

Optimization of local heating for a spherical shell made of titanium alloy BT-23

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THE PROBLEM of optimization of local heating for a spherical shell made of titanium alloy BT-23 in order to reduce maximal transient or residual stresses is posed and solved. The effect of the volume changes occurring during phase transition is taken into account. The minimum of the transient or residual stresses is adopted as an optimization criterion for determination of optimal temperature fields.

1. Introduction

THIN-WALLED SHELL constructions during manufacturing and exploitation often undergo local heating, which is used for various technological reasons. This process produces the stressed state of construction which causes appearing of the cracks, plastic deformation etc. Therefore the task of local heating optimization is very important. Minimization of the stresses is performed by choosing such heating regime which leads to the stress distribution similar to a homogenous one. As it is shown in [1], non-optimal local heating may significantly reduce effectiveness of the heating.

Solution of the problem of local optimal heating is connected with using constitutive equations of the thermoelastic theory [2, 3], and the calculation methods of constructions exposed to various thermal and thermo-mechanical influences [4, 5]. Mathematical theory of optimal local heating is considered in [6] and [7–10]. However, those approaches are valid for the materials and heating regimes if there are no structural transformations.

In this paper we consider the problem of optimization of local heating for thin-walled shell constructions. In contrast to the papers [6–10], the following problems have been analyzed:

1. Phase transition during heating, and casting during cooling are taken into account.
2. The problem of choice of optimal heating is reduced to calculation of temperature fields which minimize the transient and residual stresses. Minimization of residual stresses was provided by preliminary optimization of heterogeneous phase distribution and calculation of the appropriate temperature field.

In direct solving and optimization of the problem of local heating, the residual stresses are calculated which are produced by heterogeneous phase distribution due to casting, but not by thermoplastic deformation. The authors have not

found the above formulation of the problem of local optimal heating in the literature. The inhomogeneous distribution of phases was established, what is a novelty in local heating problems.

2. The subject of the paper and constitutive equations

The purpose of this paper is to define the stress state of a spherical shell made of titanium alloy BT-23 during local equatorial heating, and to optimize it. We assume that the shell is free from any external mechanical influences and no stresses have been introduced by previous technological operations during its production and processing. Local heating of the shell is a technological process of heating of its equator from the initial temperature of 20°C to the temperature previously defined (in our case that temperature was 720°C). We consider the heating to depend upon the radial coordinate.

Alloy BT-23 during heating in the temperature range of $520 - 920^{\circ}\text{C}$ undergoes phase transition. During rapid cooling it is cast and the phase state is fixed, which is different from the one appearing during very slow cooling. During ordinary cooling of locally heated shell in the air, the casting conditions for the heated zone are provided because the cooling is not only due to the heat exchange with environment but also to the intensive heat flow to cooler parts of the shell.

We consider two reasons of appearing the stresses in the shell:

1. The stresses are produced by thermal dilatation of phase transitions. In this case the stresses are simple functions of temperature and can be defined by thermoelastic methods [2, 3].

2. After cooling, the material is cast and the spatially inhomogeneous phase state is fixed. The residual stresses are the result of the difference of the specific volumes of the phases. The phase state of the material after complete cooling depends on the temperature from which the cooling was started, and on the speed of the cooling. We consider the speed of cooling when the phase state depends only on the temperature from which the cooling was initiated.

In the normal state, when temperature is 20°C , the alloy BT-23 contains 80% of α -phase (hexahedral dense-packed lattice) and 20% of β -phase (body-centered cubic lattice). This alloy has the following chemical composition: Al - 5.4%, Mo - 1.9%, V - 4.7%, Cr - 1.2%, Fe - 0.5%, Ti - 86.3%. By heating the alloy in the temperature range of $520^{\circ}\text{C} - 920^{\circ}\text{C}$, the quantity of β -phase increases to 100%. As it was shown in [2], the difference in the specific volumes between α and β phases is about 2 - 5% which influences significantly the value and distribution of the transient and residual stresses.

Figure 1 presents the diagram of linear approximation of the β -phase mass fraction (Ξ) as a function of temperature t of the uniformly heated material (curve 1), and of the experimentally determined dependence of the residual fraction of β -phase in the material on the initial temperature of cooling (curve 2).

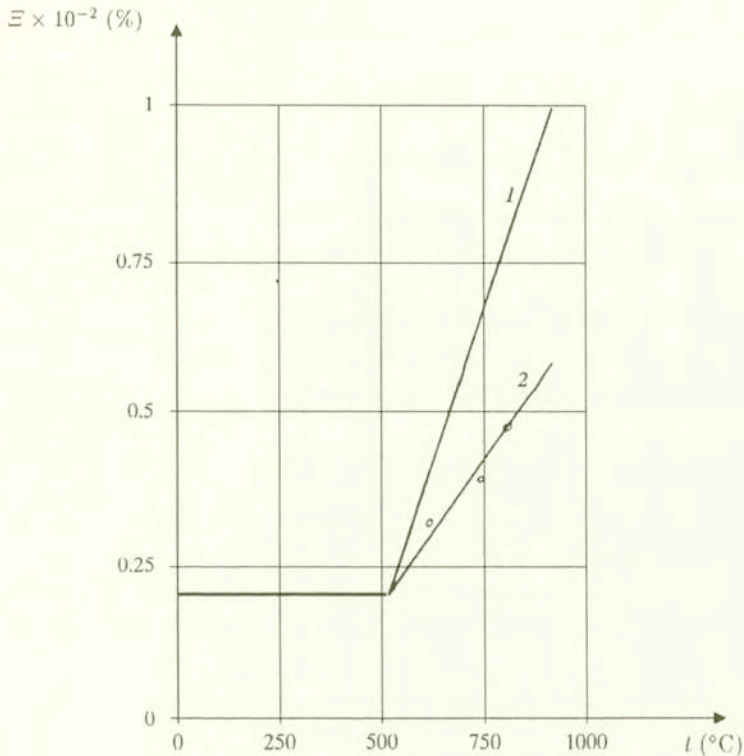


FIG. 1.

During heating the total change of shell volume de_T consists of two parts – thermal volume increment de_t and structural volume increment de_ξ :

$$(2.1) \quad de_T = de_t + de_\xi,$$

where $de_t = 3\alpha_* dt$, $de_\xi = 3\beta_* d\xi$, α_* is the linear coefficient of thermal expansion, β_* is the linear coefficient of structural dilatation (dilatation which depends on phase transition). In general, α_* and β_* are functions of temperature. It should be noticed that in the temperature range of phase transition of alloy $520^\circ \text{C} < t < 920^\circ \text{C}$, the thermal component of the volume e_t is increasing, and the structural one e_ξ is decreasing [2].

During very slow or stationary heating, at each point of the material certain phase distribution, which depends on temperature of the point, is established. Therefore, under these conditions, the phase transition fraction Ξ is a simple function of temperature t ($\Xi = \Xi(t)$), and the total change of the volume can be determined from the equation:

$$(2.2) \quad de_T = 3\alpha^*(t) dt,$$

where $\alpha^*(t) = \alpha_* + \beta_*(t) d\xi/dt$ is the generalized linear coefficient of thermal expansion which depends on all volume changes which occurred in the material

during heating. In the paper we assume:

$$(2.3) \quad \alpha^*(t) = \begin{cases} \alpha_1, & t < 520^\circ \text{C}, \\ (\alpha_1 + \alpha_2)/2 + \beta_*(t)d\varepsilon/dt, & 520^\circ \text{C} < t < 920^\circ \text{C}, \\ \alpha_2, & t > 920^\circ \text{C}, \end{cases}$$

where α_1 is the linear coefficient of thermal expansion in the temperature range of $20^\circ \text{C} < t < 520^\circ \text{C}$, α_2 is the linear coefficient of thermal expansion in the temperature range of $t > 920^\circ \text{C}$, $\beta_*(t)$ is an experimentally found function [11]. Figure 2 presents the graph of this coefficient $\alpha^*(t)$ (the graph has been taken from [11, 12]). It should be noticed that function β_* may be considered to be constant and it may be found from the equation:

$$(2.4) \quad \beta_* = (V_1 - V_2)/V_1,$$

where V_1 is the specific volume of the alloy when $t = 520^\circ \text{C}$, V_2 is the specific volume of the alloy when $t = 920^\circ \text{C}$.

After cooling, certain residual stress state is produced by inhomogeneous β -phase distribution. In this case, the volume change is defined by changing

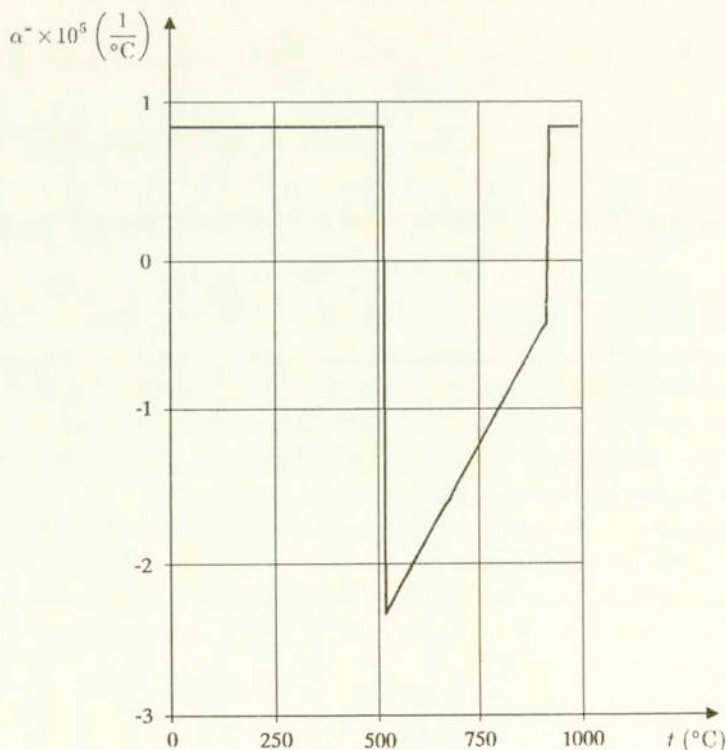


FIG. 2.

the function $d\varepsilon$:

$$(2.5) \quad de_{\varepsilon} = 3\beta_0 d\varepsilon,$$

where β_0 is the linear coefficient of structure dilatation. It was determined in [11]:

$$\beta_0 = \begin{cases} -0.013 & \text{for the points, which have experienced phase transitions,} \\ 0 & \text{for the points, which did not experience phase transitions.} \end{cases}$$

The generalized Hooke's law for the discussed case in a differential form can be written as

$$(2.6) \quad d\sigma_{ij} = \frac{E}{1+\nu} \left\{ de_{ij} + \frac{1}{1-2\nu} [\nu de - (1+\nu)de_a] \delta_{ij} \right\},$$

where $d\sigma_{ij}$ are components of the stress tensor, de_{ij} are components of the strain tensor, E is the elastic modulus, ν is Poisson's ratio, $e = e_{11} + e_{22} + e_{33}$, δ_{ij} is Kronecker's delta, $a = T, \varepsilon$, $i, j = (1, 2, 3)$.

It should be noticed that transient stresses during heating and residual stresses after heating are determined from Eq. (2.6). In this case the reason for residual stresses to appear is not the thermoplastic deformation but the inhomogeneous phase state which is fixed by rapid cooling.

From Eqs. (2.2), (2.5) and (2.6) we derive the equation for volume changes during the heating:

$$(2.7) \quad e_T = 3 \int_{t_0}^t \alpha_*(t) dt,$$

where t_0 is the initial temperature; and during the casting:

$$(2.8) \quad e_{\varepsilon} = \begin{cases} 3 \int_{\varepsilon_0}^{\varepsilon_s} \beta d\varepsilon & \text{for the points, which have experienced phase} \\ & \text{transitions,} \\ 0 & \text{for the points, which did not experience phase} \\ & \text{transitions.} \end{cases}$$

The generalized Hooke's law for the discussed case can be written in the form

$$(2.9) \quad \sigma_{ij} = \frac{E}{1+\nu} \left\{ e_{ij} + \frac{1}{1-2\nu} [\nu e - (1+\nu)e_a] \delta_{ij} \right\}.$$

3. The phase distribution and state of stress of spherical shell during and after the heating

At first we will define the state of stress of the spherical shell during heating. It is assumed that the shell is free from any external mechanical influences. Stationary, axisymmetric heating uniform across the width is provided along the equator of the sphere.

The meridional (σ_1) and circumferential (σ_2) stresses in the spherical shell are found from the relations [11]:

$$(3.1) \quad \sigma_1 = 1/(2h)(N_1 + 3\gamma/h^2 M_1), \quad \sigma_2 = 1/(2h)(N_2 + 3\gamma/h^2 M_2),$$

where γ is the distance from the middle surface of the shell ($-h < \gamma < h$),

$$N_1 = -D_1 V \operatorname{ctg} \varphi / R, \quad M_1 = -D_1 / R (d\theta / d\varphi + \nu \theta \operatorname{ctg} \varphi),$$

$$N_2 = -D_1 / R dV / d\varphi, \quad M_2 = -D_1 / R (\theta \operatorname{ctg} \varphi + \nu d\theta / d\varphi), \quad V = 1 / R (L(\theta) - \nu \theta).$$

N_1, N_2 are meridional and circumferential strains; M_1, M_2 are meridional and circumferential moments; θ is the angle of inclination of the normal to the middle surface of the shell, and it is determined from the equation [6]:

$$(3.2) \quad LL(\theta) + (D_* R^2 - \nu^2) = D_* R^2 \frac{d}{d\varphi} e_a,$$

where

$$L = \frac{d^2}{d\varphi^2} + \operatorname{ctg} \varphi \frac{d}{d\varphi} - \operatorname{ctg} \varphi,$$

φ is the arc length of the meridian, R is the radius of the middle surface of the shell, $D_* = D_0 / D_1$, $D_0 = 2Eh$ is the tensile rigidity; $D_1 = 2Eh^3 / (3(1 - \nu^2))$ is the bending rigidity, $2h$ is the thickness of the shell, $a = T, \Xi$.

During such a heating regime, the function θ will be defined when the equator does not rotate ($\theta(\pi/2) = 0$), the meridional strains are equal zero there ($N_1(\pi/2) = 0$), and meridional σ_1 and circumferential σ_2 stresses are symmetric with respect to the equatorial section.

Equation (3.1) was solved by means of the finite element method for the shell with the following physical and geometrical parameters: the radius of the middle surface of the shell $R = 0.1815$ m, the thickness $2h = 0.009$ m, $E = 106$ MPa, $\nu = 0.3$. While solving the direct problem, we used the practically local equatorial heating shown in Fig. 3, curve 1. Here $s = R\varphi$ is the distance measured along the meridian from the equator ($\varphi = 0, \pi/2$). This heating regime does not account for the phase transformation in the material. This temperature field corresponds to the transient heterogeneous phase distribution (Fig. 4, curve 1), which was defined by Fig. 1, curve 1, and the distribution of transient internal (Fig. 5, curves 1) and external (Fig. 6, curves 1) stresses; σ_1^\pm and σ_2^\pm are

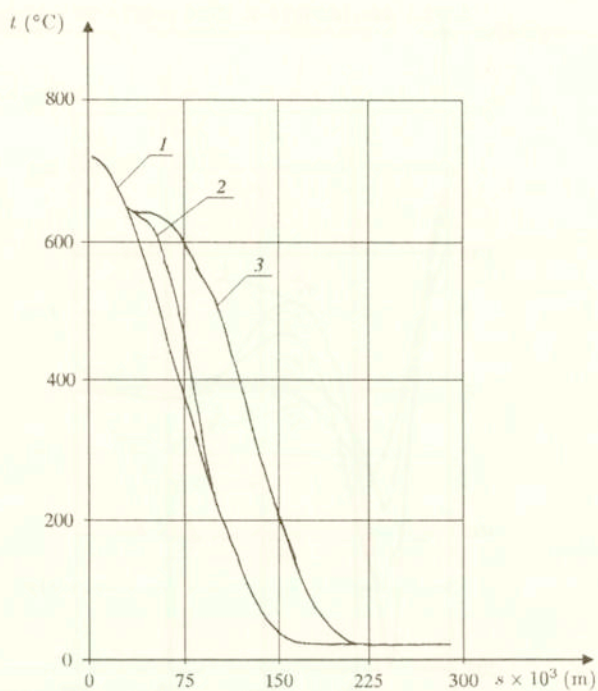


FIG. 3.

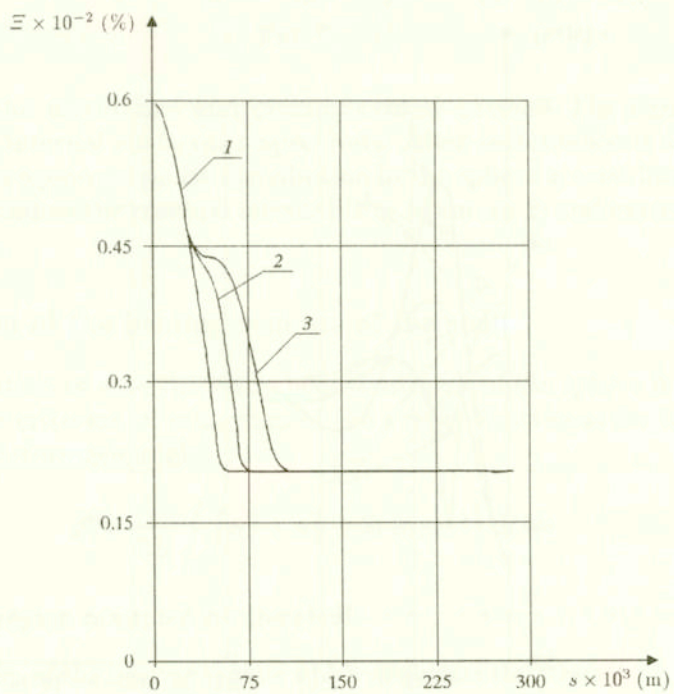


FIG. 4.

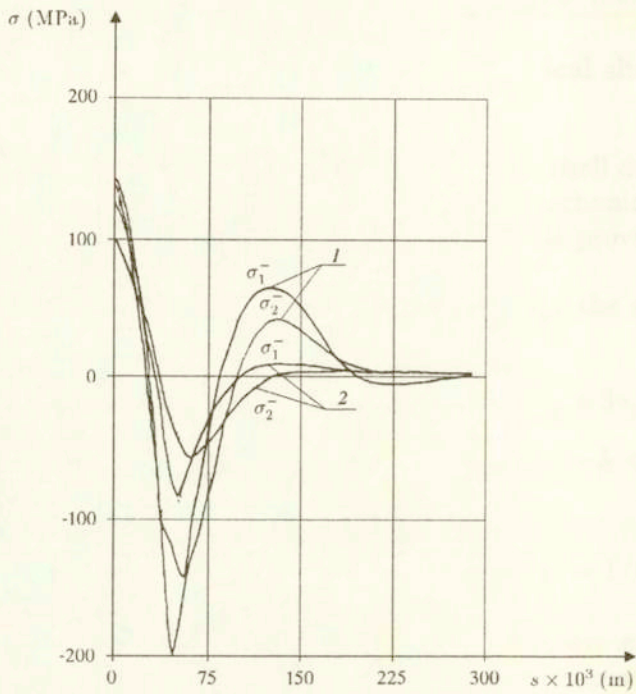


FIG. 5.

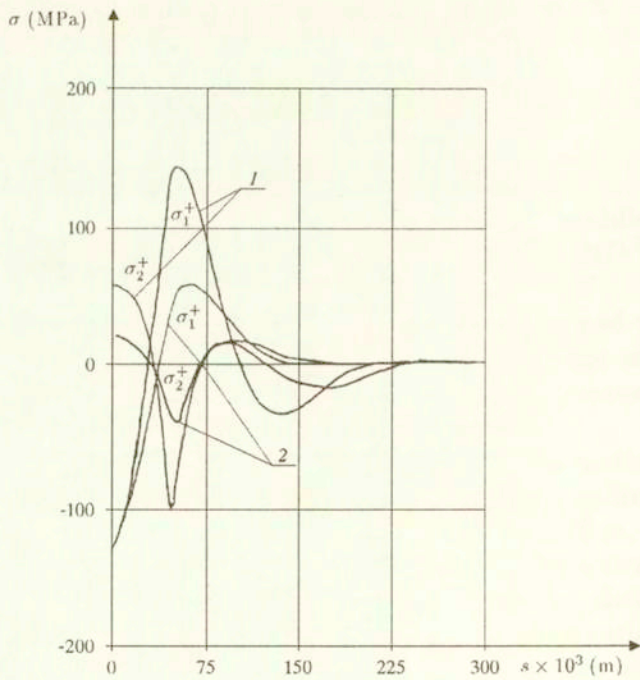


FIG. 6.

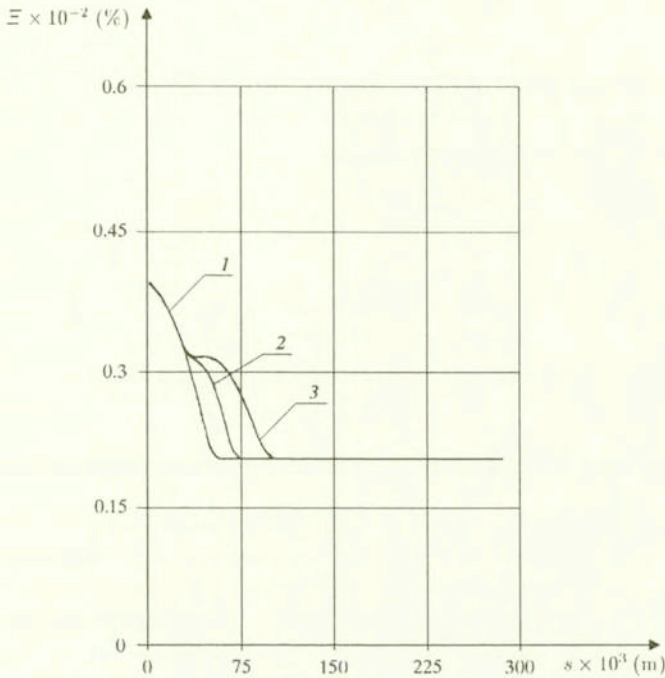


FIG. 7.

components of the meridional and circumferential stresses. The signs \pm apply to external and internal surfaces, respectively. After instantaneous stopping of heating, the heterogeneous phase distribution in the sphere is established (Fig. 7, curves 1) which causes the residual internal (Fig. 5, curves 2) and external (Fig. 6, curve 2) stresses.

4. Optimization of the heating regimes of the shell

The optimization of axi-symmetric heat treatment of the sphere is performed according to the criterion of minimum of the energy functional for transient or residual elastic deformation [8]:

$$(4.1) \quad F = \frac{1}{2E} \int_{\Omega} (\sigma_1^2 + \sigma_2^2 - 2\nu(\sigma_1\sigma_2)) d\Omega,$$

where Ω is the region occupied by the shell.

4.1. Determination of the heating regime which minimizes the stresses during the heating

The problem of determination of axi-symmetric stress during optimal heat treatment in the spherical shell with limitation imposed on temperature and

stress is considered. It is assumed that heating is homogeneous across the width. In the initial state there are no stresses in the shell. During and after the heating the shell is free from any mechanical influences. The local heating will be defined under limitation imposed on temperature ($t = t_*$) along the equator. The problem is solved under the following conditions:

$$(4.2) \quad t(\pm S_1) = t_0, \quad t(0) = t_*, \quad \left. \frac{\partial t}{\partial S} \right|_{S=\pm S} = 0,$$

where $S_1 = 0.2 \text{ m}$ is the range of heating, t_0 is the initial temperature. The optimal temperature field is the solution of the following problem: define the extremum of the elastic energy functional (4.1) which satisfies Eq. (3.2) and the condition of freedom from mechanical loading (4.2).

4.2. Determination of the heating regime which minimizes residual stresses

Residual stresses which appear in the shell after cooling are defined by inhomogeneous phase distribution. Therefore we optimize the phase state of the shell and then define the temperature field which leads to that phase state. Therefore the problem of determination of optimal phase distribution in the material subject to local heating uniformly distributed across the width is posed. There are no initial stresses in the shell. After the heating it is free from any mechanical influences. Local phase distribution will be defined with limitation of the β -phase mass fraction (Ξ_*) in the equator. The problem is solved with the following boundary conditions:

$$(4.3) \quad \Xi(\pm S_1) = \Xi_0, \quad \Xi(0) = \Xi_*, \quad \left. \frac{\partial \Xi}{\partial S} \right|_{S=\pm S} = 0,$$

where Ξ_0 is initial phase distribution in the material. The optimal phase distribution is the solution of the problem of defining the extremum of the elastic energy functional (4.1) which satisfies Eq. (3.2) and condition (4.3). The temperature field which corresponds to the optimal phase distribution is shown in Fig. 1, curve 2.

The optimization was accomplished by the Hooke-Jeeves method [13]. The temperature shown in Fig. 3, curve 1, was taken as an initial assumption in determining the optimal one, which minimizes the transient stresses in the sphere. The functional (4.1) was found from this initial approximation and its optimization was provided by variation of that temperature. Optimization of the functional was accomplished when its n -th step did not differ by more than 1% from the preceding one. The optimal temperature obtained by that method is presented in Fig. 3, curve 2. The transient phase distribution (Fig. 4, curve 2) and transient internal and external stresses (curves 1 in Fig. 8 and Fig. 9) are produced by that optimal temperature. After the cooling, the residual heterogeneous phase distribution (Fig. 7, curve 2) and the corresponding residual internal and external

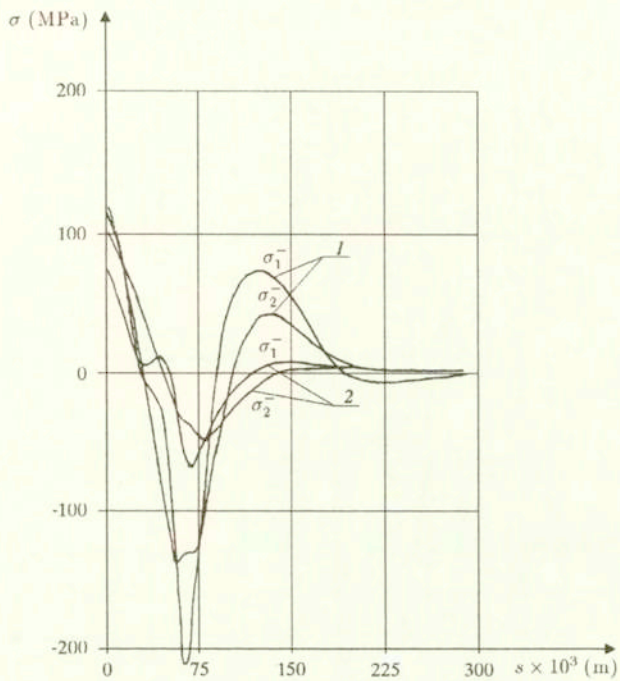


FIG. 8.

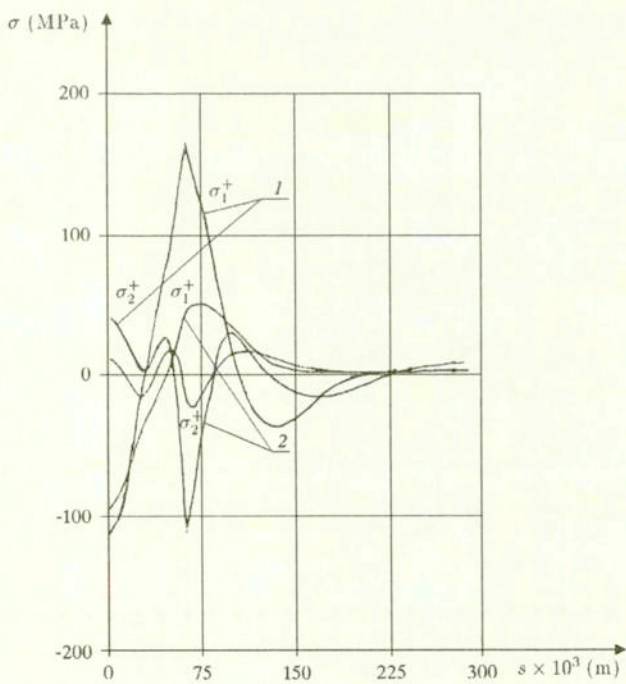


FIG. 9.

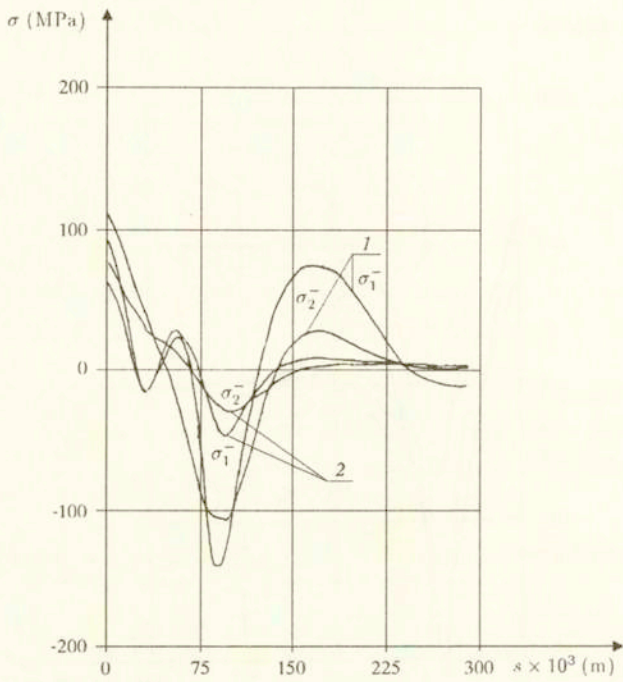


FIG. 10.

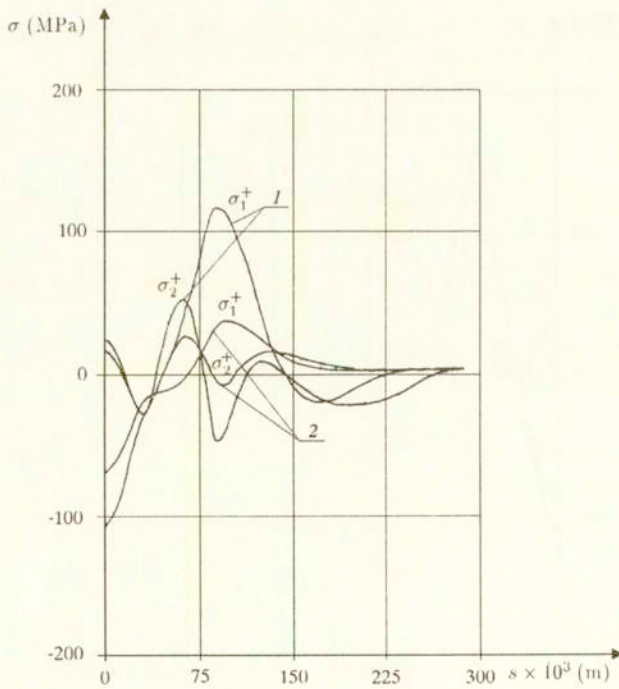


FIG. 11.

stresses (curve 2 in Fig. 8 and Fig. 9) are established. The temperature (Fig. 3, curve 2) minimizes the transient stresses in the sphere, that are the stresses which appear *during this heating regime*.

During minimization of the residual stresses, the initial functional (4.1) was defined by residual phase distribution (Fig. 7, curve 1) which was established after the same initial heating (Fig. 3, curve 1). The optimal residual phase distribution (Fig. 7, curve 3) was found by the Hooke - Jeeves method. The residual internal and external stresses (curves 2 in Fig. 10 and Fig. 11) correspond to that optimal residual phase distribution. The heating regime (Fig. 3, curve 3) which creates this phase distribution was defined by function $\Xi = \Xi(t)$ shown in Fig. 1, curve 2. The temperature (Fig. 3, curve 3) minimizes the residual stresses in the sphere which appear *after this heating regime*.

5. Conclusions

The optimal heating of the spherical shell made of titanium alloy BT-23 which undergoes phase transition exhibits some peculiarities following that phenomenon. Rapid cooling after heating and casting were taken into account by determining the residual stresses.

The determination of optimal temperature fields is performed under the assumption that temperature was not controlled once the heating had been stopped. From the obtained data we can see that optimal temperature field which minimizes the transient stresses (Fig. 3, curve 2) has a negligible effect on the stresses (compare curves 1 in Figs. 5, 6 with curves 1 in Figs. 8, 9 for transient stresses and curves 2 in Figs. 5, 6 with curves 2 in Figs. 8, 9 for residual stresses). The most significant effect was obtained by optimization of the residual phase distribution (Fig. 7, curve 3) which was established in the material after cooling. In this case we can reduce the residual stresses by approximately 35% (compare curves 2 in Figs. 5, 6 with curves 2 in Figs. 10, 11). The heat regime (Fig. 7, curve 3) which reflects that optimal residual phase distribution, was found by using the phase transition function $\Xi(t)$ presented in Fig. 1, curve 1. It turned out that, during that heat regime, the transient stresses have been reduced by approximately 20% in comparison with the initial heat treatment (Fig. 3, curve 1), compare curves 1 in Figs. 5, 6 with curves 1 in Figs. 10, 11.

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Received August 19, 1996; new version October 27, 1997.