

Existence and uniqueness result for Stokes flows in a half-plane

M. KOHR (CLUJ-NAPOCA)

THIS PAPER is concerned with the study of the two-dimensional Stokes flow past a smooth obstacle near a plane wall. Using the boundary integral formulation, the flow is represented in terms of a combined distribution of a single-layer and a double-layer potentials of Green's functions over the boundary of the obstacle. The problem is formulated as a set of Fredholm integral equations of the second kind for the density of the potentials. The existence and uniqueness results of the solution are obtained. Numerical results are presented in the case of a rigid circular obstacle moving parallel or normal to the plane wall.

1. Introduction

THE MOTION of a body of simple shape, near a plane wall, with small Reynolds number, has a long record in fluid dynamics research. For example, the motion of a rigid sphere parallel to a plane wall was treated by H. FAXEN (see [5]). The author used the method of reflections and determined a scheme of successive iterations, which solved the associated boundary value problem. The problem of a sphere moving perpendicular to a plane rigid wall was solved by H. BRENNER (see [1]). He used the general bipolar coordinate solution. By applying the method of reflections, S. J. WAKIYA (see [8]) determined the motion of a sphere toward a plane wall. H. POWER, G. MIRANDA and R. GONZALES (see [15]) obtained an exact formulation for the slow viscous flow due to the arbitrary motion of a particle of arbitrary shape near a plane wall. This formulation is given in terms of a system of Fredholm integral equations of the first kind. The authors used the Green's function, which is equal to zero on the rigid wall. R. HSU and P. GANTOS (see [9]) determined an integral equation method, based on the Green's integral representation formulae, instead of only a single-layer potential. This method solved the problem of the motion of a rigid body in a viscous fluid bounded by a plane wall. Using the method of second kind integral equations, S. J. KARRILA and S. KIM (see [10]) deduced an elegant solution for the problem of multiple particles in the unbounded flow with an exterior container as the boundary. Karrila and Kim's method was called the Completed Double-Layer Boundary Integral Equation Method, because, it uses the idea of completing the deficient range of the double-layer operator. Also, this method is a deflation procedure needed to give an integral operator with a spectral radius strictly less than one. The author shows that this method is the most efficient one when

the mobility problem is numerically solved. In such a problem the force and the torque acting on each particle are given and the unknown motion of the particle is determined.

A second kind of integral equation formulation was applied by H. POWER and B. F. POWER (see [16]), for the slow motion of an arbitrary particle near a plane wall and in a viscous fluid. Also, Y. TAKAISI (see [8]) gave an analytical solution for the motion of a rigid cylindrical obstacle, parallel to a rigid wall, in the limiting case of large values of the distance between the wall and the center of obstacle.

The goal of this paper is to present an integral formulation based on the theory of double-layer and single-layer potentials, for the slow motion of a cylindrical obstacle near a plane wall.

2. Mathematical model

We consider the problem of the slow motion of a cylindrical rigid obstacle of arbitrary shape, near a plane wall, in a viscous fluid. The cross-section of the obstacle, in the Ox_1x_2 plane, is denoted by Ω^1 , having the boundary C of Lyapunov type. We assume that the rigid wall, denoted by L , is given by the equation $x_2 = 0$.

The velocity and pressure fields of the undisturbed fluid flow are denoted by $\mathbf{U}_\infty(U_{1\infty}, U_{2\infty})$ and p_∞ . The functions \mathbf{U}_∞ and p_∞ satisfy the continuity equation and the Stokes equations in the half-plane $x_2 > 0$. Additionally, the velocity \mathbf{U}_∞ satisfies the following nonslip boundary condition:

$$(2.1) \quad \mathbf{U}_\infty(x) = \mathbf{0}, \quad \text{for } x_2 = 0.$$

Furthermore, we suppose that the rigid particle Ω^1 has the velocity $\mathbf{U}(U_1, U_2)$. Under this condition, the velocity field \mathbf{u} and the pressure field p of the resulting flow, obtained by the superposition of the undisturbed flow and the presence of the rigid obstacle Ω^1 , satisfy, in the first approximation, the following system of equations:

$$\sum_{k=1}^2 \frac{\partial^2 u_i(x)}{\partial x_k \partial x_k} = \frac{\partial p(x)}{\partial x_i}, \quad x \in \Omega,$$

(2.2)

$$\sum_{i=1}^2 \frac{\partial u_i(x)}{\partial x_i} = 0, \quad x \in \Omega.$$

Here, Ω is the domain above the wall, exterior to the particle and having C and L as boundaries. Also, we suppose that the dynamic viscosity coefficient of the flow

is equal to 1, because the general case can be reduced to this one by a coordinate transformation.

The flow satisfies the nonslip boundary condition on the wall L :

$$(2.3) \quad u_i(x) = 0, \quad \text{for } x \in L,$$

the following condition on the boundary C :

$$(2.4) \quad u_i(x) = U_i(x), \quad \text{for } x \in C,$$

and the conditions at infinity:

$$(2.5) \quad |u_i(x) - U_{i\infty}(x)| \rightarrow 0, \quad |p(x) - p_\infty(x)| \rightarrow 0, \quad \text{as } |x| \rightarrow \infty.$$

3. The Green function of the flow

Let $\mathbf{G}(G_{ij})$ and $\mathbf{q}(q_i)$ be the Green's function and the pressure vector, associated with the Stokes equations in the half-plane $x_2 > 0$. These functions correspond to the velocity and pressure fields of a Stokes flow, produced by source point or a pole placed at the point y of the above specified half-plane.

Hence, \mathbf{G} and \mathbf{q} are solutions for the following equations and conditions (see [12, 18]):

$$(3.1) \quad \sum_{k=1}^2 \frac{\partial^2 G_{ij}(x, y)}{\partial x_k \partial x_k} - \frac{\partial q_j(x, y)}{\partial x_i} = -4\pi \delta_{ij} \delta(x - y), \quad \text{for } x_2 > 0,$$

$$(3.2) \quad \sum_{i=1}^2 \frac{\partial G_{ij}(x, y)}{\partial x_i} = 0, \quad \text{for } x_2 > 0,$$

$$(3.3) \quad G_{ij}(x, y) = 0, \quad \text{for } x_2 = 0,$$

$$(3.4) \quad G_{ij}(x, y) \rightarrow 0, \quad q_j(x, y) \rightarrow 0, \quad \text{as } |x| \rightarrow \infty,$$

where $y(y_1, y_2)$ is the pole or the source point of the Green's function and δ is the two-dimensional Dirac's distribution.

The function \mathbf{G} is given by (see [17]):

$$(3.5) \quad \mathbf{G}(x, y) = \mathbf{G}^{ST}(x - y) - \mathbf{G}^{ST}(x - y^{im}) + 2y_2^2 \mathbf{G}^D(x - y^{im}) - 2y_2 \mathbf{G}^{SD}(x - y^{im}),$$

where $y^{im}(y_1, -y_2)$ is the image of the pole with respect to the wall, \mathbf{G}^{ST} is the two-dimensional Stokeslet, having the following components:

$$(3.6) \quad G_{ij}^{ST}(x) = -\delta_{ij} \ln|x| + \frac{x_i x_j}{|x|^2}, \quad x \in \mathbf{R}^2.$$

The matrices \mathbf{G}^D and \mathbf{G}^{SD} contain potential dipoles and Stokeslet doublets, respectively, and are given by (see [18]):

$$(3.7) \quad G_{ij}^D(x) = \pm \left(\frac{\delta_{ij}}{|x|^2} - 2 \frac{x_i x_j}{|x|^4} \right), \quad G_{ij}^{SD}(x) = x_2 G_{ij}^D(x) \pm \frac{\delta_{j2} x_i - \delta_{i2} x_j}{|x|^2},$$

with the plus sign for the x_1 direction and the minus sign for the x_2 direction.

The associated pressure vector \mathbf{q} is given by (see [17]):

$$(3.8) \quad \mathbf{q}(x, y) = \mathbf{q}^{ST}(x - y) - \mathbf{q}^{ST}(x - y^{im}) - 2y_2 \mathbf{q}^{SD}(x - y^{im}),$$

where

$$(3.9) \quad q_i^{ST}(x) = 2 \frac{x_i}{|x|^2}, \quad \mathbf{q}^{SD}(x) = -\frac{2}{|x|^4} (2x_1 x_2, x_1^2 - x_2^2).$$

The stress tensor \mathbf{T} , associated with the Green's function \mathbf{G} , has the following components (see [12, 18]):

$$(3.10) \quad T_{ijk}(x, y) = -q_j(x, y) \delta_{ik} + \frac{\partial G_{ij}(x, y)}{\partial x_k} + \frac{\partial G_{kj}(x, y)}{\partial x_i}, \quad \text{for } x_2 > 0.$$

The relations (3.5) – (3.9) lead to (see [17]):

$$(3.11) \quad \mathbf{T}(x, y) = \mathbf{T}^{ST} - \mathbf{T}^{ST}(x - y^{im}) + 2y_2 \mathbf{T}^D(x - y^{im}) - 2y_2 \mathbf{T}^{SL}(x - y^{im}),$$

where

$$(3.12) \quad T_{ijk}^{ST}(x) = -4 \frac{x_i x_j x_k}{|x|^4}, \quad T_{ijk}^D(x) = \frac{\partial G_{ij}^D(x)}{\partial x_k} + \frac{\partial G_{kj}^D(x)}{\partial x_i},$$

$$T_{ijk}^{SD}(x) = -q_j^{SD}(x) \delta_{ij} + \frac{\partial G_{ij}^{SD}(x)}{\partial x_k} + \frac{\partial G_{kj}^{SD}(x)}{\partial x_i}.$$

The Green's function \mathbf{G} vanishes over the wall L . When the pde y tends to a point x of L , \mathbf{G} has a singular behaviour and must vanish, because $x \in L$. Then, we obtain the following properties:

$$(3.13) \quad G_{ij}(x, y) = q_i(x, y) = T_{ijk}(x, y) = 0, \quad \text{for } y \in L.$$

Also, it is easily seen that the Green's function \mathbf{G} satisfies the symmetry property given below:

$$(3.14) \quad G_{ij}(x, y) = G_{ji}(y, x).$$

4. Integral representation of solution

The velocity field of the Stokes flow is represented in the following form:

$$(4.1) \quad u_i(x) = U_{i\infty}(x) + \int_C T_{jik}(y, x) n_k(y) \varphi_j(y) ds_y + \int_C G_{ij}(x, y) \varphi_j(y) ds_y, \quad x \in \Omega,$$

where the unknown density φ is assumed to be a continuous function on C , and the unit normal vector \mathbf{n} is directed outside of Ω^1 .

In the following we denote by $\mathcal{P}(y, x)$ the pressure field at the point y , associated with the flow $\mathbf{q}(x, y)$. Indeed, the pressure term $\mathbf{q}(x, y)$ is a solution to the equations of the Stokes flow given by a singularity with the pole at the point x (see [12, 18]). Furthermore, the pressure tensor $\mathbf{P}(x, y)$, associated with the stress tensor $\mathbf{T}(y, x)$, has the following components (for more details, see [18]):

$$(4.2) \quad T_{ij}(x, y) = -\mathcal{P}(x, y) \delta_{ij} + \frac{\partial q_i(x, y)}{\partial y_j} + \frac{\partial q_j(x, y)}{\partial y_i}.$$

By applying the above properties, we use the following integral representation for the pressure field p of the flow:

$$(4.3) \quad p(x) = p_\infty(x) + \int_C T_{ij}(x, y) n_j(y) \varphi_i(y) ds_y + \int_C q_j(x, y) \varphi_j(y) ds_y, \quad x \in \Omega.$$

By a simple computation, it can be shown that the functions \mathbf{u} and p satisfy Eqs. (2.2) of the Stokes flow. Also, from (3.3) and (3.13), we deduce that the boundary condition (2.3) is satisfied.

From the following conditions (see also, [12, 18]):

$$(4.4) \quad T_{ijk}(y, x) \rightarrow 0, \quad T_{ij}(x, y) \rightarrow 0, \quad G_{ij}(x, y) \rightarrow 0, \quad q_j(x, y) \rightarrow 0, \\ \text{as } |x| \rightarrow \infty,$$

we deduce that the conditions at infinity (2.5) hold.

The double-layer potential of the integral representation (4.1) has the following jump properties (see [12, 18]):

$$(4.5) \quad \lim_{x' \rightarrow x \in C} \int_C T_{jik}(y, x') n_k(y) \varphi_j(y) ds_y = \pm 2\pi \varphi_i(x) + \int_C^{\mathcal{PV}} T_{jik}(y, x) n_k(y) \varphi_j(y) ds_y,$$

where the plus sign applies to the external side of C , in the direction of the normal vector \mathbf{n} , and the minus sign to the internal side. The symbol $\mathcal{P}V$ means that the double-layer integral is evaluated in the principal value sense (i.e. the value of the improper but convergent double-layer integral, when $x \in C$).

By using the boundary conditions (2.4) and the above jump properties, we obtain the following integral equations:

$$(4.6) \quad 2\pi\varphi_i(x) + \int_C^{\mathcal{P}V} T_{jik}(y, x)n_k(y)\varphi_j(y)ds_y + \int_C G_{ij}(x, y)\varphi_j(y)ds_y = U_i(x) - U_{i\infty}(x).$$

In what follows, let us suppose that \mathbf{U} is a continuous function on C . Because the contour C is a Lyapunov curve, it follows that the kernel of the single-layer operator \mathbf{V}^1 , defined by:

$$(4.7) \quad [V^1\varphi]_i(x) = \int_C G_{ij}(x, y)\varphi_j(y)ds_y, \quad x \in C,$$

is weakly singular (i.e. its singularity has the type $1/(|x - y|^\lambda)$, where $0 \leq \lambda < 1$) (see, for example [13]). Thus, the single-layer operator is a compact operator on the space of continuous functions on C (see also, [13]).

With similar arguments we conclude that the double-layer operator \mathbf{V}^2 , defined by:

$$(4.8) \quad [V^2\varphi]_i(x) = \int_C^{\mathcal{P}V} T_{jik}(y, x)n_k(y)\varphi_j(y)ds_y, \quad x \in C,$$

has the kernel with a weak singularity. Then the double-layer operator is compact in the space of continuous function in C , having the values in the same Banach space (see also, [13]).

These properties show that the integral equations (4.6) determine a Fredholm integral system of the second kind. By using the Fredholm's result (see [13]), we deduce that this system has a unique continuous solution if and only if it can be proved that the associated homogeneous system has only the null solution, in the space of continuous functions on C .

Consider the following homogeneous system:

$$(4.9) \quad 2\pi\varphi_i^0(x) + \int_C^{\mathcal{P}V} T_{jik}(y, x)n_k(y)\varphi_j^0(y)ds_y + \int_C G_{ij}(x, y)\varphi_j^0(y)ds_y = 0, \quad x \in C.$$

By using a continuous solution φ^0 of the above system (4.9), we can define the velocity field $\mathbf{u}^0(u_1^0, u_2^0)$ and the pressure field p^0 , given below:

$$u_i^0(x) = \int_C T_{jik}(y, x)n_k(y)\varphi_j^0(y)ds_y + \int_C G_{ij}(x, y)\varphi_j^0(y)ds_y, \tag{4.10}$$

$x \in \Omega \cup \Omega^1,$

$$p^0(x) = \int_C T_{ij}(x, y)n_j(y)\varphi_i^0(y)ds_y + \int_C q_j(x, y)\varphi_j^0(y)ds_y,$$

$x \in \Omega \cup \Omega^1.$

Taking into account the fact that the functions (\mathbf{G}, \mathbf{q}) and (\mathbf{T}, \mathbf{P}) are solutions of the equations of the Stokes flow, we conclude that the fields \mathbf{u}^0 and p^0 determine a Stokes flow in Ω (and Ω^1 , respectively), with the zero velocity on the wall L and at infinity. Because the function φ^0 is a solution of the system (4.9), we deduce that the velocity field \mathbf{u}^0 becomes zero on the boundary C . By using the uniqueness result of the Stokes flow in Ω , we obtain the following equalities:

$$\mathbf{u}^0(x) = \mathbf{0}, \quad p^0(x) = 0, \quad \text{for all } x \in \Omega. \tag{4.11}$$

On the other hand, the continuity property of the single-layer potential across the boundary C and the jump formulas (4.5) of the double-layer potential, imply:

$$u_i^{0+}(x) - u_i^{0-}(x) = 4\pi\varphi_i^0(x), \quad x \in C, \tag{4.12}$$

where

$$u_i^{0+}(x) = \lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega}} u_i^0(x') \quad \text{and} \quad u_i^{0-}(x) = \lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega^1}} u_i^0(x').$$

From (4.11) and (4.12), we obtain:

$$u_i^{0-}(x) = -4\pi\varphi_i^0(x), \quad x \in C, \tag{4.13}$$

$$F_i^{0+}(x) = \lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega}} \left[-p^0(x)\delta_{ik} + \frac{\partial u_i^0(x')}{\partial x'_k} + \frac{\partial u_k^0(x')}{\partial x'_i} \right] n_k(x) = 0, \tag{4.14}$$

where \mathbf{F}^{0+} denotes the limiting value on C of the stress field \mathbf{F}^0 , evaluated from the external side of C and corresponding to the flow field \mathbf{u}^0 . Also, we denote by \mathbf{F}^{0-} the limiting value of \mathbf{F}^0 evaluated from the internal side of C .

By an easy computation, it follows that the stress tensor \mathbf{S}^1 , given by the single-layer potential $\mathbf{V}^1\varphi^0$, has the following components:

$$S_{ij}^1(x) = \int_C T_{ijk}(x, y)\varphi_j^0(y)ds_y, \quad x \in \Omega \cup \Omega^1. \tag{4.15}$$

The associated stress field \mathbf{F}^1 of \mathbf{S}^1 exhibits the following discontinuities across its distribution domain C (see, for example [12, 18]):

$$\begin{aligned}
 F_i^{1+}(x) &= \lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega}} S_{ik}^1(x') n_k(x) = -2\pi\varphi_i^0(x) \\
 &\quad + \int_C^{\mathcal{P}V} T_{ijk}(x, y) n_k(x) \varphi_j^0(y) ds_y, \\
 (4.16) \quad F_i^{1-}(x) &= \lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega^1}} S_{ik}^1(x') n_k(x) = 2\pi\varphi_i^0(x) \\
 &\quad + \int_C^{\mathcal{P}V} T_{ijk}(x, y) n_k(x) \varphi_j^0(y) ds_y.
 \end{aligned}$$

Therefore, we have:

$$(4.17) \quad F_i^{1+}(x) - F_i^{1-}(x) = -4\pi\varphi_i^0(x), \quad x \in C.$$

From properties (4.11) and (4.16), we deduce that the following limiting value on C exists and is finite:

$$\lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega}} F_i^{2+}(x') = 2\pi\varphi_i^0(x) - \int_C^{\mathcal{P}V} T_{ijk}(x, y) n_k(x) \varphi_j^0(y) ds_y,$$

where \mathbf{F}^2 is the stress field associated with the double-layer potentials $\mathbf{V}^2\varphi^0$.

By a Lyapunov result (see [13]), it follows that there exists an analogous limiting value from the domain Ω^1 on C , of the stress field \mathbf{F}^2 . Additionally, these limiting values are equal (see [12, 18]).

Thus, we obtain the following equalities:

$$(4.18) \quad \lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega}} F_i^{2+}(x') = \lim_{\substack{x' \rightarrow x \in C \\ x' \in \Omega^1}} F_i^{2-}(x'), \quad x \in C.$$

By using the properties (4.17) and (4.18), we obtain the following jump of the stress field \mathbf{F}^0 across C :

$$(4.19) \quad \mathbf{F}^{0+}(x) - \mathbf{F}^{0-}(x) = -4\pi\varphi^0(x), \quad x \in C.$$

From (4.14) and (4.19), we deduce:

$$(4.20) \quad \mathbf{F}^{0-}(x) = 4\pi\varphi^0(x), \quad x \in C.$$

Green's formula, applied in the bounded domain Ω^1 for the flow (\mathbf{u}^0, p^0) , implies the following equality:

$$(4.21) \quad \int_C u_i^{0-}(x) F_i^{0-}(x) ds_x = 2 \int_{\Omega^1} e_{ik}^0(x) e_{ik}^0(x) dx,$$

where $e_{ik}^0 = \frac{1}{2} \left(\frac{\partial u_i^0}{\partial x_k} + \frac{\partial u_k^0}{\partial x_i} \right)$ represents the rate of the deformation tensor.

If we use the relations (4.13), (4.20) and (4.21), we obtain:

$$(4.22) \quad \int_{\Omega^1} e_{ik}^0(x) e_{ik}^0(x) dx = -8\pi^2 \int_C \varphi_i^0(x) \varphi_i^0(x) ds_x.$$

Since the above integrals are real and non-negative, we deduce that all integrals in (4.22) become zero. Hence, $\varphi(x) = 0$, for $x \in C$.

Thus, we can formulate the following result.

THEOREM. *The Fredholm integral system (4.6) has a unique continuous solution φ . The integral representations (4.1), (4.3), with the density φ , provide the Stokes flow due to the slow motion of the rigid obstacle Ω^1 , in the presence of the wall L .*

It is easy to see that if there exists the limiting value on C of the stress field associated with the double-layer potential $\mathbf{V}^2\varphi$, then its total force and torque on C are zero. By using the jump formulas (4.16) of the surface force due to a single-layer potential, we can easily prove that the total force and torque, exerted on C by the flow (\mathbf{u}, p) , are given by the following equalities:

$$(4.23) \quad \begin{aligned} \mathcal{F}_i &= \int_C F_i(x) ds_x = -4\pi \int_C \varphi_i(x) ds_x, \\ \mathbf{M} &= \int_C \mathbf{x} \times \mathbf{F}(x) ds = -4\pi \mathbf{k} \int_C \mathbf{x} \times \varphi(x) ds_x. \end{aligned}$$

In the above equalities we have used the notation:

$$F_i(x) = \left[-p(x)\delta_{ik} + \frac{\partial u_i(x)}{\partial x_k} + \frac{\partial u_k(x)}{\partial x_i} \right] n_k(x), \quad x \in C.$$

Also \mathbf{x} is the position vector of the point x with respect to the orthogonal frame Ox_1x_2 and \mathbf{k} is the unit vector of the Ox_3 axis, which is orthogonal to the plane of the flow.

5. Numerical results

If we use properties (3.1), (3.2) of the functions \mathbf{G} and \mathbf{q} , we obtain the following property of the stress tensor \mathbf{T} (see, for example [12, 18]):

$$(5.1) \quad \int_C^{\rho V} T_{jik}(y, x) n_k(y) ds_y = -2\pi \delta_{ij}, \quad x \in C,$$

where the unit normal vector \mathbf{n} is directed outwards of C . Considering this property, the integral system (4.6) can be written in the following manner:

$$(5.2) \quad \int_C T_{jik}(y, x) n_k(y) (\varphi_j(y) - \varphi_j(x)) ds_y + \int_C G_{ij}(x, y) \varphi_j(y) ds_y = U_i(x) - U_{i\infty}(x), \quad x \in C.$$

The double-layer integrals of (5.2) are non-singular (i.e. they are regular integrals), hence the singularities of double-layer potential of system (4.6) are removed by means of the above procedure. In order to remove the singularities of single-layer potentials of system (4.6), it is sufficient to isolate these singularities and also, to integrate the singularities over the corresponding boundary element.

In the following, the numerical results are given for the case of a circular rigid obstacle with radius a , moving parallel or normal to the wall L . We solve the integral system (5.2) by means of a standard boundary element method. To this end, we consider a polygonal contour consisting of the segments C_j , $j = 1, \dots, n$, and we suppose that on each segment C_j , the functions φ_i , $i = 1, 2$, are constant and equal to their values at the middle point $x_j^*(x_{1j}^*, x_{2j}^*)$ of this segment. These values are denoted by φ_i^j , $i = 1, 2$, $j = 1, \dots, n$. Now, if we require the discretized form of (5.2) to be satisfied at each point x_m^* , $m = 1, \dots, n$, we obtain the algebraic linear system given below:

$$(5.3) \quad \sum_{p=1}^n (\varphi_j^p - \varphi_j^m) \int_{C_p} T_{jik}(y, x_m^*) n_k(y) ds_y + \sum_{p=1}^n \varphi_j^p \int_{C_p} G_{ij}(x_m^*, y) ds_y = U_i(x_m^*) - U_{i\infty}(x_m^*).$$

If $p = m$, then we have $(\varphi_j^p - \varphi_j^m) \int_{C_p} T_{jik}(y, x_m^*) n_k(y) ds_y = 0$.

In the case when $p = m$, we use the following decomposition for single-layer potential:

$$\int_{C_p} G_{ij}(x_m^*, y) ds_y = \int_{C_p} (G_{ij}(x_m^*, y) - G_{ij}^{ST}(x_m^*, y)) ds_y + \int_{C_p} G_{ij}^{ST}(x_m^*, y) ds_y.$$

In the above equality the first integral is regular and can be numerically determined by a Gaussian quadrature formula. The second integral can be evaluated accurately.

If $p \neq m$, then the double layer integrals and the single layer integrals of the system (5.3) are computed by means of the Gaussian quadrature formula.

Furthermore, after discretization, the total force and torque on C become:

$$\mathcal{F}_i = -4\pi \sum_{m=1}^n \varphi_i^m \int_{C_m} ds, \quad (5.4)$$

$$\mathcal{M} = -4\pi \mathbf{k} \sum_{m=1}^n \left\{ \varphi_2^m \int_{C_m} y_1 ds - \varphi_1^m \int_{C_m} y_2 ds \right\}.$$

In the sequel we denote by Y_0 the ratio of the distance d , between the center of the circular obstacle and the rigid wall, to the radius a .

Figure 1 illustrates the dependence between the dimensionless drag force F and Y_0 , in the case of the circular rigid obstacle moving perpendicular to the rigid wall L (i.e. $\mathbf{U}_\infty = \mathbf{0}$ and $|\mathbf{U}| = 1$).

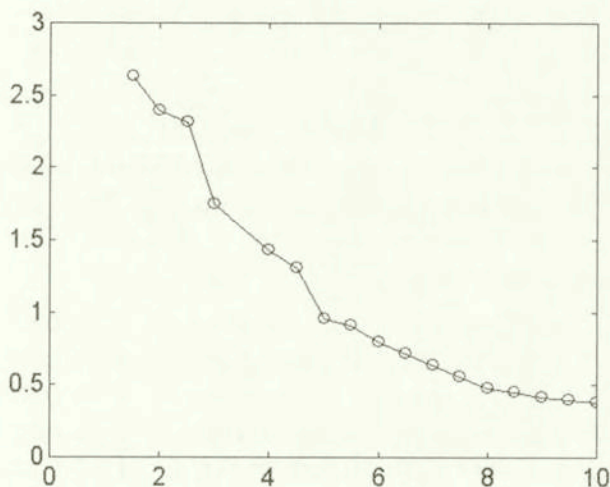


FIG. 1. Dependence of F vs. Y_0 : perpendicular motion.

Note that $F = \mathcal{F}/4\pi U$ and \mathcal{F} is the modulus of the total force \mathbb{F} on C .

Figure 2 presents the dependence between the dimensionless drag force F and Y_0 , in the case of the circular obstacle moving parallel to the wall L (i.e. $\mathbf{U}_\infty = \mathbf{0}$ and $\mathbf{U} = (1, 0)$).

Both these results show that F increases when the ratio Y_0 decreases.

The maximum value of N was assumed to be equal to 60.

In the limiting case of large values of Y_0 , it follows that the presented numerical results can be compared with the analogous results obtained by Y. TAKAISI (see [8]), where the analytical expression of the dimensionless drag force F_T was equal to

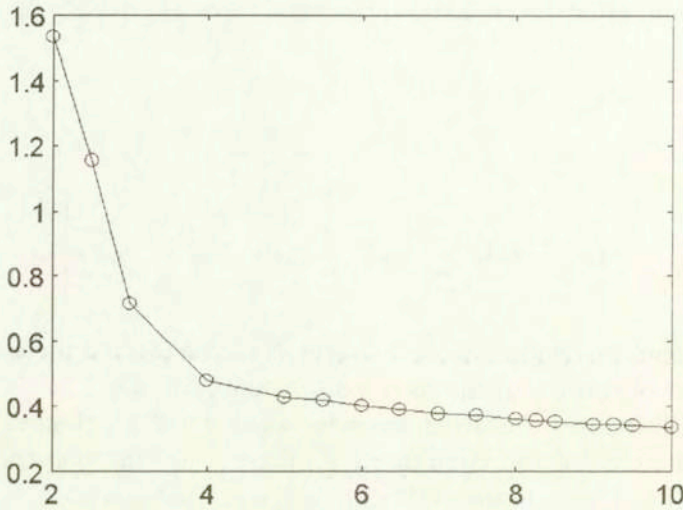


FIG. 2. Dependence of F vs. Y_0 : parallel motion.

$$F_T = \frac{\mathcal{F}_T}{4\pi\mu U} = \frac{1}{\log(2Y_0)},$$

the viscosity coefficient μ and the translational velocity U of circular obstacle in the direction of the wall being equal to 1. Note that \mathcal{F}_T is the modulus of the total force acting on on C and corresponding to Takaisi's method.

For example, we have:

$Y_0 = 6.0 \Rightarrow F_T = 0.4024296043,$	$F = 0.40312,$
$Y_0 = 6.5 \Rightarrow F_T = 0.3898712452,$	$F = 0.39012,$
$Y_0 = 7.0 \Rightarrow F_T = 0.3789231816,$	$F = 0.37901,$
$Y_0 = 7.5 \Rightarrow F_T = 0.3692693730,$	$F = 0.37002,$
$Y_0 = 8.0 \Rightarrow F_T = 0.3606737602,$	$F = 0.36107,$
$Y_0 = 8.5 \Rightarrow F_T = 0.3529561238,$	$F = 0.35301,$
$Y_0 = 9.0 \Rightarrow F_T = 0.3459762562,$	$F = 0.34599,$
$Y_0 = 9.5 \Rightarrow F_T = 0.3396232718,$	$F = 0.33969,$
$Y_0 = 10 \Rightarrow F_T = 0.3338082006,$	$F = 0.33381.$

References

1. H. BRENNER, *The slow motion of a sphere through a viscous fluid towards a plane surface*, Chem. Engrg. Sci., **16**, 242-251, 1961.
2. L. DRAGOȘ, *Principles of continuous mechanics media* [in Romanian], Ed. Tehnică, Bucharest 1983.
3. L. DRAGOȘ and A. DINU, *A direct boundary integral equations method to subsonic flow with circulation past thin airfoils in ground effect*, Comput. Methods Appl. Mech. Engrg., **127**, 357-370, 1995.

4. L. DRAGOȘ and A. DINU, *A direct boundary integral method for the three-dimensional lifting flow*, *Comput. Methods Appl. Mech. Engng.*, **127**, 357–370, 1995.
5. H. FAXEN, *Die Bewegung einer starren Kugel längs der Achse eines mit Flüssigkeit gefüllten Rohres*, *Arkiv. Mat. Astron. Fys.*, **17**, 1–28, 1923.
6. T. M. FISCHER, G. C. HSIAO and W. L. WENDLAND, *On two-dimensional slow viscous flows past obstacles in a half-plane*, *Proc. Royal Soc. Edinburgh*, **104A**, 205–215, 1986.
7. T. M. FISCHER and R. ROSENBERGER, *A boundary integral method for the numerical computation of the forces exerted on a sphere in viscous incompressible flows near a plane wall*, *ZAMP*, **38**, 339–365, 1987.
8. J. HAPPEL and H. BRENNER, *Low Reynolds number hydrodynamics*, Martinus Noordhoff, The Netherlands 1973.
9. R. HSU and P. GANATOS, *The motion of a rigid body in viscous fluid bounded by a plane wall*, *J. Fluid Mech.*, **207**, 29–72, 1989.
10. S. J. KARRILA and S. KIM, *Integral equations of the second kind for Stokes flow: Direct solution for physical variables and removal of inherent accuracy limitations*, *Chem. Engng. Comm.*, **82**, 123–161, 1989.
11. S. KIM and S. J. KARRILA, *Microhydrodynamics, principles and selected applications*, Butterworth-Heinemann, 1991.
12. M. KOHR, *The study of some viscous flows by boundary integral methods*, [in Romanian], Cluj University Press, 1997.
13. R. KRESS, *Linear integral equations*, Springer-Verlag, 1989.
14. O. A. LADYZHENSKAYA, *The mathematical theory of viscous incompressible flow*, Gordon Breach, 1963.
15. H. POWER, G. MIRANDA and R. GONZALES, *Integral equation solution for the flow due to the motion of a body of arbitrary shape near a plane wall at small Reynolds number*, *Math. Appl. Comput.*, **4**, 279–296, 1985.
16. H. POWER and B. F. POWER, *Second-kind integral equation formulation for the slow motion of a particle of arbitrary shape near a plane wall in a viscous fluid*, *SIAM J. Appl. Math.*, **53**, 1, 60–70, 1993.
17. C. POZRIKIDIS, *Creeping flow in two-dimensional channels*, *J. Fluid Mech.*, **188**, 495–514, 1987.
18. C. POZRIKIDIS, *Boundary integral and singularity methods for linearized viscous flows*, Cambridge University Press, 1992.

BABEȘ-BOLYAI UNIVERSITY
FACULTY OF MATHEMATICS

1, M. Kogălniceanu Str.
3400 Cluj-Napoca, Romania.
email:mkohr@math.ubbcluj.ro

Received December 16, 1996; new version January 7, 1998.