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Thermosolutal instability of Walters' rotating fluid (Model B′) in porous medium

R. C. SHARMA

Department of Mathematics, Himachal Pradesh University
Summer Hill, (H.P.)–171 005 India

SUNIL

Department of Applied Sciences Regional Engineering College
Hamipur, (H.P.)–177 005 India

SURESH CHAND

Department of Mathematics, Government Degree College
Palampur, (H.P.) India

THE THERMOSOLUTAL INSTABILITY of Walters' (Model B′) fluid in porous medium is considered in the presence of uniform vertical rotation. For the case of stationary convection, the stable solute gradient and rotation have stabilizing effects on the system, whereas the medium permeability has a destabilizing (or stabilizing) effect on the system under certain conditions. The dispersion relation is also analysed numerically. It has also been shown that as rotation parameter increases, the stabilizing range of medium also increases. The kinematic viscoelasticity has no effect on the stationary convection. The stable solute gradient, rotation, porosity and kinematic viscoelasticity introduce oscillatory modes in the system, which did not occur in their absence. The sufficient conditions for the non-existance of overstability are also obtained.

Key words: Thermosolutal instability, Walters' (Model B′) fluid, Rotation, Porous medium.

1. Introduction

A DETAILED ACCOUNT of the theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics and hydromagnetics has been given in the celebrated monograph by CHANDRASEKHAR [1]. The problem of thermohaline convection in a layer of fluid heated from below and subjected to a stable salinity gradient has been considered by VERONIS [2]. The physics is quite similar to the stellar case in that helium acts like salt in raising the density and in diffusing more slowly than heat. The conditions under which convective motions are important in stellar atmospheres are usually far removed from consideration of a single component fluid and rigid boundaries, and therefore it is desirable to consider a fluid acted on by a solute gradient and free boundaries. The problem of the onset of thermal instability in the presence of a solute gradient is of great importance because of its applications to atmospheric physics and astrophysics, especially in the case of the ionosphere and the outer layer of the atmosphere. The thermosolutal convection problems also arise in oceanography, limnology and engineering.

With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. The WALTERS' [3] fluid (Model B$'$) is one such fluid. In another study, SHARMA and KUMAR [4] have studies the steady flow and heat transfer of Walters' fluids (Model B$'$) through a porous pipe of uniform circular cross-section with small suction. SHARMA and KUMAR [5], recently studied the stability of the plane interface separating two viscoelastic Walters' (Model B$'$) fluids of uniform densities and found that for stable configuration, the system is stable or unstable under certain conditions.

In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. A great number of applications in geophysics may be found in a recent book by PHILIPS [6]. When the fluid permeates through a porous material, the gross effect is represented by the Darcy law. As a result of this macroscopic law, the usual viscous term in the equation of Walters' fluid (Model B$'$) motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)\mathbf{q}\right]$, where μ and μ' are the viscosity and viscoelasticity of the Walters' fluid, k_1 is the medium permeability and $\mathbf{q}$ is the Darcian (filter) velocity of the fluid. The problem of thermosolutal convection in fluids in porous medium is of great importance in geophysics, soil sciences, ground water hydrology and astrophysics. Generally, it is accepted that comets consist of a dust "snowball" made of a mixture of frozen gases which, in the process of their journey, changes from solid to gas and vice-versa. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in astrophysical context (MCDONNEL [7]). In recent study, SHARMA *et al.* [8] studied the instability of streaming Walters' viscoelastic fluid B$'$ in a porous medium. In many astrophysical situations, the effect of rotation on thermosolutal convection in a porous medium is also important. Relative to a large volume of published studies on this phenomenon in pure fluids, the thermosolutal convection in porous medium has received only attention, although it has interesting engineering applications: the migration of moisture through the air contained in fibrous insulation, grain storage installations, food processing and the underground spreading of chemical pollutants. Thermosolutal convection in porous medium is also of interest in geophysical systems, electrochemistry and metallurgy. A comprehensive review of the literature concerning thermosolutal convection in a fluid-saturated porous medium may be found in the book by NIELD and BEJAN [9].

Keeping in mind the importance of non-Newtonian fluids in geophysics, soil physics, ground water hydrology, modern technology and various applications mentioned above, the thermosolutal instability of a Walters' (Model B$'$) fluid in porous medium in the presence of uniform vertical rotation, has been considered in the present paper.

2. Formulation of the problem and perturbation equations

Here we consider an infinite, horizontal, incompressible Walters' (Model B$'$) layer of thickness d, heated and soluted from below so that the temperatures, densities and solute concentrations at the bottom surface $z = 0$ are T_0, ρ_0 and C_0, and at the upper surface $z = d$ are T_d, ρ_d and C_d, respectively, and that a uniform temperature gradient $\beta(=\mid dT/dz \mid)$ and a uniform solute gradient $\beta'(=\mid dC/dz \mid)$ are maintained. The gravity field $\mathbf{g}(0, 0, -g)$ and a uniform vertical rotation $\mathbf{\Omega}(0, 0, \Omega)$ act on the system. This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1.

Let $p, \rho, T, C, \alpha, \alpha', g$ and $\mathbf{q}(u, v, w)$ denote, respectively, the fluid pressure, density, temperature, solute concentration, thermal coefficient of expansion, an analogous solvent coefficient of expansion, gravitational acceleration and fluid velocity. The equations expressing the conservation of momentum, mass, temperature, solute concentration and equation of state of Walters' (Model B$'$) fluid are

$$\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)\mathbf{q}\right] = -\left(\frac{1}{\rho_0}\right)\nabla p + \mathbf{g}\left(1 + \frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{2}{\varepsilon}(\mathbf{q}\times\mathbf{\Omega}), \tag{2.1}$$

$$\nabla\cdot\mathbf{q} = 0, \tag{2.2}$$

$$E\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)T = \kappa\nabla^2 T, \tag{2.3}$$

$$E'\frac{\partial C}{\partial t} + (\mathbf{q}\cdot\nabla)C = \kappa'\nabla^2 C, \tag{2.4}$$

$$\rho = \rho_0\left[1 - \alpha(T - T_0) + \alpha'(C - C_0)\right], \tag{2.5}$$

where the suffix zero refers to values at the reference level $z = 0$ and in writing Eq. (2.1) use has been made of Boussinesq approximation. The kinematic viscosity v, the kinematic viscoelasticity v', the thermal diffusivity κ and the solute diffusivity κ' are all assumed to be constants. Here $E = \varepsilon + (1-\varepsilon)\left(\dfrac{\rho_s\, c_s}{\rho_0\, c_i}\right)$ is a constant and E' is a analogous to E but corresponding to solute rather than heat. ρ_s, c_s and ρ_0, c_i stand for density and heat capacity of solid (porous matrix) material and fluid, respectively. The steady state solution is

$$\text{(2.6)} \qquad \begin{aligned} &\mathbf{q} = (0,0,0)\,, && T = -\beta z + T_0\,, \\ &C = -\beta' z + C_0\,, && \rho = \rho_0(1 + \alpha\beta z - \alpha'\beta' z)\,. \end{aligned}$$

Here we use the linearized stability theory and the normal mode method. Consider a small perturbation on the steady state solution and let $\delta p, \delta\rho, \theta, \gamma$ and $\mathbf{q}(u, v, w)$ denote, respectively, the perturbation in pressure p, density ρ, temperature T, solute concentration C and velocity $\mathbf{q}(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbations θ and γ in temperature and concentration, is given by

$$\text{(2.7)} \qquad \delta\rho = -\rho_0(\alpha\theta - \alpha'\gamma)\,.$$

Then the linearized perturbation equations become

$$\text{(2.8)} \qquad \frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{\rho_0}(\nabla\delta p) - \mathbf{g}(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(v - v'\frac{\partial}{\partial t}\right)\mathbf{q} + \frac{2}{\varepsilon}(\mathbf{q}\times\mathbf{\Omega})\,,$$

$$\text{(2.9)} \qquad \nabla\cdot\mathbf{q} = 0\,,$$

$$\text{(2.10)} \qquad E\frac{\partial\theta}{\partial t} = \beta w + \kappa\nabla^2\theta\,,$$

$$\text{(2.11)} \qquad E'\frac{\partial\gamma}{\partial t} = \beta' w + \kappa'\nabla^2\gamma\,.$$

3. The dispersion relation

Analysing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$\text{(3.1)} \qquad [w, \theta, \gamma, \zeta] = [W(z), \Theta(z), \Gamma(z), Z(z)]\exp(ik_x x + ik_y y + nt)\,.$$

where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k = \sqrt{(k_x^2 + k_y^2)}$ is the resultant wave number and n is the growth rate which is, in general, a complex constant. $\zeta = \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}$ stands for the z-component of vorticity.

Expressing the coordinates x, y, z in the new unit of length d and letting $a = kd, \sigma = \dfrac{nd^2}{v}, p_1 = \dfrac{v}{\kappa}, q = \dfrac{v}{\kappa'}, F = \dfrac{v}{d^2}, P_l = \dfrac{k_1}{d^2}$, and $D = \dfrac{d}{dz}$, Eqs. (2.8)–(2.11), with the help of expression (3.1), in non-dimensional form become

$$(3.2)\qquad \left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1-\sigma F)\right](D^2-a^2)W+\frac{ga^2d^2}{\upsilon}(\alpha\Theta-\alpha'\Gamma)-\frac{2\Omega d^3}{\varepsilon\upsilon}DZ=0,$$

$$(3.3)\qquad \left[\frac{\sigma}{\varepsilon}+\frac{1}{P_l}(1-\sigma F)\right]Z=\left(\frac{2\Omega d}{\varepsilon\upsilon}\right)DW,$$

$$(3.4)\qquad (D^2-a^2-Ep_1\sigma)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)W,$$

$$(3.5)\qquad (D^2-a^2-E'q\sigma)\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)W.$$

Consider the case where both boundaries are free as well as perfect conductors of both heat and solute concentrations. The case of two free boundaries is a little artificial but it enables us to find analytical solutions and to make some qualitative conclusions. The appropriate boundary conditions, with respect to which Eqs. (3.2)–(3.5) must be solved, are (CHANDRASEKHAR [1])

$$(3.6)\qquad W=D^2W=0,\quad \Theta=0,\quad \Gamma=0,\quad DZ=0,\quad \text{at } z=0 \text{ and } 1.$$

The case of two free boundaries, though a little artificial, is the most appropriate for steller atmospheres (SPIEGEL [10]). Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for $z=0$ and *1* and hence, the proper solution of W characterizing the lowest mode is

$$(3.7)\qquad W=W_0\sin\pi z,$$

where W_0 is a constant.

Eliminating Θ, Γ and Z between Eqs. (3.2)–(3.5) and substituting the proper solution $W=W_0\sin\pi z$, in the resultant equation, we obtain the dispersion relation

$$(3.8)\qquad R_1=\left(\frac{1+x}{x}\right)\left[\frac{i\sigma_1}{\varepsilon}+\frac{1}{P}(1-i\sigma_1F)\right](1+x+iEp_1\sigma_1)$$
$$+T_{A_1}\frac{(1+x+iEp_1\sigma_1)}{x\left(\dfrac{i\sigma_1}{\varepsilon}+\dfrac{1}{P}[1-i\sigma_1F]\right)}+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q\sigma_1)},$$

where

$$R_1=\frac{g\alpha\beta d^4}{\upsilon\kappa\pi^4},\qquad S_1=\frac{g\alpha'\beta' d^4}{\upsilon\kappa'\pi^4},\qquad T_{A_1}=\frac{4\Omega^2d^4}{\upsilon^2\pi^4}=\left(\frac{2\Omega d^2}{\upsilon\pi^2}\right)^2,$$
$$x=\frac{a^2}{\pi^2},\qquad i\sigma_1=\frac{\sigma}{\pi^2}$$

and

$$P = \pi^2 P_l.$$

Equation (3.8) is the required dispersion relation including the effects of rotation, medium permeability, kinematic viscoelasticity and stable solute gradient on the thermosolutal instability of Walters' (Model B′) rotating fluid in a porous medium.

4. The stationary convection

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. Putting $\sigma = 0$, the dispersion relation (3.8) reduces to

$$R_1 = \frac{(1+x)^2}{xP} + PT_{A_1}\frac{(1+x)}{x} + S_1\,, \tag{4.1}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters S_1, T_{A_1} and P. The parameter F accounting for the kinematic viscoelasticity effect vanishes for the stationary convection.

To investigate the effects of stable solute gradient, rotation and medium permeability, we examine the behaviour of $\frac{dR_1}{dS_1}, \frac{dR_1}{dT_{A_1}}$ and $\frac{dR_1}{dP}$ analytically. Equation (4.1) yields

$$\frac{dR_1}{dS_1} = +1\,, \tag{4.2}$$

which implies that the stable solute gradient has a stabilizing effect on thermosolutal instability of Walters' (Model B′) rotating fluid in a porous medium. The reverse solute gradient has a destabilizing effect on the system since then $\frac{dR_1}{dS_1}$ becomes negative. Equation (4.1) also yields

$$\frac{dR_1}{dT_{A_1}} = \left(\frac{1+x}{x}\right)P\,. \tag{4.3}$$

The rotation, therefore, has always a stabilizing effect on the thermosolutal instability of Walters' (Model B′) rotating fluid in a porous medium.

The dispersion relation (4.1) is analysed numerically. In Fig. 1, R_1 is plotted against x for $P = 10, T_{A_1} = 5$; $S_1 = 10$ (for curve *1*), $S_1 = 20$ (for curve *2*) and $S_1 = 30$ (for curve *3*). The stabilizing role of the stable solute gradient is clear from the increase of the Rayleigh number with increasing stable solute gradient

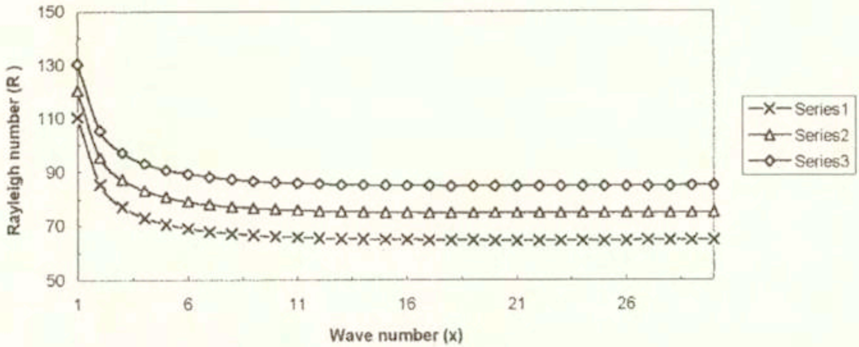

FIG. 1. The variation of Rayleigh number (R_1) with wavenumber (x) for $P = 10, T_{A_1} = 5$; $S_1 = 10$ (for curve *1*), $S_1 = 20$ (for curve *2*), and $S_1 = 30$ (for curve *3*).

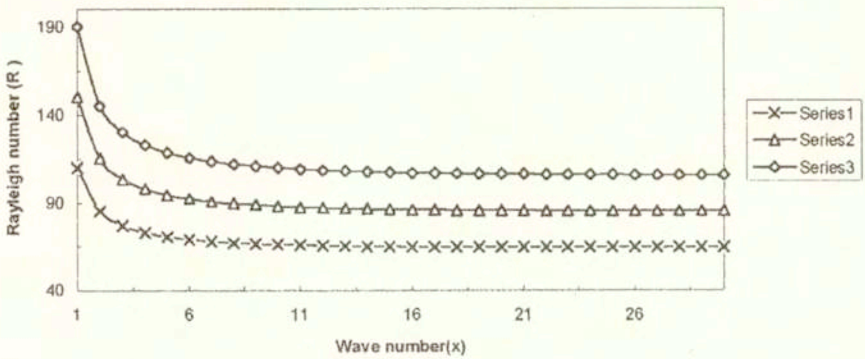

FIG. 2. The variation of Rayleigh number (R_1) with wavenumber (x) for $P = 10, S_1 = 10$; $T_{A_1} = 5$ (for curve *1*), $T_{A_1} = 7$ (for curve *2*), and $T_{A_1} = 9$ (for curve *3*).

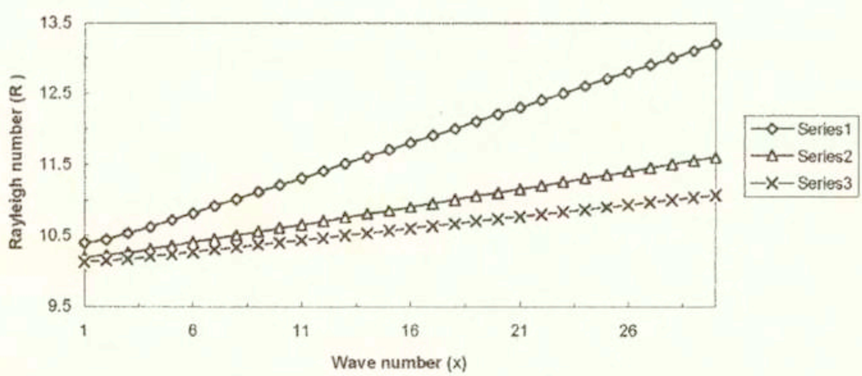

FIG. 3. The variation of Rayleigh number (R_1) with wavenumber (x) for $S_1 = 10, T_{A_1} = 0$; $P = 10$ (for curve *1*), $P = 20$ (for curve *2*), and $P = 30$ (for curve *3*).

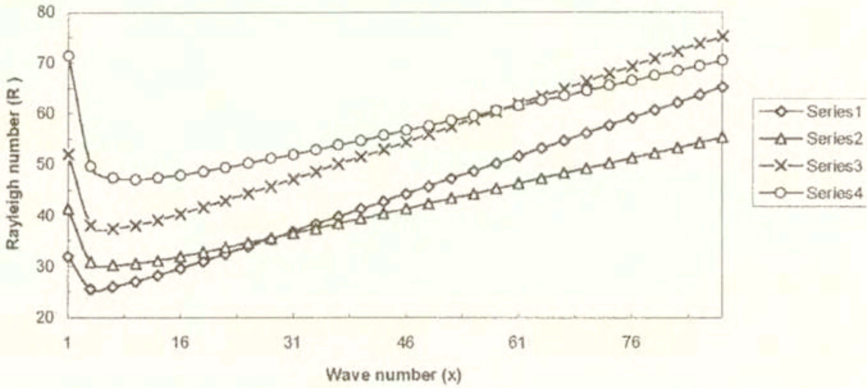

FIG. 4. The variation of Rayleigh number (R_1) with wavenumber (x) for $S_1 = 10$; $P = 2, T_{A_1} = 5$ (for curve *1*), $P = 3, T_{A_1} = 5$ (for curve *2*), $P = 2, T_{A_1} = 10$ (for curve *3*) and $P = 3, T_{A_1} = 10$ (for curve *4*).

parameter S_1 value. Figure 2 gives R_1 plotted against x for $P = 10, S_1 = 10$; $T_{A_1} = 5$ (for curve *1*), $T_{A_1} = 7$ (for curve *2*) and $T_{A_1} = 9$ (for curve *3*). Here we also find the stabilizing role of the rotation as the Rayleigh number increases with the increase in rotation parameter T_{A_1} value. It is evident from (4.1) that

$$\frac{dR_1}{dP} = -\left(\frac{1+x}{x}\right)\left[\frac{1+x}{P^2} - T_{A_1}\right]. \tag{4.4}$$

In the absence of rotation ($T_{A_1} \to 0$), $\dfrac{dR_1}{dP}$ is given by

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{xP^2}, \tag{4.5}$$

which is always negative. The medium permeability, therefore, has a destabilizing effect on thermosolutal instability of Walters' (Model B′) fluid in the absence of rotation. In the presence of rotation, the medium permeability has a destabilizing (or stabilizing) effect on the system if

$$T_{A_1} < (\text{or} >) \frac{1+x}{P^2}. \tag{4.6}$$

It has also been shown graphically that for

i) $S_1 = 10, T_{A_1} = 0$ (i.e. in the absence of rotation); $P = 10$ (for curve *1*), $P = 20$ (for curve *2*) and $P = 30$ (for curve *3*); the medium permeability has always a destabilizing effect (Fig. 3).

ii) $S_1 = 10, T_{A_1} = 5$; $P = 2$, (for curve *1*), $P = 3$, (for curve *2*); the medium permeability has a stabilizing influence for $x < 29$, and for $x > 29$ it has a destabilizing effect (Fig. 4).

iii) $S_1 = 10, T_{AI} = 10; P = 2$, (for curve *3*) and $P = 3$, (for curve *4*); the medium permeability has a stabilizing influence for $x < 59$ and for $x > 59$ it has a destabilizing effect (Fig. 4).

In addition, it has also been shown that as the rotation parameter increases, the stabilizing range of medium premeability also increases (Fig. 4).

5. Stability of the system and oscillatory modes

Here we examine the possibility of oscillatory modes, if any, in stability problem due to the presence of kinematic viscoelasticity, stable solute gradient and rotation. Multiplying (3.2) by W^*, the complex conjugate of W, and using (3.3)–(3.5) together with the boundary conditions (3.6), we obtain

$$\left[\frac{\sigma}{\varepsilon} + \frac{1}{P_l}(1 - \sigma F)\right] I_1 + \left(\frac{g\alpha'\kappa' a^2}{v\beta'}\right)\left[I_4 + E' q \sigma^* I_5\right] + d^2\left[\frac{\sigma^*}{\varepsilon} + \frac{1}{P_l}(1 - \sigma^* F)\right] I_6 - \left(\frac{g\alpha\kappa a^2}{v\beta}\right)\left[I_2 + E p_1 \sigma^* I_3\right] = 0, \tag{5.1}$$

where

$$\begin{aligned} I_1 &= \int_0^1 \left(|DW|^2 + a^2 |W|^2\right) dz, & I_2 &= \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2\right) dz, \\ I_3 &= \int_0^1 \left(|\Theta|^2\right) dz, & I_4 &= \int_0^1 \left(|D\Gamma|^2 + a^2 |\Gamma|^2\right) dz, \\ I_5 &= \int_0^1 \left(|\Gamma|^2\right) dz, & I_6 &= \int_0^1 \left(|Z|^2\right) dz. \end{aligned} \tag{5.2}$$

The integrals $I_1, \ldots, I_6$ are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$ and equating the real and imaginary parts of equation (5.1), we obtain

$$\left[\left(\frac{1}{\varepsilon} - \frac{F}{P_l}\right) I_1 + \frac{g\alpha'\kappa' a^2}{v\beta'} E' q I_5 + d^2\left(\frac{1}{\varepsilon} - \frac{F}{P_l}\right) I_6 - \frac{g\alpha\kappa a^2}{v\beta} E p_1 I_3\right] \sigma_r = -\left[\frac{I_1}{P_l} + \frac{g\alpha'\kappa' a^2}{v\beta'} I_4 + \frac{d^2}{P_l} I_6 - \frac{g\alpha\kappa a^2}{v\beta} I_2\right], \tag{5.3}$$

$$(5.4)\qquad \left[\left(\frac{1}{\varepsilon}-\frac{F}{P_l}\right)I_1-\frac{g\alpha'\kappa'a^2}{\upsilon\beta'}E'qI_5-d^2\left(\frac{1}{\varepsilon}-\frac{F}{P_l}\right)I_6 +\frac{g\alpha\kappa a^2}{\upsilon\beta}Ep_1I_3\right]\sigma_i=0\,.$$

It is evident from (5.3) that σ_r is positive or negative. The system is, therefore, stable or unstable. It is clear from (5.4) that σ_i may be zero or non-zero, meaning that the modes may be non-oscillatory or oscillatory. The oscillatory modes are introduced due to the presence of kinematic viscoelasticity, stable solute gradient and rotation, which were non-existent in their absence.

6. The case of overstability

Here we discuss the possibility of whether instability may occur as overstability. Since we wish to determine the Rayleigh number for the onset of instability via a state of pure oscillations, it suffices to find conditions for which (3.8) will admit the solutions with σ_1 real.

If we equate real and imaginary parts of (3.8) and eliminate R_1 between them, we obtain

$$(6.1)\qquad A_2c_1^2+A_1c_1+A_0=0\,,$$

where we have put $c_1=\sigma_1^2, b=1+x$ and

$$(6.2)\qquad A_2=b\left(1-\frac{\varepsilon F}{P}\right)^2E'^2q^2\left[b\left(1-\frac{\varepsilon F}{P}\right)+\frac{\varepsilon Ep_1}{p}\right],$$

$$(6.3)\qquad A_1=\Bigg\{\left[\left(1-\frac{\varepsilon F}{P}\right)\left(1-\frac{2\varepsilon F}{P}\right)\right]b^4+\left[\frac{\varepsilon}{P}Ep_1\left(1-\frac{2\varepsilon F}{P}+\frac{\varepsilon^2F^2}{P^2}\right)\right]b^3 +\left[\frac{\varepsilon^2}{P^2}E'^2q^2\left(1-\frac{\varepsilon F}{P}\right)\right]b^2+\Bigg[\varepsilon^2E'^2q^2\left(\frac{\overline{\varepsilon Ep_1}}{p^3}-T_{A_1}+\frac{\varepsilon F}{P}T_{A_1}\right) +\varepsilon(b-1)S_1\left(1-\frac{\varepsilon F}{P}\right)\left(Ep_1-E'q+\frac{\varepsilon F}{P}E'q\right)\Bigg]b +\left[\frac{\varepsilon^3}{P}T_{A_1}Ep_1E'^2q^2\right]\Bigg\},$$

$$(6.4)\qquad A_0 = \varepsilon^2 b\Biggl\{\left[\frac{1}{P^2}\left(1-\frac{\varepsilon F}{P}\right)\right]b^3 + \left[\left(\overline{\frac{\varepsilon E p_1}{p^3} - T_{A_1}} + \frac{\varepsilon F}{P}T_{A_1}\right)\right]b^2 + \left[\frac{\varepsilon}{P}T_{A_1}Ep_1\right]b + \left[\frac{\varepsilon}{P^2}(b-1)S_1(Ep_1 - E'q)\right]b\Biggr\}.$$

Since σ_1 is real for overstability, both the values of $c_1(=\sigma_1^2)$ are positive. Equation (6.1) is quadratic in c_1 and does not involve any of its roots to be positive if

$$(6.5)\qquad Ep_1 > E'q\,, \qquad Ep_1 > \frac{P^3 T_{A_1}}{\varepsilon} \quad \text{and} \quad \frac{F}{P} < \frac{1}{\varepsilon},$$

what implies

$$(6.6)\qquad E'\kappa < E\kappa'\,, \qquad \kappa < \frac{\varepsilon v^3 d^2 E}{(2\Omega\pi)^2 k_1^3} \quad \text{and} \quad v < \frac{k_1}{\varepsilon}.$$

Thus $E'\kappa < E\kappa', \kappa < \dfrac{\varepsilon v^3 d^2 E}{(2\Omega\pi)^2 k_1^3}$ and $v < \dfrac{k_1}{\varepsilon}$ are the sufficient conditions for the nonexistance of overstability, the violation of which does not necessarily imply the occurence of overstability.

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