the buoyancy parameter ( $\xi$  or  $\xi_1$ ) or  $u_w/u_\infty$ . The correlation equations for estimating the local Nusselt number have been obtained, and they agree very well with the predicted results. There is an overshoot in the velocity profiles if the buoyancy parameter or the wall velocity exceeds a certain value.

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