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Power evaluation of the influence of roughness on the value of contact stress for interaction of rough cylinders

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THE PAPER deals with mathematical modelling of contact interaction of rough cylindrical bodies, what significantly reduces the complexity of investigation of contact stress in practice. The contact problem for a rough elastic disk and isotropic plate with a rough cylindrical hole has been considered. The explicit approximate solution of the integral equation is presented in this paper. It allows us to determine the influence of the rough layer characteristics on the distribution of contact stresses.

NOTATIONS

U	energy of elastic deformation of a rough layer
P	load intensity
R and r	radii of cylindrical bodies
E_i, $(i = 1, 2)$	Young's moduli
ν_i, $(i = 1, 2)$	Poisson's ratios
a	half-length of contact
$P(t)$	normal contact pressure
$u_m, v_m (m = \overline{1,2})$	components of displacements for the plate with a hole ($m = 1$), for an elastic disk ($m = 2$)
δ	displacement of disk center
L	area of contact in non-Hertz theory of interior interaction of cylinders
$\sigma_r(\theta)$	normal contact stress in non-Hertz theory of interior interaction of cylinders
$\mu_m, (m = \overline{1,2})$	Lamé's coefficient
$\varphi_m(W), \psi_m(W)$	Kolosov-Mushelishvili complex potentials
$z = x + iy$	complex variable in frame X0Y connected with the center of hole
$s = x' + iy'$	complex variable in a frame X'0Y' connected with a center of disk and belonging to its exterior area
$\Phi_m(W) = \varphi'_m(W)$	
$\Psi_m(W) = \psi'_m(W)$	
$\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5$	coefficients of the integral equation
$\tau = t = Re^{i\theta}$	
$\zeta = h = re^{i\theta}$	
α_0	contact half-angle
$p_{\max}$	maximum contact pressure

For the state of plane deformation:

$$G_{1m} = (1 - \nu_m^2),\ G_{2m} = (1 + \nu_m);\ \kappa_m = (3 - 4\nu_m)$$

For the state of plane stress:

$$G_{1m} = G_{2m} = 1;\ \kappa_m = (3 - \nu_m)/(1 + \nu_m).$$

1. Introduction

THE CONTACT STRESS is one of the major factors leading to the wear of working surfaces and determining limit loads of machine parts [1]. However, roughness of real bodies has an essential influence on the distribution of the contact stress [1 – 6]. It considerably reduces the pressure which becomes much smaller than for smooth bodies in the case of small loads. In addition, the area of contact considerably increases [4 – 6]. Therefore the principal attention is directed to the determination of elastic interaction of rough bodies. Many authors studied this problem, which is connected with solving both the applied and fundamental problems [2, 3].

It is necessary to point out that the mechanics of rough surfaces contains several peculiarities. They are due to the fact that the roughness, formed after technological processing, has various heights distribution [3, 6]. The irregularity of roughness leads to the necessity of application of the probability methods for the determination of rigidity of the element of a rough surface [3, 6, 7] where the pressure is constant. Just the similarity of dimensions of the element and contact a ea explains methodical complexities of solving contact problems for real bodies.

The posed problem was solved in several investigations under the assumption, that distribution of pressure becomes parabolic, while the deformations of elastic half-space in comparison with the deformations of a rough layer can be neglected [6]. This approach leads to essential increasing of the complexity of integral equations and enables us to perform only the numerical analysis of the influence of roughness on the stress [4]. But the curvature of loaded surfaces is transformed and decreases at the expense of deformations of microirregularities. It essentially enables us to simplify the derivation of analytical solutions in the case of interaction on the elliptic area of the contact of elastic bodies. It is possible to carry out this research by estimating the influence of energy of elastic deformation of a rough layer on the displacement in the area of contact. This approach is explained in this paper for the case of contact of two rough cylinders.

2. Power evaluation of the influence of roughness on the deformed surface of interaction of two semi-infinite bodies

Let us consider an analog of the Hertz problem for a case of contact interaction of two elastic plane semi-infinite bodies S_1 and S_2 (Fig. 1). The frame XOY is selected so, that the line of contact L is a segment [-a, a] of the real axis OX. Average Steklov's values of deformation and stress [8] are

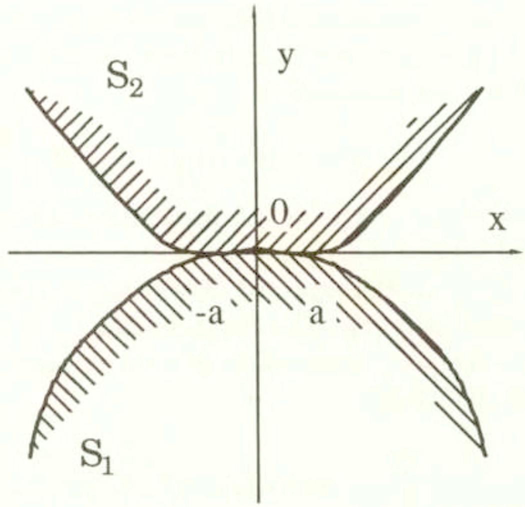

FIG. 1. Contact interaction of two elastic plane semi-infinite bodies.

$$\varepsilon_b^*(t) = \begin{cases} \dfrac{1}{2b} \displaystyle\int_{a-2b}^{a} \varepsilon^*(x)dx, & t \in [a-b, a], \\ \dfrac{1}{2b} \displaystyle\int_{t-b}^{t+b} \varepsilon^*(x)dx, & t \in [-a+b, a-b], \\ \dfrac{1}{2b} \displaystyle\int_{-a}^{-a+2b} \varepsilon^*(x)dx, & t \in [-a, -a+b], \end{cases} \tag{2.1}$$

$$\sigma_b^*(t) = \begin{cases} \dfrac{1}{2b} \displaystyle\int_{a-2b}^{a} \sigma^*(x)dx, & t \in [a-b, a], \\ \dfrac{1}{2b} \displaystyle\int_{t-b}^{t+b} \sigma^*(x)dx, & t \in [-a+b, a-b], \\ \dfrac{1}{2b} \displaystyle\int_{-a}^{-a+2b} \sigma^*(x)dx, & t \in [-a, -a+b], \end{cases} \tag{2.2}$$

where $2b(a \geqslant b)$ is the base length of a measurement of characteristics of micro-deviations, $[-a, a]$ is the segment of contact, σ^*, ε^* – microstress and microdeformation for a rough layer (σ^*, ε^* – integrable functions), and [3]

$$\varepsilon_b^*(t) = (C_0 \sigma_b^*(t))^{\chi}, \tag{2.3}$$

where C_0, χ are real and rational constants defined by parameters of microdeviations. Some authors [3] did not use (2.3) in their research work but applied the relation between stress and deformation

$$\varepsilon(t) = (C_0\sigma(t))^{\chi}.$$

It is incorrect in view of the probability of distribution of the material in a rough layer and since a and b are of the same order of smallness. It is important to note the functions (2.1), (2.2) are the expected distributions of the contact stress and deformation in the contact problem for rough bodies.

Then the energy of elastic deformation of a rough layer U will be expressed in the following way ([9], (2.3)):

$$U = \frac{\chi}{(1+\chi)C_0} 2h \int_0^a \varepsilon_b^*(s)^{\chi+1/\chi} ds = \frac{\chi C_0^{\chi}}{(1+\chi)} 2h \int_0^a \sigma_b^*(s)^{\chi+1} ds \tag{2.4}$$

$$\approx \frac{\chi C_0^{\chi}}{(1+\chi)} V(\tilde{\sigma}_b^*)^{\chi+1} \left[1 + O\left(\frac{(\chi+1)\chi}{2a} \int_0^a \left(\frac{\sigma_b^*(s)}{\tilde{\sigma}_b^*} - 1 \right)^2 ds \right) \right],$$

where $V = 2ah$, h is the thickness of the rough layer, and $\tilde{\sigma}_b^* = \dfrac{1}{a}\displaystyle\int_0^a \sigma_b^*(s) ds$.

On the other hand, from (1.3) we obtain

$$\tilde{\varepsilon}_b^* \approx (C_0\tilde{\sigma}_b^*)^{\chi} \left(1 + O\left(\frac{(\chi-1)\chi}{2a} \int_0^a \left(\frac{\sigma_b^*(s)}{\tilde{\sigma}_b^*} - 1 \right)^2 ds \right) \right); \tag{2.5}$$

here $\tilde{\varepsilon}_b^* = \dfrac{1}{a}\displaystyle\int_0^a \varepsilon_b^*(s) ds$.

Since the bodies are in the elastic equilibrium, then

$$\varepsilon_b^*(t) = \varepsilon_b(t), \qquad \sigma_b^*(t) = \sigma_b(t),$$

where $\varepsilon_b(t)$, $\sigma_b(t)$ are the average Steklov values of deformation $\varepsilon(t)$ and stress $\sigma(t)$ within the limits of the base length for smooth bodies. Therefore equalities (2.4) and (2.5) hold for these functions too.

Functions are continuous on $[-a, a]$ and differentiable on $]-a, a[$. Then taking into account (2.4) – (2.5), we can obtain the following approximate equality:

$$U \approx \frac{\chi C_0^{\chi}}{(1+\chi)} V(\tilde{\sigma})^{\chi+1}[1 + O(H_1)], \tag{2.6}$$

$$\tilde{\varepsilon} \approx (C_0\tilde{\sigma})^{\chi}(1 + O(H_2)), \tag{2.7}$$

where

$$H_1 = \frac{(\chi+1)\chi}{2a} \int\limits_0^a \left(\frac{\sigma_b(s)}{\tilde{\sigma}_b} - 1\right)^2 ds + \frac{(\chi+1)b^2}{2a} \left|\frac{\sigma'(a-b)}{\tilde{\sigma}}\right|,$$

$$H_2 = \frac{(1-\chi)\chi}{2a} \int\limits_0^a \left(\frac{\sigma_b(s)}{\tilde{\sigma}_b} - 1\right)^2 ds + \frac{\chi b^2}{2a} \left|\frac{\sigma'(a-b)}{\tilde{\sigma}}\right| + \frac{b^2}{2a}\left|\frac{\varepsilon'(a-b)}{\tilde{\varepsilon}}\right|,$$

$$\tilde{\sigma} = \frac{1}{a}\int\limits_0^a \sigma(s)ds, \qquad \tilde{\varepsilon} = \frac{1}{a}\int\limits_0^a \varepsilon(s)ds.$$

Thus the energy U of deformation of a rough layer in the case of contact interaction of rough bodies within H_1 and $\tilde{\varepsilon}$ within H_2 are fixed if the value of $\tilde{\sigma}$ is fixed.

3. Generalization of the Hertz theory to the case of contact of rough cylinders

Let us assume that there is no friction in the area of contact L of bodies S_1 and S_2 (Fig. 1). The stresses and rotations are equal to zero at infinity [10].

The equations of the boundaries of bodies before the deformation are [10]

$$y_1 = -\frac{t^2}{2R_1}, \qquad y_2 = \frac{t^2}{2R_2},$$

and after the deformation we have

$$v_1 - v_2 = \left(\frac{t^2}{2R_1} + \frac{t^2}{2R_2}\right) + v^*, \qquad t \in L$$

$$v^* = \frac{\Delta}{2}(a^2 - t^2),$$

where v^* is the displacement of the rough layer, Δ is a constant determined by the parameters of the rough layer. Then from (2.7) we obtain

$$\Delta \approx \frac{3}{a^2} h \left(C_0 \frac{P}{2a}\right)^{\chi}.$$

Taking into account all the conditions we obtain, after some transformations, the following solution in case when the contact pressure $p(t)$ $(p(t) = -\sigma(t))$ is equal to zero at points -a and a :

$$p(t) = \left(\frac{R + r - \Delta Rr}{Rr}\right) \frac{1}{K} \sqrt{(a^2 - t^2)},$$

$$K = \frac{\kappa_1 + 1}{4\mu_1} + \frac{\kappa_2 + 1}{4\mu_2},$$

where $\kappa_m = (3 - 4\nu_m)$ for the state of plane deformation; $\kappa_m = (3 - \nu_m)/(1 + \nu_m)$ for the state of plane stress and a is the solution of nonlinear equation

$$a^2 - 3\frac{Rrh}{(r + R)}\left(C_0 \frac{P}{2a}\right)^{\chi} = \frac{2PrRK}{\pi(r + R)}; \tag{3.1}$$

here P is the intensity of the load; R, r are radii of the curvature of the interacting bodies.

The results of numerical analysis of (3.1) show that the roughness has an essential role only for small operating loads, and with their growth the influence of roughness decreases (Fig. 2). It is necessary to point out that relative errors H_1 and H_2 from (2.6), (2.7) do not exceed 9% and 15% of the examined values.

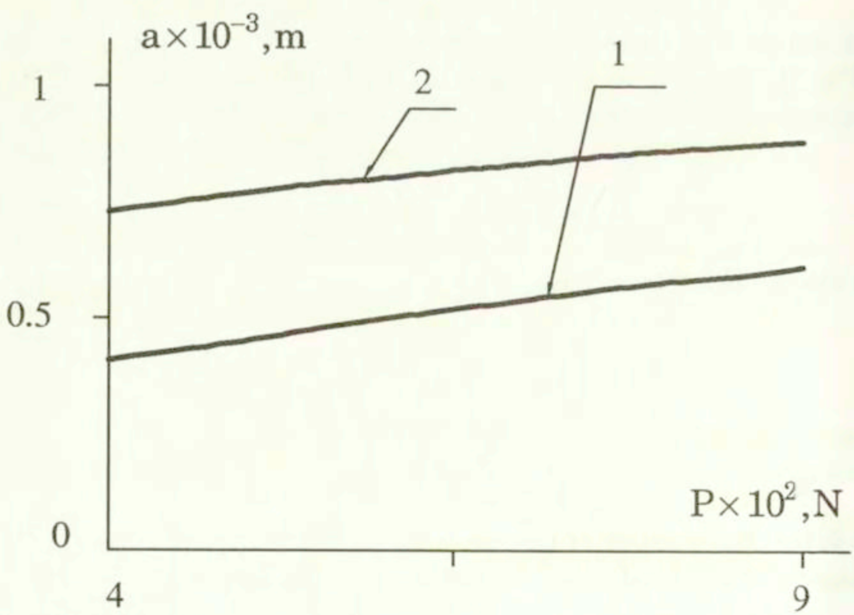

FIG. 2. Relation between a and P: 1 – for smooth cylinders; 2 – for rough cylinders ($C_0 = 3.812 \cdot 10^{-11}$ m^2/N, $K = 7.289 \cdot 10^{-11}$ m^2/N, $R/r = 10$, $\chi = 2/9$).

4. Interior contact of rough elastic disk and plate with cylindrical hole

Consider an elastic isotropic plate with a cylindrical hole of radius R. An elastic isotropic disk of radius r is inserted into the hole. It will be assumed that ε^2, ε/R ($\varepsilon = R - r > 0$) are small values. Force P acts along the y - axis (Fig. 3). Due to the fact that displacements in the area of contact L are negligible in comparison with the dimensions of the bodies, one obtains:

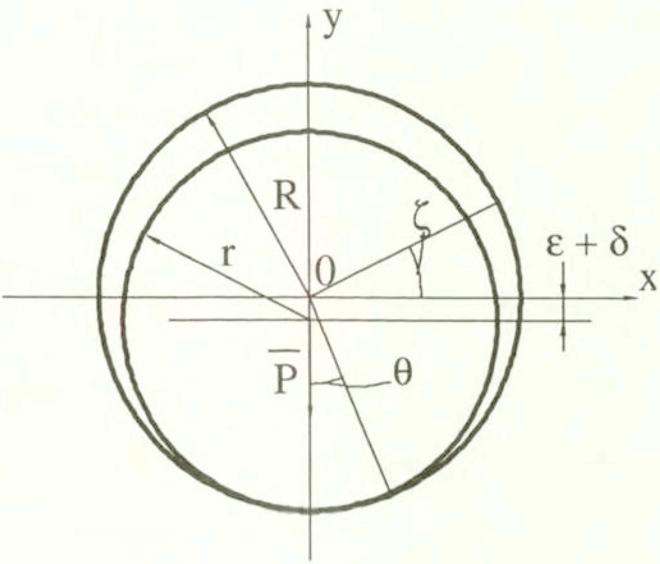

FIG. 3. Scheme of bodies location

(4.1) $$(x_1 + u_1^{**})^2 + (y_1 + v_1^{**})^2 = (x_2 + u_2^{**})^2 + (y_2 + v_2^{**} - \delta)^2,$$

where

$$x_1 = R\cos(\zeta), \qquad y_1 = R\sin(\zeta),$$

$$x_2 = r\cos(\zeta), \qquad y_2 = r\sin(\zeta) - \varepsilon,$$

u_m^{**}, v_m^{**}($m = \overline{1,2}$) are components of the displacements of the plate with the hole ($m = 1$), for the elastic disk ($m = 2$); δ is the displacement of the disk center. It is easy to see that equation (4.1) reduces to

$$\varepsilon + u_1^{**}\cos(\zeta) + v_1^{**}\sin(\zeta) = u_2^{**}\cos(\zeta) + (v_2^{**} - \delta - \varepsilon)\sin(\zeta).$$

Let

$$u_m^{**} = u_m^{*} + u_m, \qquad v_m^{**} = v_m^{*} + v_m,$$

where u_m, v_m, u_m^*, v_m^* are displacements of the basic material and the surface rough layer, respectively.

We shall assume, that the elastic radial displacement in the area of contact being determined by the deformation of a micro-irregularity, is given by the following expression [11]:

$$v_r^* = u_1^* \cos(\zeta) + v_1 \sin(\zeta) - u_2^* \cos(\zeta) - v_2^* \sin(\zeta) = \Delta(\cos(\zeta) - \cos(\alpha_0)).$$

We obtain, similarly to Sec. 1, that

$$\Delta \approx \frac{\alpha_0}{\sin(\alpha_0) - \alpha_0 \cos(\alpha_0)} h \left(C_0 \frac{P}{2R\sin(\alpha_0)} \right)^{\chi},$$

where α_0 is the contact half-angle. After transformation we obtain

$$(4.2) \qquad (\varepsilon - \Delta\cos(\alpha_0)) - 2\frac{\partial u_1}{\partial \zeta}\sin(\zeta) + 2\frac{\partial v_1}{\partial \zeta}\cos(\zeta) + \left(\frac{\partial^2 u_1}{\partial \zeta^2}\cos(\zeta) + \frac{\partial^2 v_1}{\partial \zeta^2}\sin(\zeta)\right) = -2\frac{\partial u_2}{\partial \zeta}\sin(\zeta) + 2\frac{\partial v_2}{\partial \zeta}\cos(\zeta) + \left(\frac{\partial^2 u_2}{\partial \zeta^2}\cos(\zeta) + \frac{\partial^2 v_2}{\partial \zeta^2}\sin(\zeta)\right).$$

But on the contour of the hole we have [12]

$$(4.3) \qquad \frac{1}{R_m}\left(\frac{\partial v_{\zeta m}}{\partial \theta} + v_m\right) = \frac{1}{E_m}(G_{1m}\sigma_{\zeta m} - \nu_m G_{2m}\sigma_r),$$

where $R_m = R(m=1)$ and $R_m = r(m=2)$; $\nu_m, (m=\overline{1,2})$ is Poisson's ratio; $E_m, (m=\overline{1,2})$ is Young's modulus; $G_{1m} = (1-\nu_m^2)$, $G_{2m} = (1+\nu_m)$ for the state of plane deformation; $G_{1m} = G_{2m} = 1$ for the state of plane stress; $\sigma_{\zeta m}$, σ_r are normal components of stress. Then, using (4.2), (4.3) we obtain:

$$(4.4) \qquad \varepsilon - \Delta\cos(\alpha_0) + \frac{R}{E_1}(G_{11}\sigma_{\zeta 1} - \nu_1 G_{21}\sigma_r) + \frac{\partial}{\partial \zeta}\left(\frac{\partial u_1}{\partial \zeta}\cos(\zeta) + \frac{\partial v_1}{\partial \zeta}\sin(\zeta)\right) = \frac{r}{E_2}(G_{12}\sigma_{\zeta 2} - \nu_2 G_{22}\sigma_r) + \frac{\partial}{\partial \zeta}\left(\frac{\partial u_2}{\partial \zeta}\cos(\zeta) + \frac{\partial v_2}{\partial \zeta}\sin(\zeta)\right).$$

It is known that [10]

$$(4.5) \qquad \begin{aligned} \sigma_{\zeta m} + \sigma_r &= 2\left[\Phi_m(W) + \overline{\Phi_m(W)}\right], \\ \sigma_{\zeta m} - \sigma_r + 2i\tau_{r\zeta m} &= 2e^{2i\zeta}\left[\overline{W}\Phi'(W) + \Psi(W)\right], \\ 2\mu_m(u_m + iv_m) &= \kappa_m\varphi_m(W) - W\overline{\Phi_m(W)} - \overline{\psi_m(W)}. \end{aligned}$$

Here $w = z(m = 1)$, $w = s(m = 2)$; $i = \sqrt{-1}$; $\varphi_m(W)$, $\psi_m(W)$ are Kolosov-Mushelishvili complex potentials; $\mu_m, (m = \overline{1,2})$ is Lamé's coefficient; $\varphi'_m(W) = \Phi_m(W), \psi'_m(W) = \Psi_m(W)$.

From (4.4), (4.5) we obtain

$$
\begin{aligned}
(4.6)\qquad &(\varepsilon - \Delta\cos(\alpha_0)) + \frac{R}{E_1}(2G_{11}[\Phi_1(t) + \overline{\Phi_1(t)}] - (G_{11} + \nu_1 G_{21})\sigma_r)\\
&+R\frac{\partial}{\partial\zeta}\left(\frac{(\kappa_1+1)}{4\mu_1} i[\Phi_1(t) - \overline{\Phi_1(t)}]\right) = \frac{r}{E_2}(2G_{12}[\Phi_2(h) + \overline{\Phi_2(h)}]\\
&-(G_{12} + \nu_2 G_{22})\sigma_r) + r\frac{\partial}{\partial\zeta}\left(\frac{(\kappa_2+1)}{4\mu_2} i[\Phi_2(h) - \overline{\Phi_2(h)}]\right),\\
&1/h = r/(rt), \qquad t = Rh/r - i\varepsilon.
\end{aligned}
$$

Then using the Eqs. (4.6), (4.7) [10, 13]

$$
(4.7)\qquad
\begin{aligned}
\Phi_1(z) &= \frac{\kappa_1}{2\pi(1+\kappa_1)}\frac{iP}{z} - \frac{1}{2\pi i}\int\limits_L \frac{\sigma_r(\tau)d\tau}{\tau - z},\\
\Phi_2(s) &= \frac{-iP}{2\pi(1+\kappa_2)}\frac{1}{s} - \frac{iP}{2\pi(1+\kappa_2)}\frac{s}{r^2} + \frac{1}{2\pi i}\int\limits_L \frac{\sigma_r(\xi)d\xi}{\xi - s}\\
&\qquad - \frac{1}{4\pi i}\int\limits_L \frac{\sigma_r(\xi)d\xi}{\xi},
\end{aligned}
$$

we arrive at the integral equation:

$$
(4.8)\qquad \frac{t}{\pi i}\int\limits_L \frac{\sigma'_r(\tau)d\tau}{\tau - t} = \gamma_1\sigma_r(\tau) - \frac{iP}{\pi}\gamma_2\left(\frac{1}{t} - \frac{t}{R^2}\right) - \gamma_3\frac{P}{\pi} - \gamma_4 b - \gamma_5(\varepsilon - \Delta\cos(\alpha_0)),
$$

where

$$
\gamma_1 = \frac{(G_{12} - \nu_2 G_{22})E_1 Rr - (G_{11} - \nu_1 G_{21})E_2 R^2}{2(R^2 E_2 G_{11} + r^2 E_1 G_{12})}
$$

$$
\gamma_2 = \frac{(1+\nu_2)E_1 Rr + \kappa_1(1+\nu_1)E_2 R^2}{4(R^2 E_2 G_{11} + r^2 E_1 G_{12})},
$$

$$
\gamma_3 = \frac{G_{12}\varepsilon R E_1}{2r(1+\kappa_2)(R^2 E_2 G_{11} + r^2 E_1 G_{12})},
$$

$$\gamma_4 = \frac{G_{11}E_2}{(R^2E_2G_{11} + r^2E_1G_{12})},$$

$$\gamma_5 = \frac{E_1E_2}{2(R^2E_2G_{11} + r^2E_1G_{12})},$$

$$\frac{b}{R^2} = -\frac{1}{2\pi i}\int\limits_L \frac{\sigma_\tau}{\tau}d\tau, \qquad t = Re^{i\zeta}.$$

The results of the investigations [10, 14] show that the approximate solution of (4.8) can be reduced to the following form:

$$\text{(4.9)} \quad \sigma_r(\theta) = -P\frac{\sqrt{2}}{R}\left[\gamma_2\frac{2}{\pi} + \frac{\gamma_1}{\alpha_0 - \cos(\alpha_0)\sin(\alpha_0)}\right]\sqrt{\cos(\theta) - \cos(\alpha_0)}$$

$$\cos(\theta/2) + 2\left[P\left(\frac{\gamma_3}{\pi} + \frac{\gamma_1\cos(\alpha_0)}{R(\alpha_0 - \cos(\alpha_0)\sin(\alpha_0))}\right) + \gamma_4 b\right.$$

$$\left. + \gamma_5(\varepsilon - \Delta\cos(\alpha_0))\right] \times \ln\left[\frac{\sqrt{1+\cos(\theta)} - \sqrt{\cos(\theta) - \cos(\alpha_0)}}{\sqrt{1+\cos(\alpha_0)}}\right];$$

here

$$P = -2R\int\limits_0^{\alpha_0} \sigma_r(\theta)\cos(\theta)d\theta, \qquad b = -\frac{R^2}{\pi}\int\limits_0^{\alpha_0}\sigma_r(\theta)d\theta.$$

It is necessary to emphasize that for elastic constants of isotropic materials, which are widely used in machines, the error of approximation (4.9) of the solution of the equation (4.8) with respect to $\sigma_r^{\max}$ is less than 4%.

It has been established that the obtained dependence of the half-angle of contact on the non-dimensional parameter, introduced by I.Y. Staerman, is analogous to the dependences established by M. I. TEPLY for the state of plane deformation [13]. This confirms a high accuracy of the approximate solution (4.9).

The obtained solutions for smooth bodies can be used as a zero approximation in the analysis of the influence of roughness on the distribution of normal radial stresses. The obtained results show that in the case of interaction of rough bodies, the contact half-angle increases in comparison with the contact half-angle for smooth machine parts, and the greatest contact stress decreases (Figs. 4, 5). It is necessary to note that in this case, the relative errors H_1 and H_2 from (2.6), (2.7) do not exceed 10% and 11% of the investigated values.

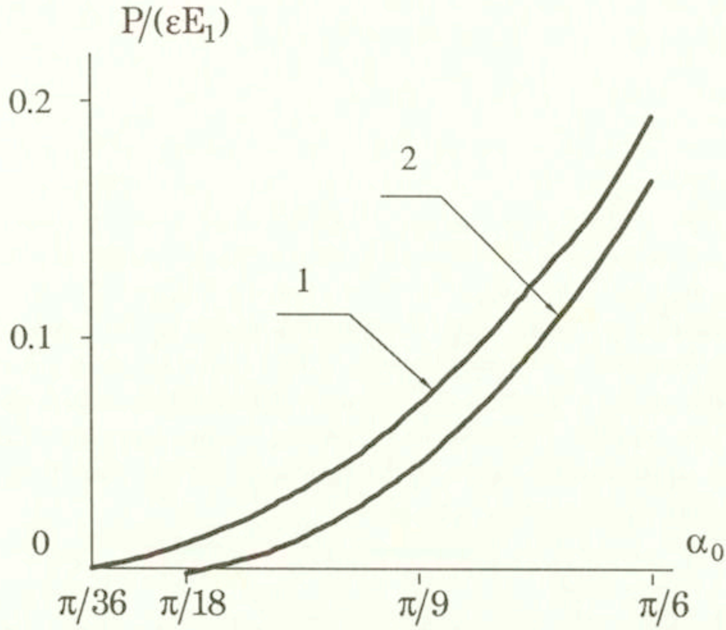

FIG. 4. The relation between α_0 and parameter $P/\varepsilon E$: 1 – for a smooth hole; 2 – for a rough hole.

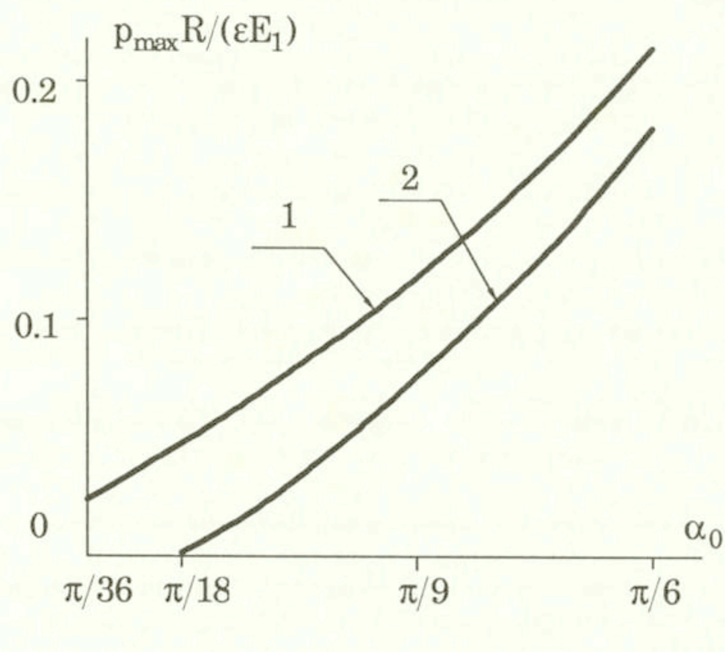

FIG. 5. The relation between α_0 and parameter $p_{\max}R/(\varepsilon E_1)(p_{\max} = -\sigma_r^{\max})$: 1 – for a smooth hole; 2 – for a rough hole.

5. Conclusions

The problems of elastic contact interaction of rough cylindrical bodies is solved by taking into account not only their geometry and relative location, but also the geometric characteristics of their surfaces. It allows us to determine the influence of basic parameters of the problem on the contact stress. It considerably reduces the complexity of investigating contact stresses in practice.

The comparison of the data of the stress analysis in the area of contact for various combinations of elastic characteristics of interacting bodies with the results of paper [13] confirms high effectiveness of the approach proposed here.

The suppositions made here and the conclusions drawn from the experimental research enable us to take into account the geometric peculiarities of the surfaces of interacting bodies.

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