# 2-D boundary value problems of thermoelasticity in a multi-wedge – multi-layered region
# Part 1. Sweep method

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A METHOD PROPOSED earlier to solve the BVP for Poisson's equation in a domain consisting of wedges and plane layers, is discussed and applied to 2D thermoelasticity problems. Linear conditions of general form are prescribed along the exterior boundaries as well as at all the interfaces. The essence of the method consists in combining the Fourier and Mellin transforms along the common interface. This allows to reduce the boundary value problems to special systems of singular equations. The analysis is significantly simplified by incorporating the fact that layers and wedges represent chain-like systems. In the paper, relations between the Fourier (Mellin) transformations of solutions for the layered (wedge-shaped) part of the domain are found by using the sweep method of LINKOV and FILIPPOV (1991, Mecchanica, 26, 195–209). All matrix-functions in the relations are slowly increasing ones. Their asymptotic behaviour is analyzed depending on the types of the exterior boundary conditions.

## 1. Introduction

ELASTICITY PROBLEMS for inhomogeneous bodies of regular structures (for example layered media) were intensively investigated in [2, 4, 5, 6, 7, 11, 12, 27, 32, 34, 39, 40]. We do not discuss here the problems and methods of their solution for composite laminates or periodic composite plates (see for this purpose [13, 31]). The important point of the mentioned bodies is the fact that they have chain-like structures. Independently of the applied technique (FEM, BEM, Fourier transform, etc.) in each layer, this made it possible to use the methods for chain-like systems of rods and beams to solve the problems under consideration. One of the most commonly encountered methods is the so-called "transfer-matrix" method having various modifications (see for example [1, 9, 17, 19, 30, 33]).

However, in the process of adaptation of this method, an intrinsic defect often occurs: the square matrices are ill-conditioned, and this defect is redoubled for products of matrices. Shortages of the "transfer-matrix" method are discussed in details in [16, 17, 19].

To eliminate such difficulties, many authors used special modifications of the method [18, 19, 38]. The modifications were based on the explicit forms of the boundary conditions between the layers.

As it was noted in [16], all the mentioned modifications were particular cases of a "sweep method" in fact. "General sweep method" for layered medium with

arbitrary boundary conditions along the interfaces was proposed and investigated in [16]. It consists in reduction of the problems to three-points difference equations. Corresponding results are then based on the theory of difference equations (see for example [10]). The stability of the proposed method is investigated, and conclusions of its efficiency and interconnections with the other methods are presented in [16], depending on the number of the layers and some types of the intermediate conditions.

For the case of multi-wedge bodies analogous results are obtained in [3]. Besides, the last work allows us to build stable algorithm to calculate parameters determining singularity of the gradient of solutions near the common corner tip for an arbitrary number of the wedges.

In [23, 24] classical two-dimensional boundary value problems for Poisson's equation in multi-layered – multi-wedge regions are investigated. Then the Fourier and Mellin transforms are applied to based domains (layer and wedge), respectively. As the simplest example of such geometry we can note elasticity problems for a crack normally terminating at the layered media, which were investigated by different techniques in [14, 15, 21, 36] and others.

Previously the idea of using the Fourier and Mellin transforms simultaneously to solve some plane and Mode III problems of linear elasticity for layered media with a notch or, in particular case, a crack was presented in [21, 22]. The notch (crack) was symmetric with respect to the normal to the interfaces, but intermediate boundary conditions were of the "ideal type" (defined by given discontinuities of displacements and tractions along the interfaces). At that time, explicit form of the interconnection formulae for the arising matrices (which takes into account "ideal" type of interfacial conditions) was very important. This made it possible to reduce the problems to a special class of systems of singular integral equations with fixed point singularities, and to investigate symbols of the corresponding systems in some Banach spaces with a relevant weight. The justification of the method [22] in a relevant space of distributions is presented from [25].

In the papers [23, 24] arbitrary numbers of the layers and wedges as well as the types of intermediate and external boundary conditions are considered. At that time, the "sweep method" proposed in [16] plays an important role. Moreover, in Appendix [23] exact asymptotic formulas (which are absent from [16]) for arising functions are obtained for all possible types of exterior and interior boundary conditions. This allows us to reduce the problems by the method [22] to the mentioned class of systems of integral equations and to investigate its symbols. Besides, in Appendix [24] it is shown that general partial differential equations of the divergent form (not only Poisson's equation) in the mentioned regions can be analogously solved.

In this paper, two-dimensional boundary value problems of thermoelasticity (see [28, 29]) in the multi-layered – multi-wedge region are considered. In the first part, necessary formulas derived in the process of solution of differ-

ent two-dimensional boundary value problems of thermoelasticity in layered (wedge-shaped) media by the "sweep method" are presented. They are generalizations of the formulas obtained in [3, 16]. Besides, asymptotic expansions of resultant matrix-functions near zero and infinity points, similar to those derived in [23], are found and justified for all types of linear interior and exterior boundary conditions. Then in the next part of the paper we shall make it possible to reduce all problems under consideration to systems of integral equations by the method of integral transforms.

General formulations of the problems are presented exactly in the second section of the paper. In the next two sections, the "sweep method" proposed in [16] is consequently applied in the layered and wedge-shaped parts of the domain. The main results consist in the Lemma 1 and Lemma 2.

## 2. Problem formulation

Let us consider the infinite domain presented in Fig. 1 consisting of a layered part $\Omega_L = \bigcup_{i=1}^{n} \Omega_i$ and two wedge parts $\Omega^+ = \bigcup_{j=1}^{m_+} \Omega_j^+$, $\Omega^- = \bigcup_{k=1}^{m_-} \Omega_k^-$.

$$\Omega_i = \{(x_1, x_2) :  x_1 \in \mathbb{R}, \ x_2 \in (y_{i-1}, y_i)\}, \quad i = 1, 2, \ldots, n,$$
$$\Omega_j^+ = \{(r, \theta) :  r \in \mathbb{R}_+, \ \theta \in (\theta_{j-1}^+, \theta_j^+)\}, \quad j = 1, 2, \ldots, m_+,$$
$$\Omega_k^- = \{(r, \theta) :  r \in \mathbb{R}_+, \ \theta \in (\theta_{k-1}^-, \theta_k^-)\}, \quad k = 1, 2, \ldots, m_-.$$

By $\Gamma_i$ ($i = 1, 2, ..., n-1$) we denote an interior boundary between the regions $\Omega_i$ and $\Omega_{i+1}$. Similarly, $\Gamma_j^+$ ($j = 1, 2, ..., m_+ - 1$) and $\Gamma_k^-$ ($k = 1, 2, ..., m_- - 1$) be the interior boundaries between the corresponding wedges.

$$0 = y_0 < \ldots < y_i < y_{i+1} < \ldots < y_n \le \infty, \qquad h_i = y_i - y_{i-1},$$

$$\pi = \theta_0^- < \ldots < \theta_k^- < \theta_{k+1}^- < \ldots < \theta_{m_-}^- = -\frac{\pi}{2} + \phi_*^-, \qquad \phi_k^- = \theta_k^- - \theta_{k-1}^-,$$

$$-\frac{\pi}{2} + \phi_*^+ = \theta_0^+, \ldots, < \theta_j^+ < \theta_{j+1}^+ < \ldots < \theta_{m_+}^+ = 0, \qquad \phi_j^+ = \theta_j^+ - \theta_{j-1}^+.$$

Thus, by $\Gamma_n$, $\Gamma_0^+$ and $\Gamma_{m_-}^-$ we denote the exterior boundaries of the layered region ($\Omega_L$), or the wedge-shaped regions ($\Omega^\pm$), respectively. Besides, let $\Gamma_0 = \Gamma_{m_+}^+ \cup \Gamma_0^-$ be the interior boundary between the different parts of domain $\Omega$.

We shall seek for the vector of displacements $\mathbf{u}(x_1, x_2)$ and the tensor of stresses $\underline{\sigma}(x_1, x_2)$ with components satisfying the equilibrium equations:

(2.1)           $$\sigma_{\beta\alpha,\beta} + X_\alpha = 0, \qquad \alpha, \beta = 1, 2,$$

and the Duhamel–Neumann relations (see [28]):

(2.2)           $$\sigma_{\alpha\beta} = \mu(u_{\alpha,\beta} + u_{\beta,\alpha}) + (\lambda\varepsilon - \gamma\Theta)\delta_{\alpha\beta}, \qquad \varepsilon = u_{\alpha,\alpha},$$

where $X_\alpha$ are the defined internal forces, $\mu, \lambda$ – Lamé constants, $\gamma = (2\mu + 3\lambda)\alpha_t$ – thermoelasticity constant ($\alpha_t$ is the coefficient of linear thermal expansion). Besides, if $\nu$ is the Poisson coefficient then $\lambda = 2\mu\nu^*/(1 - 2\nu^*)$, where $\nu^* = \nu$ under plane stress conditions, $\nu^* = \nu/(1 - \nu)$ under plane strain conditions. Further on we omit the upper index $*$ in the symbol $\nu^*$. All constants are different inside the regions $\Omega_i$, $\Omega_j^+$, $\Omega_k^-$, in general. We assume here that the temperature $\Theta(x_1, x_2) = T(x_1, x_2) - T(0, 0)$ is a known function. Corresponding boundary value problems for Poisson's equation for the function $\Theta(x_1, x_2)$ in a similar domain with different boundary and interfacial conditions have been solved in [23, 24].
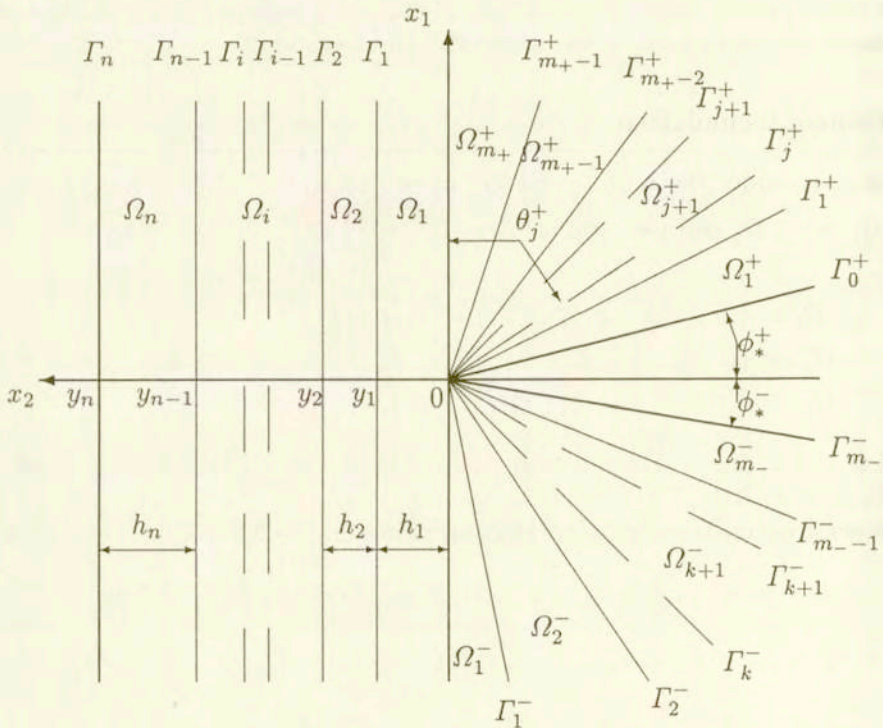


FIG. 1. The domain $\Omega$ under consideration.

Let us introduce the following symbols:

(2.3)

$$\mathbf{u} = \begin{cases} \mathbf{u}^{(i)} \\ \mathbf{v}^{(j)} \\ \mathbf{w}^{(k)} \end{cases}, \quad \underline{\boldsymbol{\sigma}} = \begin{cases} \underline{\boldsymbol{\sigma}}^{(i)} \\ \underline{\mathbf{p}}^{(j)} \\ \underline{\mathbf{q}}^{(k)} \end{cases}, \quad \mu = \begin{cases} \mu_i \\ \mu_j^+ \\ \mu_k^- \end{cases},$$

$$\lambda = \begin{cases} \lambda_i \\ \lambda_j^+ \\ \lambda_k^- \end{cases}, \quad \beta = \begin{cases} \beta_i \\ \beta_j^+ \\ \beta_k^- \end{cases}, \quad (x_1, x_2) \in \begin{cases} \Omega_i \\ \Omega_j^+ \\ \Omega_k^- \end{cases}.$$

Besides, we shall use Cartesian coordinates in the layered part and polar coordinates in the wedge parts of the domain.

Along the interior boundaries of layered domain $\Omega_L$ the conditions hold:

$$(2.4) \quad \begin{aligned} (\mathbf{u}^{(i+1)} - \mathbf{u}^{(i)} - \boldsymbol{\tau}_i\,\boldsymbol{\sigma}^{(i)})|_{\Gamma_i} &= \delta \mathbf{u}_i(x_1), & x_1 \in \mathbb{R}, & \quad i = 1, 2, ..., n-1, \\ (\boldsymbol{\sigma}^{(i+1)} - \boldsymbol{\sigma}^{(i)})|_{\Gamma_i} &= \delta\,\boldsymbol{\sigma}_i(x_1), & x_1 \in \mathbb{R}, & \end{aligned}$$

where $\mathbf{u}^{(i)} = (u_{x_2}^{(i)}, u_{x_1}^{(i)})^\top$, but $\boldsymbol{\sigma}^{(i)}$ is vector of stresses along boundary $\Gamma_i$; $\boldsymbol{\tau}_i$ is a diagonal matrix with positive constant components, but $\delta\boldsymbol{\sigma}_i$, $\delta\mathbf{u}_i$ are some known vector-functions. Analogous relations for the interior boundaries of wedge domains $\Omega^\pm$ are given in the form:

$$(2.5) \quad \begin{aligned} (\mathbf{v}^{(j+1)} - \mathbf{v}^{(j)} - r\boldsymbol{\tau}_j^+\,\mathbf{p}^{(j)})|_{\Gamma_j^+} &= \delta\mathbf{v}_j(r), & r \in \mathbb{R}_+, & \quad j = 1, 2, ..., m_+ - 1, \\ (\mathbf{p}^{(j+1)} - \mathbf{p}^{(j)})|_{\Gamma_j^+} &= \delta\mathbf{p}_j(r), & r \in \mathbb{R}_+; & \end{aligned}$$

$$(2.6) \quad \begin{aligned} (\mathbf{w}^{(k+1)} - \mathbf{w}^{(k)} - r\boldsymbol{\tau}_k^-\,\mathbf{q}^{(k)})|_{\Gamma_k^-} &= \delta\mathbf{w}_k(r), & r \in \mathbb{R}_+, & \quad k = 1, 2, ..., m_- - 1, \\ (\mathbf{q}^{(k+1)} - \mathbf{q}^{(k)})|_{\Gamma_k^-} &= \delta\mathbf{q}_k(r), & r \in \mathbb{R}_+, & \end{aligned}$$

where $\boldsymbol{\tau}_j^+$, $\boldsymbol{\tau}_k^-$ are diagonal matrices similar to $\boldsymbol{\tau}_i$.

Finally, the last of the interfacial conditions between different geometry regions (along the boundaries $\Gamma_{m_+}^+$, $\Gamma_0^-$) are characterized by given discontinuities of the displacements and tractions:

$$(2.7) \quad \begin{aligned} (\mathbf{u}^{(1)} - \mathbf{v}^{(m_+)})|_{\Gamma_{m_+}^+} &= \delta\mathbf{u}_+(x_1), \\ (\boldsymbol{\sigma}^{(1)} - \mathbf{p}^{(m_+)})|_{\Gamma_{m_+}^+} &= \delta\boldsymbol{\sigma}_+(x_1), \quad x_1 > 0; \end{aligned}$$

$$(2.8) \quad \begin{aligned} (\mathbf{u}^{(1)} - \mathbf{w}^{(1)})|_{\Gamma_0^-} &= \delta\mathbf{u}_-(-x_1), \\ (\boldsymbol{\sigma}^{(1)} + \mathbf{q}^{(1)})|_{\Gamma_0^-} &= \delta\boldsymbol{\sigma}_-(-x_1), \quad x_1 < 0. \end{aligned}$$

The direction of normals to the boundaries is taken into account in (2.8).

Now we define the exterior boundary conditions for the domain $\Omega$. So, on the wedge boundaries $\Gamma_0^+$, $\Gamma_{m_-}^-$ one from the following relations holds:

$$(2.9) \quad \begin{aligned} &\text{(a)} & \mathbf{v}^{(1)}|_{\Gamma_0^+} &= \delta\mathbf{v}_0(r), & & & r \in \mathbb{R}_+, \\ &\text{(b)} & \mathbf{p}^{(1)}|_{\Gamma_0^+} &= \delta\mathbf{p}_0(r), & & & r \in \mathbb{R}_+, \\ &\text{(c)} & v_\theta^{(1)}|_{\Gamma_0^+} &= \delta v_3(r), & p_{r\theta}^{(1)}|_{\Gamma_0^+} &= \delta p_3(r), & r \in \mathbb{R}_+, \\ &\text{(d)} & v_r^{(1)}|_{\Gamma_0^+} &= \delta v_4(r), & p_{\theta\theta}^{(1)}|_{\Gamma_0^+} &= \delta p_4(r), & r \in \mathbb{R}_+; \end{aligned}$$

$$\text{(2.10)}\quad\begin{aligned}
\text{(a)}\quad & \mathbf{w}^{(m-)}\big|_{\Gamma_{m_-}^-} = -\delta\mathbf{w}_0(r), & & r\in\mathbb{R}_+, \\[4pt]
\text{(b)}\quad & \mathbf{q}^{(m-)}\big|_{\Gamma_{m_-}^-} = -\delta\mathbf{q}_0(r), & & r\in\mathbb{R}_+, \\[4pt]
\text{(c)}\quad & w_\theta^{(m-)}\big|_{\Gamma_{m_-}^-} = -\delta w_3(r), \quad q_{r\theta}^{(m-)}\big|_{\Gamma_{m_-}^-} = -\delta q_3(r), & & r\in\mathbb{R}_+, \\[4pt]
\text{(d)}\quad & w_r^{(m-)}\big|_{\Gamma_{m_-}^-} = -\delta w_4(r), \quad q_{\theta\theta}^{(m-)}\big|_{\Gamma_{m_-}^-} = -\delta q_4(r), & & r\in\mathbb{R}_+.
\end{aligned}$$

Let us note that in the limiting case of crack $(\theta_0^+ = \theta_{m_-}^-)$ there is no contact between the crack surfaces. Some of such problems are considered in [20, 35] for homogeneous unbounded media.

On the exterior boundary $\Gamma_n$ we shall consider conditions (a), (b), (c), (d) analogous to (2.9), (2.10) and the relation (e):

$$\text{(2.11)}\quad\begin{aligned}
\text{(a)}\quad & \mathbf{u}^{(n)}\big|_{\Gamma_n} = -\delta\mathbf{u}_0(x_1), & & x_1\in\mathbb{R}, \\[4pt]
\text{(b)}\quad & \boldsymbol{\sigma}^{(n)}\big|_{\Gamma_n} = -\delta\boldsymbol{\sigma}_0(x_1), & & x_1\in\mathbb{R}, \\[4pt]
\text{(c)}\quad & u_{x_2}^{(n)}\big|_{\Gamma_n} = -\delta u_3(x_1), \quad \sigma_{x_1 x_2}^{(n)}\big|_{\Gamma_n} = -\delta\sigma_3(x_1), & & x_1\in\mathbb{R}, \\[4pt]
\text{(d)}\quad & u_{x_1}^{(n)}\big|_{\Gamma_n} = -\delta u_4(x_1), \quad \sigma_{x_2 x_2}^{(n)}\big|_{\Gamma_n} = -\delta\sigma_4(x_1), & & x_1\in\mathbb{R}, \\[4pt]
\text{(e)}\quad & \lim_{x_2\to\infty}\mathbf{u}^{(n+1)} = 0. & & x_1\in\mathbb{R}.
\end{aligned}$$

In the last case (e) we assume that the region $\Omega_{n+1}$ is a half-plane. Then the condition $(2.11)_e$ means that the solution of the problem decreases to zero in the direction $x_2\to\infty$ as well as in the other one $x_1\to\infty$. Consequently, we have here ninety different combinations of the exterior conditions. The corresponding problems (2.1)–(2.8) with the boundary conditions from (2.9)–(2.11) will be denoted by $(\mathcal{J}^+, \mathcal{J}^-, \mathcal{J})$, where $(\mathcal{J}^+ = 1 - 4,\ \mathcal{J}^- = 1 - 4,\ \mathcal{J} = 1 - 5)$. Here the value of $\mathcal{J}^+$ is equal to 1 (2, 3, 4) if the conditions $(2.9)_a$ $((2.9)_b,\ (2.9)_c,$ $(2.9)_d)$ hold. In an analogous way one can define the values of $\mathcal{J}^-,\ \mathcal{J}$ from the conditions (2.10) and (2.11), respectively.

We assume that all of the known functions which are presented in the equations and the boundary conditions are sufficiently smooth:

$$\begin{aligned}
\text{(2.12)}\quad & X_\alpha\in\mathbf{C}(\overline{G}), \qquad \Theta\in\mathbf{C}^1(\overline{G}), \qquad \delta\mathbf{u}_i\in\mathbf{C}_2^2(\mathbb{R}), \\[4pt]
& \qquad\qquad\qquad\qquad\qquad \delta\boldsymbol{\sigma}_i\in\mathbf{C}_2^1(\mathbb{R}), \qquad i=0,1,...,n-1, \\[4pt]
& \delta u_3,\delta u_4\in\mathbf{C}^2(\mathbb{R}), \qquad \delta\mathbf{v}_j,\delta\mathbf{w}_k,\delta\mathbf{u}_\pm\in\mathbf{C}_2^2(\mathbb{R}_+), \qquad j=0,1,...,m_+-1, \\[4pt]
& \delta\sigma_3,\sigma_4\in\mathbf{C}^1(\mathbb{R}), \qquad \delta\mathbf{p}_j,\delta\mathbf{q}_k,\delta\boldsymbol{\sigma}_\pm\in\mathbf{C}_2^1(\mathbb{R}_+), \qquad k=0,1,...,m_--1, \\[4pt]
& \delta v_3,\delta v_4,\delta w_3,\delta w_4\in\mathbf{C}^2(\mathbb{R}_+), \qquad \delta p_3,\delta p_4,\delta q_3,\delta q_4\in\mathbf{C}^1(\mathbb{R}_+), \\[4pt]
& \delta\mathbf{u}_i,r\delta\boldsymbol{\sigma}_i,\delta u_3,\delta u_4,\delta\mathbf{v}_j,\delta\mathbf{w}_k,\delta\mathbf{u}_\pm,r\delta\sigma_3,r\sigma_4,r\delta\mathbf{p}_j,r\delta\mathbf{q}_k,r\delta\boldsymbol{\sigma}_\pm,\delta v_3, \\[4pt]
& \quad \delta v_4,\delta w_3,\delta w_4,r\delta p_3,r\delta p_4,r\delta q_3,r\delta q_4,r\Theta,r^2 X_\alpha = o(r^{-1-\varepsilon}), \quad r\to\infty, \\[4pt]
& \delta\mathbf{u}_\pm,\delta\mathbf{v}_0,\delta\mathbf{w}_0,\delta v_3,\delta v_4,\delta w_3,\delta w_4 = o(r^{1+\varepsilon}), \quad r\to 0, \\[4pt]
& \delta\boldsymbol{\sigma}_\pm,\delta\mathbf{p}_0,\delta\mathbf{q}_0,\delta p_3,\delta p_4,\delta q_3,\delta q_4,\Theta,r X_\alpha = o(r^\varepsilon), \quad r\to 0.
\end{aligned}$$

Here $G$ denotes any region $\Omega_j$, $\Omega_j^{\pm}$ from $\Omega$, but $\varepsilon$ is some positive constant. The suppositions concerning the defined functions are assumed in the forms (2.12) in order to exhibit the singularities of solutions, connected with the internal properties of the problems only.

REMARK 1. As it follows from [23, 24], the temperature $\Theta$ is a function of the class $\mathbf{C}^2(G)$, at least, in any region $G$, and the asymptotics is true:

$$\Theta(r,\theta) = a + b\ln r + f_1^{\infty}(\theta)r^{-\omega_1^{\infty}} + f_2^{\infty}(\theta)r^{-\omega_2^{\infty}} + o(r^{-2-\varepsilon}), \qquad r \to \infty,$$

$$\Theta(r,\theta) = \begin{cases} f_0^0(\theta)r^{\omega_0^0} + \mathcal{O}(r), \\ f_1^0(\theta)r^{\omega_1^0} + f_2^0(\theta)r^{\omega_2^0} + o(r), & r \to 0, \\ f_3^0(\theta)r\ln r + \mathcal{O}(r^2), \end{cases}$$

where $\omega_1^{\infty}$, $\omega_2^{\infty} - 1$, $\omega_i^0 \in (0,1)$ are certain constants; functions $f_i^{\infty}$, $f_i^0$ depend on the geometry and exterior boundary conditions. Three different forms of the asymptotics of the temperature $\Theta$ near zero point depend on the type of interfacial boundary conditions. The first one corresponds to "ideal" interfacial conditions; the second – to "nonideal" contact through a thin heat conducting wedge; and, finally, the third term corresponds to interaction between the materials through a thin heat conducting layer. Besides, the constant $b = 0$ when there is a balance of the heat flow. When all known functions are equal to zero, except $\Theta$, we can easily find particular solutions of the problems $(2.1)-(2.11)$ in the neighbourhood of zero and infinity points using suitable asymptotics. Then using the property of linearity of the problems, we obtain the solutions of the initial problems as a sum of the mentioned solution and the solution of the problems $(2.1)-(2.11)$ under assumptions (2.12).

We shall seek for the classical solutions of the problems $(\mathcal{J}^+, \mathcal{J}^-, \mathcal{J})$ in a class of vector-functions $\mathbf{LW}(\Omega)$ such that $\mathbf{u} \in \mathbf{LW}(\Omega)$ if the following relations are true:

(2.13)

1.    $\mathbf{u}_{|G} \in \mathbf{C}^2(G)$,

2.    $\mathbf{u}(r,\theta) = \mathcal{O}(r^{-\gamma_1})$,     $\boldsymbol{\sigma}(r,\theta) = \mathcal{O}(r^{-\gamma_2-1})$,    $r \to \infty$,

3.    $\mathbf{u}(r,\theta) = \mathbf{u}_* + \mathcal{O}(r^{\gamma_0})$,    $\boldsymbol{\sigma}(r,\theta) = \mathcal{O}(r^{\gamma_0-1})$,    $r \to 0$,

where, as before, by $G$ we denote any region from $\Omega$, and $\gamma_0$, $\gamma_1$, $\gamma_2$ are certain constants such that $0 < \gamma_0 < 1$, $\gamma_1$, $\gamma_2 > 0$. Besides, in the cases when displacements (or one of its components) are prescribed at least on one of the wedge boundaries, the corresponding components of the vector $\mathbf{u}_*$ are equal to zero due to the assumptions (2.12) (the corresponding relations are presented in (3.29)). Precise values of the parameters $\gamma_0 = \gamma_0(\mathcal{J}^+, \mathcal{J}^-, \mathcal{J})$, $\gamma_1 = \gamma_1(\mathcal{J}^+, \mathcal{J}^-, \mathcal{J})$, $\gamma_2 = \gamma_2(\mathcal{J}^+, \mathcal{J}^-, \mathcal{J})$ and $\mathbf{u}_*$ will be obtained by solving the problems. Let us note that the value of $\gamma_0$ defines the order of stress singularity in the neighbourhood of zero and plays an important role in physical applications [8, 26].

REMARK 2. It can be shown that the solution of the problem $(\mathcal{J}^+, \mathcal{J}^-, \mathcal{J})$ from $\mathbf{LW}(\Omega)$ with the values of the parameters as given above belongs to an energetic space of the corresponding linear boundary problem [26]. Therefore, the problems in the class $\mathbf{LW}(\Omega)$ have unique solutions.

REMARK 3. Instead of the interfacial conditions (2.7), (2.8) between the domains of different geometry (layer and wedges), the other ones can be considered which are generalizations of the conditions (2.4), (2.5), (2.6). But, as it has been shown in [24] just for Mode III problem, this significantly complicates the problems, and such new conditions should be investigated separately.

## 3. Sweep method in the layered domain

Applying to Eqs. (2.1), (2.2) the Fourier transform of the form:

$$(3.1) \qquad \overline{f}(\lambda, x) = \mathcal{F}[f(x_1, x_2); \; x_1 \to \lambda] \equiv \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} \exp(i\lambda x_1) u(x_1, x_2) \, dx_1,$$

we obtain (see [37]) the following relations inside each of the layered domains $\Omega_j$, $(j = 1, ..., n)$:

$$(3.2) \qquad -i\lambda \overline{\sigma}_{1\alpha}^{(j)} + \frac{\partial}{\partial x_2} \overline{\sigma}_{2\alpha}^{(j)} + \overline{X}_\alpha^{(j)} = 0, \qquad \alpha = 1, 2;$$

$$(3.3) \qquad \begin{aligned}
-i\lambda \overline{u}_1^{(j)} &= \frac{1}{2\mu_j} \left[ \overline{\sigma}_{11}^{(j)} - \nu_j \overline{\sigma}^{(j)} \right] + \beta_j \overline{\Theta}^{(j)}, \\
\frac{\partial}{\partial x_2} \overline{u}_2^{(j)} &= \frac{1}{2\mu_j} \left[ \overline{\sigma}_{22}^{(j)} - \nu_j \overline{\sigma}^{(j)} \right] + \beta_j \overline{\Theta}^{(j)}, \\
\frac{\partial}{\partial x_2} \overline{u}_1^{(j)} - i\lambda \overline{u}_2^{(j)} &= \frac{1}{\mu_j} \overline{\sigma}_{12}, \qquad \sigma^{(j)} = \sigma_{11}^{(j)} + \sigma_{22}^{(j)},
\end{aligned}$$

where $\beta_j = \gamma_j / (2\mu_j + \lambda_j)$, but the constant $\nu_j$ is defined in (2.2). For the function $\overline{\sigma}_{22}^{(j)}$ the equation can be found:

$$(3.4) \qquad \left( \frac{\partial^2}{\partial x_2^2} - \lambda^2 \right)^2 \overline{\sigma}_{22}^{(j)} = \left( \frac{\partial^2}{\partial x_2^2} - \lambda^2 \right) g_1^{(j)} + \lambda^2 g_2^{(j)},$$

where

$$g_1^{(j)} = -\frac{\partial}{\partial x_2} \overline{X}_2^{(j)} - i\lambda \overline{X}_1^{(j)} + \lambda^2 \frac{2\mu_j \beta_j}{1 - \nu_j} \overline{\Theta}^{(j)}, \qquad g_2^{(j)} = \frac{1}{1 - \nu_j} \left[ \frac{\partial}{\partial x_2} \overline{X}_2^{(j)} - i\lambda \overline{X}_1^{(j)} \right].$$

The corresponding solution is of the form:

$$(3.5) \qquad \overline{\sigma}_{22}^{(j)} = C_1^j(\lambda)e^{-|\lambda|x_2} + x_2 C_2^j(\lambda)e^{-|\lambda|x_2} + C_3^j(\lambda)e^{|\lambda|x_2} + x_2 C_4^j(\lambda)e^{|\lambda|x_2} + \sigma_*^{(j)},$$

where

$$\sigma_*^{(j)} = -\frac{1}{2} \int\limits_{y_{j-1}}^{x_2} (\xi - x_2)g_2^{(j)}(\lambda,\xi)\text{ch}[(\xi - x_2)\lambda]\,d\xi$$

$$+ \frac{1}{2\lambda} \int\limits_{y_{j-1}}^{x_2} g_2^{(j)}(\lambda,\xi)\text{sh}[(\xi - x_2)\lambda]\,d\xi - \frac{1}{\lambda} \int\limits_{y_{j-1}}^{x_2} g_1^{(j)}(\lambda,\xi)\text{sh}[(\xi - x_2)\lambda]\,d\xi.$$

Using relations (3.2) and (3.3), Fourier transforms of all the remaining components of stress tensor and vector of displacements can be calculated in terms of functions $C_k^j$ $(k = 1, ..., 4)$.

Following [16], we denote new unknown vector-functions:

$$(3.6) \qquad \begin{aligned} \overline{\boldsymbol{\sigma}}_t^j(\lambda) &= \overline{\boldsymbol{\sigma}}^{(j)}(\lambda, x_2)_{|_{\Gamma_j}}, & \overline{\mathbf{u}}_t^j(\lambda) &= \overline{\mathbf{u}}^{(j)}(\lambda, x_2)_{|_{\Gamma_j}}, \\ \overline{\boldsymbol{\sigma}}_b^j(\lambda) &= \overline{\boldsymbol{\sigma}}^{(j)}(\lambda, x_2)_{|_{\Gamma_{j-1}}}, & \overline{\mathbf{u}}_b^j(\lambda) &= \overline{\mathbf{u}}^{(j)}(\lambda, x_2)_{|_{\Gamma_{j-1}}}, & j = 1, 2, ..., n. \end{aligned}$$

Further on we omit all overbars and upper brackets.

From *a priori* estimates (2.13) for vector-functions of class $\mathbf{LW}(\Omega)$ and from the properties of the Fourier transform it can be shown that the vector-functions defined above should satisfy the relations:

$$[\mathbf{u}_{t(b)}^j(\lambda)]_+ = \begin{cases} \mathcal{O}(\lambda^{\gamma_1 - 1}), & 0 < \gamma_1 < 1, \\ \mathcal{O}(\ln(\lambda)), & \gamma_1 = 1, \\ \text{Const} + \mathcal{O}(\lambda^{\gamma_1 - 1}), & \gamma_1 > 1, \quad \lambda \to 0; \end{cases}$$

$$[\mathbf{u}_{t(b)}^j(\lambda)]_- = \mathcal{O}(\lambda^{\gamma_1 - 1}), \quad \lambda \to 0;$$

$$[\boldsymbol{\sigma}_{t(b)}^j(\lambda)]_+ = \text{Const} + \mathcal{O}(\lambda^{\gamma_2}), \qquad [\boldsymbol{\sigma}_{t(b)}^j(\lambda)]_- = \mathcal{O}(\lambda^{\gamma_2}), \quad \lambda \to 0;$$

$$(3.7) \qquad \lambda\frac{\partial}{\partial\lambda}\boldsymbol{\sigma}_{t(b)}^j(\lambda) = \mathcal{O}(\lambda^{\gamma_2}), \qquad \lambda \to 0, \quad j = 1, 2, ..., n;$$

$$\lambda\mathbf{u}_{t(b)}^j, \; \lambda\mathbf{u}_t^1, \; \boldsymbol{\sigma}_{t(b)}^j, \; \boldsymbol{\sigma}_t^1, \; \lambda\frac{\partial}{\partial\lambda}\boldsymbol{\sigma}_{t(b)}^j, \; \lambda\frac{\partial}{\partial\lambda}\boldsymbol{\sigma}_t^1 = \mathcal{O}(\lambda^{-2}),$$

$$\lambda \to \infty, \quad j = 2, 3, ..., n;$$

$$\lambda\mathbf{u}_b^1(\lambda), \; \boldsymbol{\sigma}_b^1(\lambda), \; \lambda\frac{\partial}{\partial\lambda}\boldsymbol{\sigma}_b^1(\lambda) = \mathcal{O}(\lambda^{-\gamma_0}), \qquad \lambda \to \infty.$$

Here by $[\mathbf{f}(\lambda)]_\pm$ we understand the even (odd) part of a vector-function $\mathbf{f}$. All constants in (3.7) are different, in general.

Substituting the relations (3.6) in (3.2), (3.3) and eliminating the functions of $C_k^j(\lambda)$, we obtain the relations between vector-functions $\mathbf{u}_{t(b)}^j$ and $\boldsymbol{\sigma}_{t(b)}^j$ in the form:

$$
(3.8) \qquad
\begin{aligned}
\mathbf{u}_t^j &= \mathbf{R}_{tt}^j \boldsymbol{\sigma}_t^j + \mathbf{R}_{tb}^j \boldsymbol{\sigma}_b^j + \mathbf{u}_{t0}^j, \\
\mathbf{u}_b^j &= \mathbf{R}_{bt}^j \boldsymbol{\sigma}_t^j + \mathbf{R}_{bb}^j \boldsymbol{\sigma}_b^j + \mathbf{u}_{b0}^j, \qquad j = 1, 2, ..., n,
\end{aligned}
$$

where coefficients are calculated in the following way:

$$
\mathbf{R}_{tt}^j(\lambda) = \frac{1}{2i\lambda\mu_j} \mathbf{E}_2 - \frac{1-\nu_j}{\lambda^2\mu_j} \mathbf{R}_1^j \mathbf{E}_1,
$$

$$
\mathbf{R}_{bb}^j(\lambda) = \frac{1}{2i\lambda\mu_j} \mathbf{E}_2 + \frac{1-\nu_j}{\lambda^2\mu_j} \mathbf{E}_1 \mathbf{R}_1^j,
$$

$$
\mathbf{R}_{tb}^j(\lambda) = \frac{1-\nu_j}{\lambda^2\mu_j} \mathbf{R}_2^j \mathbf{E}_1,
$$

$$
\mathbf{R}_{bt}^j(\lambda) = -\frac{1-\nu_j}{\lambda^2\mu_j} \mathbf{E}_1 \mathbf{R}_2^j, \qquad
\mathbf{E}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
$$

$$
(3.9) \ \ \mathbf{R}_1^j = r_j(\lambda) \begin{pmatrix} h_j + \dfrac{1}{2\lambda}\mathrm{sh}2\lambda h_j & \dfrac{i}{\lambda}\mathrm{sh}^2\lambda h_j \\[2mm] \dfrac{i}{\lambda}\mathrm{sh}^2\lambda h_j & h_j - \dfrac{1}{2\lambda}\mathrm{sh}2\lambda h_j \end{pmatrix}, \qquad
\mathbf{E}_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix},
$$

$$
\mathbf{R}_2^j = r_j(\lambda) \begin{pmatrix} h_j\mathrm{ch}\lambda h_j + \dfrac{1}{\lambda}\mathrm{sh}\lambda h_j & -ih_j\mathrm{sh}\lambda h_j \\[2mm] ih_j\mathrm{sh}\lambda h_j & h_j\mathrm{ch}\lambda h_j - \dfrac{1}{\lambda}\mathrm{sh}\lambda h_j \end{pmatrix},
$$

$$
r_j(\lambda) = \frac{\lambda^2}{\lambda^2 h_j^2 - \mathrm{sh}^2\lambda h_j}, \qquad
\mathbf{u}_{t(b)0}^j = \frac{1-\nu_j}{\lambda^2\mu_j} r_j(\lambda)
$$

$$
\times \begin{pmatrix} f_{t(b)}^1 + \left[ \dfrac{1}{2(1-\nu_j)r_j(\lambda)} - \dfrac{\mathrm{sh}^2\lambda h_j}{\lambda^2} \right] \overline{X}_{2t(b)} - \dfrac{h_j}{\lambda}\mathrm{sh}\lambda h_j \overline{X}_{2b(t)} \\[3mm] i\left( f_{t(b)}^2 \pm \left[ h_j - \dfrac{\mathrm{sh}2\lambda h_j}{2\lambda} \right] \overline{X}_{2t(b)} \mp \left[ h_j\mathrm{ch}\lambda h_j - \dfrac{1}{\lambda}\mathrm{sh}\lambda h_j \right] \overline{X}_{2b(t)} \right) \end{pmatrix},
$$

where $y_b = y_{i-1}$, $y_t = y_i$, $\overline{X}_{2t(b)} = \overline{X}_2^{(j)}(\lambda, y_{t(b)})$, but the remaining functions are defined by

$$
f_{t(b)}^1(\lambda) = \int_{y_{j-1}}^{y_j} [g_2 - 2g_1](\lambda, \xi) \left\{ \frac{h_j}{2\lambda}\mathrm{sh}[\lambda(\xi - y_{t(b)})] + \frac{\mathrm{sh}\lambda h_j}{2\lambda^2}\mathrm{sh}[\lambda(\xi - y_{b(t)})] \right\} d\xi
$$

$$
- \int_{y_{j-1}}^{y_j} g_2(\lambda, \xi) \left\{ \frac{\mathrm{sh}\lambda h_j}{2\lambda}(\xi - y_{t(b)})\mathrm{ch}[\lambda(\xi - y_{b(t)})] + \frac{h_j}{2}(\xi - y_{b(t)})\mathrm{ch}[\lambda(\xi - y_{t(b)})] \right\} d\xi,
$$

$$f^2_{t(b)} = \frac{h_j}{2} \int\limits_{y_{j-1}}^{y_j} (\xi - y_{b(t)}) g_2 \text{sh}[\lambda(\xi - y_{t(b)})] d\xi - \frac{\text{sh}\lambda h_j}{\lambda^2} \int\limits_{y_{j-1}}^{y_j} g_1 \text{ch}[\lambda(\xi - y_{b(t)})] d\xi$$

$$-\frac{\text{sh}\lambda h_j}{2\lambda} \int\limits_{y_{j-1}}^{y_j} (\xi - y_{t(b)}) g_2 \text{sh}[\lambda(\xi - y_{b(t)})] d\xi + \frac{h_j}{\lambda} \int\limits_{y_{j-1}}^{y_j} g_1 \text{ch}[\lambda(\xi - y_{t(b)})] d\xi.$$

Note that the matrix-functions $\mathbf{R}^j_{t(b)t(b)}(\lambda)$ and the vector-functions $\mathbf{u}^j_{b(t)0}(\lambda)$ can be estimated like $\mathcal{O}(\lambda^{-4})$ when $\lambda \to 0$. But in view of (3.7), the unknown vectors $\mathbf{u}^j_{t(b)}(\lambda)$, $\boldsymbol{\sigma}^j_{t(b)}(\lambda)$ are bounded near this point. Hence, by investigating the main terms of asymptotics ($\lambda \to 0$) of the right-hand side of relations (3.8) it can be obtained that vector-functions $\boldsymbol{\sigma}^j_{t(b)}(\lambda)$ should satisfy the following additional relations:

(3.10)
$$\boldsymbol{\sigma}^j_t(0) - \boldsymbol{\sigma}^j_b(0) + \int\limits_{y_{j-1}}^{y_j} \overline{\mathbf{X}}^j(0, \xi) d\xi = 0,$$

$$\mathbf{X}^j(x_1, x_2) = [X_2^{(j)}(x_1, x_2), X_1^{(j)}(x_1, x_2)]^\top,$$

$$[1, 0] i \frac{d}{d\lambda} \left\{ \boldsymbol{\sigma}^j_t(\lambda) - \boldsymbol{\sigma}^j_b(\lambda) + \int\limits_{y_{j-1}}^{y_j} \overline{\mathbf{X}}^j(\lambda, \xi) d\xi \right\}\bigg|_{\lambda=0}$$

$$+ [0, 1] \left\{ y_j \boldsymbol{\sigma}^j_t(0) - y_{j-1} \boldsymbol{\sigma}^j_b(0) + \int\limits_{y_{j-1}}^{y_j} \xi \overline{\mathbf{X}}^j(0, \xi) d\xi \right\} = 0.$$

Let us note, that the mentioned equations are the usual equilibrium conditions of the $j$-th layer.

Now we apply the Fourier transform to the interfacial contact conditions along boundaries $\Gamma_j$, $j = 1, 2, ..., n-1$. The corresponding equations can be written in terms of vector-functions $\mathbf{u}^j_{t(b)}$, $\boldsymbol{\sigma}^j_{t(b)}$ defined above

(3.11)
$$\mathbf{u}^{j+1}_b - \mathbf{u}^j_t - \boldsymbol{\tau}_j \boldsymbol{\sigma}^j_t = \Delta \mathbf{u}_j,$$

$$\boldsymbol{\sigma}^{j+1}_b - \boldsymbol{\sigma}^j_t = \Delta \boldsymbol{\sigma}_j, \qquad j = 1, 2, ..., n-1.$$

Here $\Delta \mathbf{u}_j(\lambda) = \mathcal{F}[\delta \mathbf{u}_j](\lambda)$, $\Delta \boldsymbol{\sigma}_j(\lambda) = \mathcal{F}[\delta \boldsymbol{\sigma}_j](\lambda)$ are the Fourier transforms of known functions.

As it has been shown in [16], relations (3.8) and (3.11) make it possible to eliminate the unknown functions, either $\mathbf{u}^j_{t(b)}$ or $\boldsymbol{\sigma}^j_{t(b)}$, and to obtain formulas for the remaining ones. We present the relations for vector-functions $\boldsymbol{\sigma}^j_{t(b)}$. Substitute (3.8) in (3.11), then two systems of difference equations (3.12)$_a$, (3.12)$_b$ for

vector-functions $\boldsymbol{\sigma}_{t(b)}^j$ are obtained

$$(3.12) \quad
\begin{aligned}
&\text{(a)} \quad \mathbf{A}_\sigma^j \boldsymbol{\sigma}_t^{j-1} - \mathbf{C}_\sigma^j \boldsymbol{\sigma}_t^j + \mathbf{B}_\sigma^j \boldsymbol{\sigma}_t^{j+1} + \mathbf{F}_{\sigma t}^j = 0, \qquad j = 2, 3, ..., n-1, \\
&\text{(b)} \quad \mathbf{A}_\sigma^j \boldsymbol{\sigma}_b^j - \mathbf{C}_\sigma^j \boldsymbol{\sigma}_b^{j+1} + \mathbf{B}_\sigma^j \boldsymbol{\sigma}_b^{j+2} + \mathbf{F}_{\sigma b}^j = 0, \qquad j = 1, 2, ..., n-2,
\end{aligned}$$

where

$$(3.13) \quad
\begin{aligned}
\mathbf{A}_\sigma^j &= -\mathbf{R}_{tb}^j, \qquad \mathbf{C}_\sigma^j = \boldsymbol{\tau}_j + \mathbf{R}_{tt}^j - \mathbf{R}_{bb}^{j+1}, \qquad \mathbf{B}_\sigma^j = \mathbf{R}_{bt}^{j+1}, \\
\mathbf{F}_{\sigma b}^j &= \mathbf{u}_{b0}^{j+1} - \mathbf{u}_{t0}^j - \Delta \mathbf{u}_j + (\mathbf{R}_{tt}^j + \boldsymbol{\tau}_j)\Delta\boldsymbol{\sigma}_j - \mathbf{R}_{bt}^{j+1}\Delta\boldsymbol{\sigma}_{j+1}, \\
\mathbf{F}_{\sigma t}^j &= \mathbf{u}_{b0}^{j+1} - \mathbf{u}_{t0}^j - \Delta \mathbf{u}_j + \mathbf{R}_{bb}^{j+1}\Delta\boldsymbol{\sigma}_j - \mathbf{R}_{tb}^j\Delta\boldsymbol{\sigma}_{j-1}.
\end{aligned}$$

Equations (3.12) are identical in the case when all jumps of vector-functions $\boldsymbol{\sigma}_j$ across the interfaces ($\Delta\boldsymbol{\sigma}_j = 0$, $j = 1, 2, ..., n-1$) are equal to zero.

In order to solve any of these systems of difference equations, it is necessary to have exterior boundary ("initial" in this sense) conditions for the first and the last surface of the package of layers. At boundary $\Gamma_n$ one of the conditions from (2.11) is defined. Applying the Fourier transform to the corresponding boundary condition and taking into account (3.8), (3.11), we rewrite it in a form similar to (3.12):

$$(3.14) \quad \mathbf{A}_\sigma^n \boldsymbol{\sigma}_t^{n-1} - \mathbf{C}_\sigma^n \boldsymbol{\sigma}_t^n + \mathbf{F}_{\sigma t}^n = 0,$$

where the functions $\mathbf{A}_\sigma^n$, $\mathbf{C}_\sigma^n$, $\mathbf{F}_{\sigma t}^n$ are defined for each of the conditions $(2.11)_a$ – $(2.11)_e$:

$$\mathcal{J} = 1:$$
$$\mathbf{A}_\sigma^n = -\mathbf{R}_{tb}^n, \qquad \mathbf{C}_\sigma^n = \mathbf{R}_{tt}^n,$$
$$\mathbf{F}_{\sigma t}^n = -\Delta \mathbf{u}_n - \mathbf{u}_{t0}^n - \mathbf{R}_{tb}^n \Delta\boldsymbol{\sigma}_{n-1},$$

$$\mathcal{J} = 2:$$
$$\mathbf{A}_\sigma^n = 0, \qquad \mathbf{C}_\sigma^n = \mathbf{I}, \qquad \mathbf{F}_{\sigma t}^n = -\Delta\boldsymbol{\sigma}_n,$$

$$\mathcal{J} = 3, 4:$$
$$\mathbf{A}_\sigma^n = -\begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} \mathbf{R}_{tb}^n,$$

$$(3.15) \qquad \mathbf{C}_\sigma^n = \begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} \mathbf{R}_{tt}^n + \begin{pmatrix} \delta_{\mathcal{J}4} & 0 \\ 0 & \delta_{\mathcal{J}3} \end{pmatrix},$$

$$\mathbf{F}_{\sigma t}^n = -\begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} \left( \mathbf{u}_{t0}^n + \mathbf{R}_{tb}^n \Delta\boldsymbol{\sigma}_{n-1} \right) - \Delta \mathbf{h}_{\mathcal{J}},$$

$$\Delta \mathbf{h}_{\mathcal{J}} = \begin{pmatrix} \overline{\delta u_{\mathcal{J}}(\lambda)} \\ \overline{\delta \sigma_{\mathcal{J}}(\lambda)} \end{pmatrix},$$

$$\mathcal{J} = 5:$$
$$\mathbf{A}_\sigma^n = -\mathbf{R}_{tb}^n, \qquad \mathbf{C}_\sigma^n = \boldsymbol{\tau}_n + \mathbf{R}_{tt}^n - \mathbf{R}_\infty^{n+1},$$
$$\mathbf{F}_{\sigma t}^n = -\Delta \mathbf{u}_n - \mathbf{u}_{t0}^n - \mathbf{R}_{tb}^n \Delta\boldsymbol{\sigma}_{n-1} + \mathbf{R}_\infty^{n+1}\Delta\boldsymbol{\sigma}_n + \mathbf{u}_\infty^{n+1},$$

where $\delta_{Jj}$ is the Kronecker symbol, but

$$\mathbf{R}_\infty^{n+1}(\lambda) = \lim_{h_{n+1}\to\infty} \mathbf{R}_{bb}^{n+1}(\lambda, h_{n+1}) = -\frac{1-\nu_{n+1}}{\mu_{n+1}|\lambda|}\mathbf{I} + i\frac{1-2\nu_{n+1}}{2\mu_{n+1}\lambda}\mathbf{E}_2,$$

$$\mathbf{u}_\infty^{n+1}(\lambda) = \lim_{h_{n+1}\to\infty} \mathbf{u}_{b0}^{n+1}(\lambda) = \frac{1-\nu_{n+1}}{2\mu_{n+1}\lambda^2}$$

$$\times \begin{pmatrix} \displaystyle\int_{y_n}^\infty [g_2 - 2g_1](\lambda,\xi)e_n\,d\xi + |\lambda|\int_{y_n}^\infty g_2(\lambda,\xi)(\xi - y_n)e_n\,d\xi + \frac{3-2\nu_{n+1}}{1-\nu_{n+1}}\overline{X}_{2b} \\[2mm] \displaystyle i\,\mathrm{sign}\lambda\left[2\int_{y_n}^\infty g_1(\lambda,\xi)e_n\,d\xi - |\lambda|\int_{y_n}^\infty g_2(\lambda,\xi)(\xi - y_n)e_n\,d\xi - 2\overline{X}_{2b}\right] \end{pmatrix},$$

where $e_n = e^{|\lambda|(y_n-\xi)}$.

Let us note that relation (3.14) corresponding to (9e) is obtained by passing to the limit $h_{n+1} \to \infty$ in (3.8) and taking into account boundary condition (3.12) for $j = n$. This fact allows us to consider this condition in a common scheme with the other ones.

In order to complete the difference equations (3.12) it is necessary to know the second boundary condition along boundary $\Gamma_0$. Such a condition is absent (the solution along $\Gamma_0$ is not known and is connected with the solutions inside the wedge-shaped regions $\Omega_{m_+}^+$, $\Omega_0^-$). To overcome the mentioned difficulty, let us assume that this condition has the form:

$$(3.16) \qquad \mathbf{z}(x_1) = \boldsymbol{\sigma}^{(1)}(x_1, x_2)|_{\Gamma_a},$$

with some unknown vector-function $\mathbf{z}(x_1)$. Then the missing relation can be written:

$$(3.17) \qquad -\mathbf{C}_\sigma^0\boldsymbol{\sigma}_b^1 + \mathbf{B}_\sigma^0\boldsymbol{\sigma}_b^2 + \mathbf{F}_{\sigma b}^0 = 0,$$

where

$$\mathbf{C}_\sigma^0 = \mathbf{I}, \qquad \mathbf{B}_\sigma^0 = 0, \qquad \mathbf{F}_{\sigma b}^0 = \overline{\mathbf{z}}_+(\lambda) + \overline{\mathbf{z}}_-(\lambda).$$

Here we present the vector-function $\mathbf{z}(x_1)$ as a sum of even and odd vector-functions $\mathbf{z}_+(x_1)$, $\mathbf{z}_-(x_1)$.

As one could expect, the boundary conditions (3.14) and (3.17) are prescribed for different vector-functions $\boldsymbol{\sigma}_b^j$, $\boldsymbol{\sigma}_t^j$. In order to solve any of the systems of difference equations $(3.12)_a$, $(3.12)_b$, these equations should be rewritten in terms of the common type vector-functions. So (3.17) can be written in the form:

$$(3.18) \qquad -\mathbf{C}_\sigma^0\boldsymbol{\sigma}_t^0 + \mathbf{B}_\sigma^0\boldsymbol{\sigma}_t^1 + \mathbf{F}_{\sigma t}^0 = 0,$$

where $\mathbf{C}_\sigma^0$, $\mathbf{B}_\sigma^0$ are defined above, $\mathbf{F}_{\sigma t}^0 = \mathbf{F}_{\sigma b}^0$, and the vector-function $\boldsymbol{\sigma}_t^0$ is defined by the relation similar to $(3.11)_2$:

$$(3.19) \qquad \boldsymbol{\sigma}_t^0 = \boldsymbol{\sigma}_b^1 \qquad (\Delta\boldsymbol{\sigma}_0 = 0).$$

Now we can solve the difference equation $(3.12)_a$ with boundary conditions (3.14), (3.18) by the sweep method. Following [16], we define the auxiliary matrix-functions $\boldsymbol{\alpha}_\sigma^j$ and vector-functions $\boldsymbol{\beta}_\sigma^j$:

$$(3.20) \qquad \boldsymbol{\alpha}_\sigma^n = (\mathbf{C}_\sigma^n)^{-1}\mathbf{A}_\sigma^n, \qquad \boldsymbol{\beta}_\sigma^n = (\mathbf{C}_\sigma^n)^{-1}\mathbf{F}_{\sigma t}^t,$$

and in the next steps

$$(3.21) \qquad \begin{aligned} \boldsymbol{\alpha}_\sigma^j &= (\mathbf{C}_\sigma^j - \mathbf{B}_\sigma^j\boldsymbol{\alpha}_\sigma^{j+1})^{-1}\mathbf{A}_\sigma^j, \\ \boldsymbol{\beta}_\sigma^j &= (\mathbf{C}_\sigma^j - \mathbf{B}_\sigma^j\boldsymbol{\alpha}_\sigma^{j+1})^{-1}(\mathbf{F}_{\sigma t}^j + \mathbf{B}_\sigma^j\boldsymbol{\beta}_\sigma^{j+1}), \qquad j = n-1, ..., 2, 1. \end{aligned}$$

Then the solutions of this problem are in the form:

$$(3.22) \qquad \boldsymbol{\sigma}_t^0 = \boldsymbol{\beta}_\sigma^0 = \bar{\mathbf{z}}_+(\lambda) + \bar{\mathbf{z}}_-(\lambda),$$

$$(3.23) \qquad \boldsymbol{\sigma}_t^j = \boldsymbol{\alpha}_\sigma^j\boldsymbol{\sigma}_t^{j-1} + \boldsymbol{\beta}_\sigma^j, \qquad j = 1, 2, ..., n.$$

If vector-functions $\bar{\mathbf{z}}_+(\lambda)$, $\bar{\mathbf{z}}_-(\lambda)$ are known, then the values of $\boldsymbol{\sigma}_t^j$ will be found from (3.22), (3.23). Moreover, the values of $\boldsymbol{\sigma}_b^j$ and $\mathbf{u}_{b(t)}^j$ can be obtained from (3.8), $(3.11)_2$. Corresponding relations are of the form

$$(3.24) \qquad \begin{aligned} \mathbf{u}_t^j &= \mathbf{D}_{\sigma t}^j\boldsymbol{\sigma}_t^j + \mathbf{d}_{\sigma t}^j, \qquad j = 1, 2, ..., n, \\ \mathbf{u}_b^j &= \mathbf{D}_{\sigma b}^j\boldsymbol{\sigma}_t^j + \mathbf{d}_{\sigma b}^j, \qquad j = 1, 2, ..., n-1, \end{aligned}$$

where

$$\begin{aligned} \mathbf{D}_{\sigma t}^j &= \mathbf{R}_{tt}^j + \mathbf{R}_{tb}^j(\boldsymbol{\alpha}_\sigma^j)^{-1}, \qquad \mathbf{d}_{\sigma t}^j = \mathbf{u}_{t0}^j + \mathbf{R}_{tb}^j(\Delta\sigma_{j-1} - (\boldsymbol{\alpha}_\sigma^j)^{-1}\boldsymbol{\beta}_\sigma^j), \\ \mathbf{D}_{\sigma b}^j &= \mathbf{R}_{bt}^j + \mathbf{R}_{bb}^j(\boldsymbol{\alpha}_\sigma^j)^{-1}, \qquad \mathbf{d}_{\sigma b}^j = \mathbf{u}_{b0}^j + \mathbf{R}_{bb}^j(\Delta\sigma_{j-1} - (\boldsymbol{\alpha}_\sigma^j)^{-1}\boldsymbol{\beta}_\sigma^j). \end{aligned}$$

Further, we shall need the relation between the Fourier transforms of vector-functions $\mathbf{u}_b^1$, and tractions $\boldsymbol{\sigma}_b^1$ ($= \boldsymbol{\sigma}_t^0$), along the exterior (with respect to the layered part of the domain) boundary $\Gamma_0$:

$$(3.25) \qquad \mathbf{u}_b^1 = \mathbf{M}_\sigma\boldsymbol{\sigma}_b^1 + \mathbf{m}_\sigma = \mathbf{M}_\sigma(\lambda)(\bar{\mathbf{z}}_+(\lambda) + \bar{\mathbf{z}}_-(\lambda)) + \mathbf{m}_\sigma,$$

where matrix-function $\mathbf{M}_\sigma$ and vector-function $\mathbf{m}_\sigma$ are of the form

$$(3.26) \qquad \mathbf{M}_\sigma = \mathbf{R}_{bt}^1\boldsymbol{\alpha}_\sigma^1 + \mathbf{R}_{bb}^1, \qquad \mathbf{m}_\sigma = \mathbf{u}_{b0}^1 + \mathbf{R}_{bt}^1\boldsymbol{\beta}_\sigma^1.$$

Relations (3.25), (3.26) will be necessary to satisfy the contact conditions along the boundary $\Gamma_0$.

LEMMA 1. Matrix-function $\mathbf{M}_\sigma(\lambda)$ has negative components on the main diagonal ($m_{\sigma kk}(\lambda) < 0$, $k = 1, 2$), and is nondegenerate ($\det \mathbf{M}_\sigma(\lambda) > 0$) for arbitrary $\lambda \in \mathbb{R}_+$. Both the matrix-function $\mathbf{M}_\sigma$ and vector-function $\mathbf{m}_\sigma$ belong

to the corresponding class $\mathbf{C}^\infty(\mathbb{R}_+)$, and the following estimates at infinity are valid for any exterior boundary conditions under consideration ($\mathcal{J} = 1 - 5$):

$$\mathbf{M}_\sigma(\lambda) = -\frac{1-\nu_1}{\mu_1|\lambda|}\mathbf{I} + i\frac{1-2\nu_1}{2\mu_1\lambda}\mathbf{E}_2 + \mathcal{O}(P_3(|\lambda|)e^{-2|\lambda|h_1}),$$
$$\mathbf{m}_\sigma(\lambda) = o(|\lambda|^{-3}), \qquad |\lambda| \to \infty;$$

but in the neighbourhood of zero point ($\lambda \to 0$), they depend on the exterior boundary conditions in the following manner:

$\mathcal{J} = 1:$
$$\mathbf{M}_\sigma(\lambda) = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda) \\ \mathcal{O}(\lambda) & \mathcal{O}(1) \end{pmatrix}, \qquad \mathbf{m}_\sigma(\lambda) = \mathcal{O}(1),$$

$\mathcal{J} = 2:$
$$\mathbf{M}_\sigma(\lambda) = c_2\lambda^{-4}\begin{pmatrix} 2 + \mathcal{O}(\lambda^2) & i\lambda a_2 + \mathcal{O}(\lambda^3) \\ -i\lambda a_2 + \mathcal{O}(\lambda^3) & \lambda^2 b_2 + \mathcal{O}(\lambda^4) \end{pmatrix},$$
$$\mathbf{m}_\sigma(\lambda) = -c_2\lambda^{-4}\begin{pmatrix} 2\xi_1 + 2i\lambda\Upsilon_L - i\lambda h_n\xi_2 + \mathcal{O}(\lambda^2) \\ -i\lambda a_2\xi_1 + \lambda^2 a_2\Upsilon_L + \lambda^2(b_2 - y_n a_2)\xi_2 + \mathcal{O}(\lambda^3) \end{pmatrix},$$

$\mathcal{J} = 3:$
$$\mathbf{M}_\sigma(\lambda) = c_3\lambda^{-2}\begin{pmatrix} \mathcal{O}(\lambda^2) & i\lambda a_3 + \mathcal{O}(\lambda^3) \\ -i\lambda b_3 + \mathcal{O}(\lambda^3) & 1 + \mathcal{O}(\lambda^2) \end{pmatrix},$$
$$\mathbf{m}_\sigma(\lambda) = -c_3\lambda^{-2}\begin{pmatrix} i\lambda a_3\xi_2 + \mathcal{O}(\lambda^2) \\ \xi_2 + \mathcal{O}(\lambda) \end{pmatrix},$$

$\mathcal{J} = 4:$
$$\mathbf{M}_\sigma(\lambda) = c_4\lambda^{-4}\begin{pmatrix} 1 + \mathcal{O}(\lambda^2) & i\lambda a_4 + \mathcal{O}(\lambda^3) \\ -i\lambda a_4 + \mathcal{O}(\lambda^3) & \lambda^2 a_4^2 + \mathcal{O}(\lambda^4) \end{pmatrix},$$
$$\mathbf{m}_\sigma(\lambda) = -c_4\lambda^{-4}\begin{pmatrix} \xi_1 + i\lambda\Upsilon_L + \mathcal{O}(\lambda^2) \\ -i\lambda a_4\xi_1 + \lambda^2 a_4\Upsilon_L + \mathcal{O}(\lambda^3) \end{pmatrix},$$

$\mathcal{J} = 5:$
$$\mathbf{M}_\sigma(\lambda) = c_5\lambda^{-2}\begin{pmatrix} |\lambda|(1 + \mathcal{O}(|\lambda|)) & i\lambda a_5(1 + \mathcal{O}(|\lambda|)) \\ -i\lambda a_5(1 + \mathcal{O}(|\lambda|)) & |\lambda|(1 + \mathcal{O}(|\lambda|)) \end{pmatrix},$$
$$\mathbf{m}_\sigma(\lambda) = -c_5\lambda^{-2}\begin{pmatrix} |\lambda|\xi_1 + i\lambda a_5\xi_2 + \mathcal{O}(\lambda^2) \\ -i\lambda a_5\xi_1 + |\lambda|\xi_2 + \mathcal{O}(\lambda^2) \end{pmatrix}.$$

Here constants $a_\mathcal{J} = a_\mathcal{J}^{(1)}$, $b_\mathcal{J} = b_\mathcal{J}^{(1)}$, $c_\mathcal{J} = c_\mathcal{J}^{(1)}$ ($\mathcal{J} = 2,3,4$) are defined from the recurrent relations:

$$c_2^{(j)} = c_2^{(j+1)}\left(1 + c_2^{(j+1)}\left[2(a_2^{(j+1)})^2 + 2h_j a_2^{(j)} + 2a_2^{(j+1)}a_2^{(j)}\right]\frac{\mu_j h_j}{3(1-\nu_j)}\right)^{-1},$$
$$a_2^{(j)} = a_2^{(j+1)} + 2h_j, \qquad b_2^{(j)} = b_2^{(j+1)} + h_j\left(a_2^{(j)} + a_2^{(j+1)}\right),$$
$$a_2^{(n)} = h_n, \qquad b_2^{(n)} = \frac{2}{3}h_n^2, \qquad c_2^{(n)} = -\frac{3(1-\nu_n)}{\mu_n h_n^3},$$

$$a_3^{(j)} = a_3^{(j+1)} - h_j \frac{3 + \nu_j}{1 - \nu_j}, \qquad b_3^{(j)} = b_3^{(j+1)} + h_j,$$

$$c_3^{(j)} = c_3^{(j+1)} h_j^{-1} \left[1 - 2c_3^{(j+1)}\right]^{-1},$$

$$a_3^{(n)} = b_3^{(n)} = -h_n \frac{3 + \nu_n}{1 - \nu_n}, \qquad c_3^{(n)} = -\frac{1 - \nu_n}{2\mu_n h_n},$$

$$c_4^{(j)} = c_4^{(j+1)}(1 - \nu_j)h_j^{-1}\left(1 - \nu_j - \mu_j c_4^{(j+1)}\left[2a_4^{(j+1)}a_4^{(j)} + \frac{2}{3}h_j^2\right]\right)^{-1},$$

$$a_4^{(j)} = a_4^{(j+1)} + h_j, \qquad a_4^{(n)} = h_n, \qquad c_4^{(n)} = -\frac{3(1 - \nu_n)}{2\mu_n h_n^3},$$

$$a_5 = \frac{1 - 2\nu_{n+1}}{2(1 - \nu_{n+1})}, \qquad c_5 = -\frac{(1 - \nu_{n+1})}{\mu_{n+1}}.$$

Besides, vector $\Xi_L$ and constant $\Upsilon_L$ from the right-hand sides of the relations are:

$$\Xi_L = [\xi_1, \xi_2]^\top = \sum_{j=1}^n \int_{y_{j-1}}^{y_j} \bar{\mathbf{X}}^j(0, x)\, dx - \sum_{j=1}^n \Delta\sigma_j(0),$$

$$\Upsilon_L = i[1, 0]\frac{\partial}{\partial\lambda}\left(\sum_{j=1}^n \Delta\boldsymbol{\sigma}j(\lambda) - \sum_{j=1}^n \int_{y_{j-1}}^{y_j} \bar{\mathbf{X}}^j(\lambda, x)\, dx\right)\Big|_{\lambda=0}$$

$$- [0, 1]\left(\sum_{j=1}^n (y_n - y_j)\Delta\boldsymbol{\sigma}_j(0) + \sum_{j=1}^n \int_{y_{j-1}}^{y_j} (x - y_n)\bar{\mathbf{X}}^j(0, x)\, dx\right),$$

for all types of the exterior boundary conditions along the boundary $\Gamma_n$ ($\mathcal{J} = 2, 3, 4$). But in the case $\mathcal{J} = 5$ (when the last layer is a half-plane), in these relations it is necessary to replace number $n$ by $n + 1$, and to assume that $\Delta\sigma_{n+1} = 0$, $y_{n+1} = \infty$.

To prove this Lemma 1 we need recurrent relations for the matrix-function $\mathbf{M}_\sigma = \mathbf{M}_\sigma^1$ and the vector-function $\mathbf{m}_\sigma = \mathbf{m}_\sigma^1$ in the form:

$$\mathbf{M}_\sigma^j = -\mathbf{R}_{bt}^j\left[\boldsymbol{\tau}_j + \mathbf{R}_{tt}^j - \mathbf{M}_\sigma^{j+1}\right]^{-1}\mathbf{R}_{tb}^j + \mathbf{R}_{bb}^j,$$

$$\mathbf{M}_\sigma^n = \mathbf{R}_{bt}^n \boldsymbol{\alpha}_\sigma^n + \mathbf{R}_{bb}^n, \qquad j = n - 1, ..., 2, 1;$$

$$\mathbf{m}_\sigma^j = \mathbf{u}_{b0}^j - (\mathbf{M}_\sigma^j - \mathbf{R}_{bb}^j)(\mathbf{R}_{tb}^j)^{-1}(\mathbf{m}_\sigma^{j+1} + \mathbf{M}_\sigma^{j+1}\Delta\sigma_j - \mathbf{u}_{t0}^j - \Delta\mathbf{u}_j),$$

$$\mathbf{m}_\sigma^n = \mathbf{R}_{bt}^n \boldsymbol{\beta}_\sigma^n + \mathbf{u}_{b0}^n, \qquad j = n - 1, ..., 2, 1.$$

Here $\boldsymbol{\alpha}_\sigma^n, \boldsymbol{\beta}_\sigma^n$ are defined in (3.20). Then the results follow by induction on $j$. We do not present all the details, taking into account the volume of the paper and its technical character.

In the process of proving Lemma 1 we also obtain all properties of the even and odd components $\mathbf{M}_\sigma^\pm$, $\mathbf{m}_\sigma^\pm$ of matrix-function $\mathbf{M}_\sigma$ and vector-function $\mathbf{m}_\sigma$. Thus, matrix $\mathbf{M}_\sigma^+$ has only nonzero elements on the main diagonal and they are real even functions of $\lambda$, but $\mathbf{M}_\sigma^-$ has nonzero elements on the second diagonal and they are imaginary odd functions of $\lambda$.

COROLLARY 1. From (3.7) and Lemma 1 we can rewrite *a priori* estimates for the unknown vector-functions $\mathbf{z}_+$, $\mathbf{z}_-$:

$$\bar{\mathbf{z}}_\pm(\lambda), \quad \lambda\frac{\partial}{\partial\lambda}\bar{\mathbf{z}}_\pm(\lambda) = \mathcal{O}(\lambda^{-\gamma_0}), \qquad \lambda\to\infty;$$

$$\bar{\mathbf{z}}_+(\lambda) = \mathbf{z}_*^+ + \mathcal{O}(\lambda^{\gamma_2^+}), \qquad \lambda\frac{\partial}{\partial\lambda}\bar{\mathbf{z}}_\pm(\lambda) = \mathcal{O}(\lambda^{\gamma_2^\pm}), \qquad \lambda\to 0;$$

$(3.27)_1$
$$\bar{\mathbf{z}}_-(\lambda) = \mathbf{z}_*^-\lambda^{\gamma_2^-} + \mathcal{O}(\lambda^{\gamma_2^-+\varepsilon}), \qquad \lambda\to 0;$$

$$\lambda(\mathbf{M}_\sigma^+(\lambda)\bar{\mathbf{z}}_+(\lambda) + \mathbf{M}_\sigma^-(\lambda)\bar{\mathbf{z}}_-(\lambda) + \mathbf{m}_\sigma^+(\lambda)) = \mathcal{O}(\lambda^{\min\{1,\gamma_1^+\}}), \qquad \lambda\to 0;$$

$$\lambda(\mathbf{M}_\sigma^+(\lambda)\bar{\mathbf{z}}_-(\lambda) + \mathbf{M}_\sigma^-(\lambda)\bar{\mathbf{z}}_+(\lambda) + \mathbf{m}_\sigma^-(\lambda)) = \mathcal{O}(\lambda^{\gamma_1^-}), \qquad \lambda\to 0;$$

$$\lambda(\mathbf{M}_\sigma^+(\lambda)\bar{\mathbf{z}}_\pm(\lambda) + \mathbf{M}_\sigma^-(\lambda)\bar{\mathbf{z}}_\mp(\lambda) + \mathbf{m}_\sigma^\pm(\lambda)) = \mathcal{O}(\lambda^{\gamma_0}), \qquad \lambda\to\infty;$$

here

$$\mathcal{J} = 1: \quad \gamma_1^+ \geq 1, \qquad \gamma_1^- = 2,$$

$$\mathcal{J} = 2: \quad \begin{cases} \gamma_2^+ = 2, \quad \gamma_2^- = 1, \\ \mathbf{z}_*^+ = \Xi_L, \quad [1,0]\mathbf{z}_*^- = i\Upsilon_L - i[0,1]y_n\Xi_L, \end{cases}$$

$(3.27)_2$
$$\mathcal{J} = 3: \quad \begin{cases} \gamma_2^- = \gamma_1^+ = 1, \\ [0,1](\mathbf{z}_*^+ - \Xi_L) = 0, \end{cases}$$

$$\mathcal{J} = 4: \quad \begin{cases} \gamma_2^+ = 2, \quad \gamma_2^- = 1, \\ [1,0](\mathbf{z}_*^+ - \Xi_L) = 0, \quad [1,0]\mathbf{z}_*^- = i\Upsilon_L - i[0,1]y_n\Xi_L, \end{cases}$$

$$\mathcal{J} = 5: \quad \begin{cases} \gamma_2^+ = \gamma_1^+, \quad \gamma_2^- = 1, \quad \gamma_1^- \geq 1, \\ \mathbf{z}_*^+ = \Xi_L. \end{cases}$$

Let us note that the even and odd components of the solution decrease at infinity in a different way (the corresponding orders are $\gamma_1^\pm$, $\gamma_2^\pm$). So by $\gamma_1$, $\gamma_2$ in (2.13) we shall understand the largest of them. Besides, the values of constant vector $\mathbf{z}_*^+$ in the case $\mathcal{J} = 1$, and the first (second) component of $\mathbf{z}_*^+$ in the case $\mathcal{J} = 3$ ($\mathcal{J} = 4$) are unknown as yet and will be obtained below. This is important to note that the corresponding relations (3.27) present the usual equilibrium conditions for the layered part of the domain and consequently, vector $\mathbf{z}_*^+$ (or one of its components) is defined from *a priori* estimations. They follow from (3.10) and the interfacial conditions (3.11):

$$\sigma_t^n(0) - \sigma_b^1(0) + \sum_{j=1}^n \int_{y_{j-1}}^{y_j} \bar{\mathbf{X}}(0, x)dx - \sum_{j=1}^{n-1} \Delta\boldsymbol{\sigma}_j(0) = 0,$$

$$i[1,0]\frac{\partial}{\partial\lambda}\left(\sigma_t^n(\lambda) - \sigma_b^1(\lambda) + \sum_{j=1}^{n}\int_{y_{j-1}}^{y_j}\bar{\mathbf{X}}^j(\lambda,x)dx - \sum_{j=1}^{n-1}\Delta\sigma j(\lambda)\right)|_{\lambda=0}$$

$$+ [0,1]\left(\sum_{j=1}^{n}\int_{y_{j-1}}^{y_j}x\bar{\mathbf{X}}^j(0,x)dx - \sum_{j=1}^{n-1}y_j\Delta\sigma_j(0) + y_n\sigma_t^n(0)\right) = 0.$$

The second equation can also be rewritten in an equivalent form:

$$i[1,0]\frac{\partial}{\partial\lambda}\left(\sigma_t^n(\lambda) - \sigma_b^1(\lambda) + \sum_{j=1}^{n}\int_{y_{j-1}}^{y_j}\bar{\mathbf{X}}^j(\lambda,x)dx - \sum_{j=1}^{n-1}\Delta\sigma j(\lambda)\right)|_{\lambda=0}$$

$$+ [0,1]\left(\sum_{j=1}^{n}\int_{y_{j-1}}^{y_j}(x-y_n)\bar{\mathbf{X}}^j(0,x)dx + \sum_{j=1}^{n-1}(y_n-y_j)\Delta\sigma_j(0) + y_n\sigma_b^1(0)\right) = 0.$$

Constant $u_*$ from (2.13) can be calculated from the equation:

$$(3.28) \qquad \mathbf{u}_* = 2\int_0^\infty (\mathbf{M}_\sigma^+(\lambda)\bar{\mathbf{z}}_+(\lambda) + \mathbf{M}_\sigma^-(\lambda)\bar{\mathbf{z}}_-(\lambda) + \mathbf{m}_\sigma^+(\lambda))d\lambda.$$

Moreover, if any component of the displacement is prescribed along one of the exterior wedge surfaces, then we have (see (2.13)) the additional relation for the corresponding component of vector $\mathbf{u}_*$:

$$(3.29) \qquad \begin{aligned}
\mathbf{u}_*(1,\mathcal{J}^-,\mathcal{J}) &= 0, \quad \mathbf{u}_*(\mathcal{J}^+,1,\mathcal{J}) = 0, \quad \mathcal{J}^\pm = 1-4, \quad \mathcal{J} = 1-5, \\
[1,0]\mathbf{S}(\theta_0^+)\mathbf{u}_*(3,\mathcal{J}^-,\mathcal{J}) &= 0, \quad \mathcal{J}^- = 2-4, \quad \mathcal{J} = 1-5, \\
[0,1]\mathbf{S}(\theta_0^+)\mathbf{u}_*(4,\mathcal{J}^-,\mathcal{J}) &= 0, \quad \mathcal{J}^- = 2-4, \quad \mathcal{J} = 1-5, \\
[1,0]\mathbf{S}(\theta_{m_-}^-)\mathbf{u}_*(\mathcal{J}^+,3,\mathcal{J}) &= 0, \quad \mathcal{J}^+ = 2-4, \quad \mathcal{J} = 1-5, \\
[0,1]\mathbf{S}(\theta_{m_-}^-)\mathbf{u}_*(\mathcal{J}^+,4,\mathcal{J}) &= 0, \quad \mathcal{J}^+ = 2-4, \quad \mathcal{J} = 1-5.
\end{aligned}$$

Here matrix-function $\mathbf{S}(\phi)$ is defined in (4.9). Besides, for the next problems we can conclude that for any $\mathcal{J} = 1-5$

$$\mathbf{u}_*(3,4,\mathcal{J}) = 0, \quad \mathbf{u}_*(4,3,\mathcal{J}) = 0, \quad \theta_0^+ \neq \theta_{m_-}^- + \pi/2,$$

$$\mathbf{u}_*(3,3,\mathcal{J}) = 0, \quad \mathbf{u}_*(4,4,\mathcal{J}) = 0, \quad \theta_0^+ \neq \theta_{m_-}^-.$$

Finally, if equalities arise instead of the inequalities in the last four problems, only one of the respective conditions (3.29) which should be satisfied is linearly independent.

## 4. Sweep method in the wedge-shaped domain

Rewriting Eqs. (2.1), (2.2) in the polar coordinates and applying the Mellin transform in the form:

$$\bar{\mathbf{u}}(s,\theta) = \int_0^\infty \mathbf{u}(r,\theta) r^{s-1} dr,$$

$$\tilde{\boldsymbol{\sigma}}, \tilde{\Theta}(s,\theta) = \int_0^\infty \boldsymbol{\sigma}, \Theta(r,\theta) r^s dr,$$

$$\tilde{\mathbf{X}}(s,\theta) = \int_0^\infty \mathbf{X}(r,\theta) r^{s+1} dr,$$

we obtain [37] the following relations in the respective regions:

(4.1)
$$-s\tilde{\sigma}_{rr} + \frac{\partial}{\partial\theta}\tilde{\sigma}_{r\theta} - \tilde{\sigma}_{\theta\theta} + \tilde{X}_r = 0,$$
$$-(s-1)\tilde{\sigma}_{r\theta} + \frac{\partial}{\partial\theta}\tilde{\sigma}_{\theta\theta} + \tilde{X}_\theta = 0;$$

(4.2)
$$-s\tilde{u}_r = \frac{1}{2\mu}[\tilde{\sigma}_{rr} - \nu\tilde{\sigma}] + \beta\tilde{\Theta},$$
$$\tilde{u}_r + \frac{\partial}{\partial\theta}\bar{u}_\theta = \frac{1}{2\mu}[\tilde{\sigma}_{\theta\theta} - \nu\tilde{\sigma}] + \beta\tilde{\Theta},$$
$$\frac{\partial}{\partial\theta}\bar{u}_r - (s+1)\tilde{u}_\theta = \frac{1}{\mu}\tilde{\sigma}_{r\theta}, \qquad \sigma = \sigma_{rr} + \sigma_{\theta\theta},$$

where constants $\mu$, $\beta = \gamma/(2\mu + \lambda)$, and $\nu$ (defined in (2.2)) are different in regions $\Omega_j^+$, $\Omega_j^-$. For function $\tilde{\sigma}_{\theta\theta}$ the equation can be found:

(4.3)
$$\left(\frac{\partial^2}{\partial\theta^2} + (s+1)^2\right)\left(\frac{\partial^2}{\partial\theta^2} + (s-1)^2\right)\tilde{\sigma}_{\theta\theta}$$
$$= \left(\frac{\partial^2}{\partial\theta^2} + (s+1)^2\right)h_1 + s(s-1)h_2,$$

where

$$h_1(s,\theta) = -\frac{\partial}{\partial\theta}\tilde{X}_\theta - (s-1)\tilde{X}_r - s(s-1)\frac{2\mu\beta}{1-\nu}\tilde{\Theta},$$
$$h_2(s,\theta) = \frac{1}{1-\nu}\left[(s+1)\tilde{X}_r - \frac{\partial}{\partial\theta}\tilde{X}_\theta\right].$$

The corresponding solution is of the form:

$$(4.4) \quad \tilde{\sigma}_{\theta\theta}^{(j\pm)}(\theta, s) = C_1^{j\pm}(s)\cos[\theta(s+1)] + C_2^{j\pm}(s)\cos[\theta(s-1)]$$

$$+ C_3^{j\pm}(s)\sin[\theta(s+1)] + \frac{s-1}{4(s+1)}\int\limits_{\theta_{j-1}^{\pm}}^{\theta} h_2^{(j\pm)}(s,\phi)\sin[(\phi-\theta)(s+1)]d\phi$$

$$+ C_4^{j\pm}(s)\sin[\theta(s-1)] - \frac{1}{4}\int\limits_{\theta_{j-1}^{\pm}}^{\theta}\left[h_2^{(j\pm)} + \frac{4}{(s-1)}h_1^{(j\pm)}\right]\sin[(\phi-\theta)(s-1)]d\phi.$$

Using a line of reasoning similar to that applied in (3.8), we can obtain the relations between the Mellin transforms of the vectors of displacements and tractions along the corresponding boundaries of the wedges.

$$(4.5) \quad \begin{aligned} \mathbf{v}_t^j &= \mathbf{P}_{tt}^j \mathbf{p}_t^j + \mathbf{P}_{tb}^j \mathbf{p}_b^j + \mathbf{v}_{t0}^j, \\ \mathbf{v}_b^j &= \mathbf{P}_{bt}^j \mathbf{p}_t^j + \mathbf{P}_{bb}^j \mathbf{p}_b^j + \mathbf{v}_{b0}^j, \quad j = 1, 2, ..., m_+; \end{aligned}$$

$$(4.6) \quad \begin{aligned} \mathbf{w}_t^{(k)} &= \mathbf{Q}_{tt}^k \mathbf{q}_t^k + \mathbf{Q}_{tb}^k \mathbf{q}_b^k + \mathbf{w}_{t0}^k, \\ \mathbf{w}_b^k &= \mathbf{Q}_{bt}^k \mathbf{q}_t^k + \mathbf{Q}_{bb}^k \mathbf{q}_b^k + \mathbf{w}_{b0}^k, \quad k = 1, 2, ..., m_-. \end{aligned}$$

From the properties of the Mellin transform and from *a priori* estimates (2.13) of vector-functions of class $\mathbf{LW}(\Omega)$ it follows that vector-functions $\mathbf{p}_{b(t)}^j(s)$, $\mathbf{q}_{b(t)}^k(s)$ are analytical in the strip $-\gamma_0 < \Re s < \gamma_2$, but $\mathbf{w}_{b(t)}^k(s)$, $\mathbf{v}_{b(t)}^j(s)$ are analytical in domain $0 < \Re s < \gamma_1$, in general. In point $s = 0$ they can have a common simple pole. Besides, the relations hold:

$$(4.7) \quad \mathbf{v}_{t(b)}^j(s) - \frac{1}{s}\mathbf{u}_*, \quad \mathbf{w}_{t(b)}^k(s) - \frac{1}{s}\mathbf{u}_* = \mathcal{O}(1), \quad s \to 0,$$

and the vector-functions from the left-hand side of (4.7) are analytical in the whole strip $-\gamma_0 < \Re s < \gamma_1$. All of the discussed vector-functions decrease to zero along imaginary axis inside the corresponding domains.

Here the coefficients in (4.5) are found from the formulae:

$$\mathbf{P}_{tt}^j(s) = \frac{1}{2s\mu_j^+}[\mathbf{E}_2 - \mathbf{P}_1^j\mathbf{E}_1], \qquad \mathbf{P}_{tb}^j(s) = \frac{1}{2s\mu_j^+}\mathbf{P}_2^j\mathbf{E}_1,$$

$$\mathbf{P}_{bb}^j(s) = \frac{1}{2s\mu_j^+}[\mathbf{E}_2 + \mathbf{E}_1\mathbf{P}_1^j], \qquad \mathbf{P}_{bt}^j(s) = -\frac{1}{2s\mu_j^+}\mathbf{E}_1\mathbf{P}_2^j,$$

$$\mathbf{P}_1^j = p_j(s)\begin{pmatrix} s\sin 2\phi_j^+ + \sin 2s\phi_j^+ & 2\sin^2 s\phi_j^+ - 2s\sin^2 \phi_j^+ \\ 2\sin^2 s\phi_j^+ + 2s\sin^2 \phi_j^+ & s\sin 2\phi_j^+ - \sin 2s\phi_j^+ \end{pmatrix},$$

$$(4.8) \qquad p_j(s) = \frac{1 - \nu_j^+}{\sin^2 s\phi_j^+ - s^2 \sin^2 \phi_j^+},$$

$$2s\mu_j^+ \mathbf{v}_{t0}^j = \mathbf{P}_1^j \mathbf{E}_1 H_t - \mathbf{P}_2^j \mathbf{E}_1 H_b$$

$$-\frac{1}{s+1} \left( \begin{array}{c} \tilde{X}_{\theta t} - (1-\nu_j^+) \int\limits_{\theta_{j-1}^+}^{\theta_j^+} h_2 \cos[(\phi - \theta_j^+)(s+1)] d\phi \\ -(1-\nu_j^+) \int\limits_{\theta_{j-1}^+}^{\theta_j^+} h_2 \sin[(\phi - \theta_j^+)(s+1)] d\phi \end{array} \right),$$

$$2s\mu_j^+ \mathbf{v}_{b0}^j = \mathbf{E}_1 \mathbf{P}_2^j H_t - \mathbf{E}_1 \mathbf{P}_1^j H_b - \frac{1}{s+1} \left( \begin{array}{c} \tilde{X}_{\theta b} \\ 0 \end{array} \right), \qquad H_b = \frac{1}{s-1} \left( \begin{array}{c} 0 \\ \tilde{X}_{\theta b} \end{array} \right),$$

$$\mathbf{P}_2^j = p_j(s) \left( \begin{array}{cc} 2s\sin\phi_j^+ \cos s\phi_j^+ + 2\sin s\phi_j^+ \cos\phi_j^+ & -2(s-1)\sin\phi_j^+ \sin s\phi_j^+ \\ 2(s+1)\sin\phi_j^+ \sin s\phi_j^+ & 2s\sin\phi_j^+ \cos s\phi_j^+ - 2\sin s\phi_j^+ \cos\phi_j^+ \end{array} \right),$$

where $\phi_j^+ = \theta_j^+ - \theta_{j-1}^+$, $\tilde{X}_{\theta b} = \tilde{X}_\theta(s, \theta_{j-1}^+)$, $\tilde{X}_{\theta t} = \tilde{X}_\theta(s, \theta_j^+)$, the functions $h_j(s, \theta)$ and the matrices $\mathbf{E}_1$, $\mathbf{E}_2$ are defined in (4.3) and (3.8), respectively, but the remaining vector-function $H_t$ is:

$$H_t = \frac{1}{4} \left( \begin{array}{c} h_t^{(1)} \\ h_t^{(2)} \end{array} \right),$$

$$h_t^{(1)} = \frac{s-1}{s+1} \int\limits_{\theta_{j-1}^+}^{\theta_j^+} h_2 \sin[(\phi - \theta_j^+)(s+1)] d\phi$$

$$- \int\limits_{\theta_{j-1}^+}^{\theta_j^+} \left[ h_2 + \frac{4h_1}{s-1} \right] \sin[(\phi - \theta_j^+)(s-1)] d\phi,$$

$$h_t^{(2)} = \frac{4}{s-1} \tilde{X}_{\theta t} - \int\limits_{\theta_{j-1}^+}^{\theta_j^+} h_2 \cos[(\phi - \theta_j^+)(s+1)] d\phi$$

$$+ \int\limits_{\theta_{j-1}^+}^{\theta_j^+} \left[ h_2 + \frac{4h_1}{s-1} \right] \cos[(\phi - \theta_j^+)(s-1)] d\phi.$$

Note that matrix-functions $\mathbf{P}_{t(b)t(b)}^j(s)$ and vector-functions $\mathbf{v}_{b(t)0}^j(s)$ can be esti-

mated to be of $\mathcal{O}(s^{-2})$ when $s \to 0$. But the unknown vectors $\mathbf{v}^j_{t(b)}(s)$ can have in this point a simple pole only. Hence, by investigating the main terms of asymptotics ($s \to 0$) of the right-hand side of the relations (4.5) it can be obtained that the bounded vector-functions $\mathbf{p}^j_{t(b)}(s)$ should satisfy the following additional relations:

(4.9)
$$\mathbf{p}^j_t(0) = \mathbf{S}(\phi^+_j)\mathbf{p}^j_b(0) - \int_{\theta^+_{j-1}}^{\theta^+_j} \mathbf{S}(\theta^+_j - \phi)\tilde{\mathbf{X}}^j(0, \phi)d\phi,$$

$$\mathbf{S}(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix},$$

where $\mathbf{X}^j(r, \theta) = [X^j_\theta(r, \theta), X^j_r(r, \theta)]^\top$. This relation is the usual equilibrium condition of the $j_+$-th wedge. Moreover, passing to the limit $s \to 1$ in (4.5) we obtain

(4.10)
$$[1, 0]\left\{\mathbf{p}^j_t(1) - \mathbf{p}^j_b(1) - \int_{\theta^+_{j-1}}^{\theta^+_j} \tilde{\mathbf{X}}^j(1, \phi)d\phi\right\} = 0,$$

what is the torque balance condition for the $j_+$-th wedge.

What concerns the formulae for coefficients $\mathbf{Q}^j_{t(b)t(b)}(s)$, $\mathbf{w}^j_{t(b)0}(s)$ from (4.6), they are similar to those in (4.8) if all upper indices "+" are replaced by "−" (for example $\mu^+_j$, $\theta^+_j$, $\phi^+_j$ by $\mu^-_j$, $\theta^-_j$, $\phi^-_j$).

Further on we rewrite the internal boundary conditions as above:

(4.11)
$$\begin{aligned} \mathbf{v}^{j+1}_b - \mathbf{v}^j_t - \boldsymbol{\tau}^+_j\mathbf{p}^j_t &= \Delta\mathbf{v}_j, \\ \mathbf{p}^{j+1}_b - \mathbf{p}^j_t &= \Delta\mathbf{p}_j, \qquad j = 1, ..., m_+ - 1, \\ \mathbf{w}^{k+1}_b - \mathbf{w}^k_t - \boldsymbol{\tau}^-_k\mathbf{q}^k_t &= \Delta\mathbf{w}_k, \\ \mathbf{q}^{k+1}_b - \mathbf{q}^k_t &= \Delta\mathbf{q}_k, \qquad k = 1, ..., m_- - 1, \end{aligned}$$

where

(4.12)
$$\begin{aligned} \Delta\mathbf{v}_j(s) &= \mathcal{M}[\delta\mathbf{v}_j](s), & \Delta\mathbf{p}_j(s) &= \mathcal{M}[\delta\mathbf{p}_j](s+1), \\ \Delta\mathbf{w}_k(s) &= \mathcal{M}[\delta\mathbf{w}_k](s), & \Delta\mathbf{q}_k(s) &= \mathcal{M}[\delta\mathbf{q}_k](s+1), \end{aligned}$$

are the Mellin transforms of the respective vector-functions (see (2.5), (2.6)).

So the net result for wedge-shaped domain $\Omega^-$ is of the similar form as (3.22), (3.23):

(4.13)
$$\begin{aligned} \mathbf{q}^0_t &= \mathbf{q}^1_b = \tilde{\mathbf{z}}_-(s+1) - \tilde{\mathbf{z}}_+(s+1) + \Delta\mathbf{q}_*(s), \\ \mathbf{q}^k_t &= \boldsymbol{\alpha}^k_q\mathbf{q}^{k-1}_t + \boldsymbol{\beta}^k_q, \qquad k = 1, ..., m_-, \\ \mathbf{w}^k_t &= \mathbf{D}^k_{qt}\mathbf{q}^k_t + \mathbf{d}^k_{qt}, \qquad \mathbf{w}^k_b = \mathbf{D}^k_{qb}\mathbf{q}^k_t + \mathbf{d}^k_{qb}. \end{aligned}$$

Let us now write the relations necessary to calculate the coefficients of (4.13):

$$\alpha_q^{m-} = (C_q^{m-})^{-1} A_q^{m-}, \qquad \beta_q^{m-} = (C_q^{m-})^{-1} F_{qt}^{m-},$$

$$\alpha_q^k = (C_q^k - B_q^k \alpha_q^{k+1})^{-1} A_q^k,$$

$$\beta_q^k = (C_q^k - B_q^k \alpha_q^{k+1})^{-1}(F_{qt}^k + B_q^k \beta_q^{k+1}),$$

$$D_{qt}^k = Q_{tt}^k + Q_{tb}^k(\alpha_q^k)^{-1}, \qquad d_{qt}^k = w_{t0}^k + Q_{tb}^k(\Delta q_{k-1} - (\alpha_q^k)^{-1}\beta_q^k),$$

$$(4.14) \quad D_{qb}^k = Q_{bt}^k + Q_{bb}^k(\alpha_q^k)^{-1}, \qquad d_{qb}^k = w_{b0}^k + Q_{bb}^k(\Delta q_{k-1} - \alpha_q^k)^{-1}\beta_q^k),$$

$$A_q^k = -Q_{tb}^k, \qquad C_q^k = \tau_k^- + Q_{tt}^k - Q_{bb}^{k+1}, \qquad B_q^k = Q_{bt}^{k+1},$$

$$F_{qb}^k = w_{b0}^{k+1} - w_{t0}^k - \Delta w_k + (Q_{tt}^k + \tau_k^-)\Delta q_k - Q_{bt}^{k+1}\Delta q_{k+1},$$

$$F_{qt}^k = w_{b0}^{k+1} - w_{t0}^k - \Delta w_k + Q_{bb}^{k+1}\Delta q_k - Q_{tb}^k\Delta q_{k-1},$$

$$\Delta q_*(s) = \mathcal{M}[\delta\sigma_-](s+1), \qquad k = m_- - 1, ..., 1.$$

The constants at the first step of the sweep method are defined by one of boundary conditions (2.10)

$$\mathcal{J} = 1: \qquad C_q^{m-} = Q_{tt}^{m-}, \qquad A_q^{m-} = -Q_{tb}^{m-},$$

$$F_{qt}^{m-} = -\Delta w_{m-} - w_{t0}^{m-} - Q_{tb}^{m-}\Delta q_{m_- - 1},$$

$$\mathcal{J} = 2: \qquad C_q^{m-} = I, \qquad A_q^{m-} = 0, \qquad F_{qt}^{m-} = -\Delta q_{m-},$$

$$\mathcal{J} = 3, 4: \qquad C_q^{m-} = \begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} Q_{tt}^{m-} + \begin{pmatrix} \delta_{\mathcal{J}4} & 0 \\ 0 & \delta_{\mathcal{J}3} \end{pmatrix},$$

$$A_q^{m-} = -\begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} Q_{tb}^{m-}, \qquad \Delta h_{\mathcal{J}} = \begin{pmatrix} \widetilde{\delta w}_{\mathcal{J}}(s) \\ \widetilde{\delta q}_{\mathcal{J}}(s+1) \end{pmatrix},$$

$$F_{qt}^{m-} = -\begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix}\left(w_{t0}^{m-} + Q_{tb}^{m-}\Delta q_{m_- - 1}\right) - \Delta h_{\mathcal{J}}.$$

Here $\delta_{\mathcal{J}j}$ is the Kronecker symbol. The remaining initial conditions along the boundary $\Gamma_0^-$ follow from (2.5) and from assumption (3.16):

$$(4.15) \qquad C_q^0 = I, \qquad B_q^0 = 0, \qquad \Delta q_0 = 0, \qquad F_{qt}^0 = \tilde{z}_-(s+1) - \tilde{z}_+(s+1) + \Delta q_*.$$

In an analogous way we obtain the relations

$$p_b^{m_+ + 1} = p_t^{m+} = \tilde{z}_-(s+1) + \tilde{z}_+(s+1) - \Delta p_*(s),$$

$$(4.16) \qquad p_b^j = \alpha_p^j p_b^{j+1} + \beta_p^j, \qquad j = m_+, ..., 1,$$

$$v_t^j = D_{pt}^j p_b^j + d_{pt}^j, \qquad v_b^j = D_{pb}^j p_p^j + d_{pb}^j,$$

for wedge region $\Omega^+$, where the necessary coefficients are of the form:

$$\boldsymbol{\alpha}_p^1 = (\mathbf{C}_p^0)^{-1}\mathbf{B}_p^0, \qquad \boldsymbol{\beta}_p^1 = (\mathbf{C}_p^0)^{-1}\mathbf{F}_{pb}^0,$$

$$\boldsymbol{\alpha}_p^{j+1} = (\mathbf{C}_p^j - \mathbf{A}_p^j \boldsymbol{\alpha}_p^j)^{-1}\mathbf{B}_p^j,$$

$$\boldsymbol{\beta}_p^{j+1} = (\mathbf{C}_p^j - \mathbf{A}_p^j \boldsymbol{\alpha}_p^j)^{-1}(\mathbf{F}_{pb}^j + \mathbf{A}_p^j \boldsymbol{\beta}_p^j),$$

$$\mathbf{D}_{pt}^j = \mathbf{P}_{tb}^j + \mathbf{P}_{tt}^j \boldsymbol{\alpha}_p^j)^{-1}, \qquad \mathbf{d}_{pt}^j = \mathbf{v}_{t0}^j - \mathbf{P}_{tt}^j(\Delta \mathbf{p}_j + \boldsymbol{\alpha}_p^j)^{-1}\boldsymbol{\beta}_p^j),$$

(4.17) $\quad \mathbf{D}_{pb}^j = \mathbf{P}_{bb}^j + \mathbf{P}_{bt}^j \boldsymbol{\alpha}_p^j)^{-1}, \qquad \mathbf{d}_{pb}^j = \mathbf{v}_{b0}^j - \mathbf{P}_{bt}^j(\Delta \mathbf{p}_j + (\boldsymbol{\alpha}_p^j)^{-1}\boldsymbol{\beta}_p^j),$

$$\mathbf{A}_p^j = -\mathbf{P}_{tb}^j, \qquad \mathbf{C}_p^j = \boldsymbol{\tau}_j^+ + \mathbf{P}_{tt}^j - \mathbf{P}_{bb}^{j+1}, \qquad \mathbf{B}_p^j = \mathbf{P}_{bt}^{j+1},$$

$$\mathbf{F}_{pb}^j = \mathbf{v}_{b0}^{j+1} - \mathbf{v}_{t0}^j - \Delta \mathbf{v}_j + (\mathbf{P}_{tt}^j + \boldsymbol{\tau}_j^+)\Delta \mathbf{p}_j - \mathbf{P}_{bt}^{j+1}\Delta \mathbf{p}_{j+1},$$

$$\mathbf{F}_{pt}^j = \mathbf{v}_{b0}^{j+1} - \mathbf{v}_{t0}^j - \Delta \mathbf{v}_j + \mathbf{P}_{bb}^{j+1}\Delta \mathbf{p}_j - \mathbf{P}_{tb}^j \Delta \mathbf{p}_{j-1},$$

$$\Delta \mathbf{p}_*(s) = \mathcal{M}[\delta \boldsymbol{\sigma}_+](s+1), \qquad j = 1, 2, \ldots, l-1.$$

Boundary conditions for the corresponding system of difference equations follow from (2.9), (2.7)$_b$ and assumption (3.16):

$\mathcal{J} = 1:$

$$\mathbf{B}_p^0 = -\mathbf{P}_{bt}^1, \qquad \mathbf{C}_p^0 = \mathbf{P}_{bb}^1,$$

$$\mathbf{F}_{bt}^0 = \Delta \mathbf{v}_0 - \mathbf{v}_{b0}^1 + \mathbf{P}_{bt}^1 \Delta \mathbf{p}_1,$$

$\mathcal{J} = 2:$

$$\mathbf{B}_p^0 = 0, \qquad \mathbf{C}_p^0 = \mathbf{I}, \qquad \mathbf{F}_{pb}^0 = \Delta \mathbf{p}_0,$$

$\mathcal{J} = 3, 4:$

$$\mathbf{B}_p^0 = -\begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} \mathbf{P}_{bt}^1,$$

$$\mathbf{C}_p^0 = \begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} \mathbf{P}_{bb}^1 + \begin{pmatrix} \delta_{\mathcal{J}4} & 0 \\ 0 & \delta_{\mathcal{J}3} \end{pmatrix},$$

$$\mathbf{F}_{pb}^0 = -\begin{pmatrix} \delta_{\mathcal{J}3} & 0 \\ 0 & \delta_{\mathcal{J}4} \end{pmatrix} \left(\mathbf{v}_{b0}^0 - \mathbf{P}_{bt}^1 \Delta \mathbf{p}_1\right) + \Delta \mathbf{h}_{\mathcal{J}},$$

$$\Delta \mathbf{h}_{\mathcal{J}} = \begin{pmatrix} \widetilde{\delta v}_{\mathcal{J}}(s) \\ \widetilde{\delta p}_{\mathcal{J}}(s+1) \end{pmatrix},$$

$$\mathbf{C}_p^{m+} = \mathbf{I}, \qquad \mathbf{A}_p^{m+} = 0, \qquad \Delta \mathbf{p}_{m+} = 0,$$

$$\mathbf{F}_{pt}^{m+} = \tilde{\mathbf{z}}_+(s+1) + \tilde{\mathbf{z}}_-(s+1) - \Delta \mathbf{p}_*.$$

Besides we can find relations similar to (3.25) connecting the Mellin transforms of displacements and tractions along boundaries $\Gamma_0^-$, $\Gamma_{m+}^+$:

(4.18) $\quad \mathbf{v}_t^{m+} = \mathbf{M}_p \mathbf{p}_t^{m+} + \mathbf{m}_p = \mathbf{M}_p(\tilde{\mathbf{z}}_+(s+1) + \tilde{\mathbf{z}}_-(s+1) - \Delta \mathbf{p}_*(s)) + \mathbf{m}_p,$

(4.19) $\quad \mathbf{w}_b^1 = \mathbf{M}_q \mathbf{q}_b^1 + \mathbf{m}_q = \mathbf{M}_q(\tilde{\mathbf{z}}_-(s+1) - \tilde{\mathbf{z}}_+(s+1) + \Delta \mathbf{q}_*(s)) + \mathbf{m}_q,$

where

$$(4.20) \quad \begin{aligned} \mathbf{M}_p(s) &= \mathbf{P}_{tb}^{m+} \boldsymbol{\alpha}_p^{m+} + \mathbf{P}_{tt}^{m+}, & \mathbf{m}_p(s) &= \mathbf{v}_{t0}^{m+} + \mathbf{P}_{tb}^{m+} \boldsymbol{\beta}_p^{m+}, \\ \mathbf{M}_q(s) &= \mathbf{Q}_{bt}^1 \boldsymbol{\alpha}_q^1 + \mathbf{Q}_{bb}^1, & \mathbf{m}_q(s) &= \mathbf{w}_{b0}^1 + \mathbf{Q}_{bt}^1 \boldsymbol{\beta}_q^1. \end{aligned}$$

LEMMA 2. Matrix-functions $\mathbf{M}_p(s)$, $\mathbf{M}_q(s)$ and vector-functions $\mathbf{m}_p(s)$, $\mathbf{m}_q(s)$ are analytical in the strip $|\Re s| < 1$ except maybe one point $s = 0$. For matrix-functions $\mathbf{M}_p(s)$, $\mathbf{M}_q(s)$ and for their components $m_{pkk}(s)$, $m_{qkk}(s)$ $(k = 1, 2)$, the following relations are true:

$$\mathbf{M}_{p(q)}(-s) = \mathbf{M}_{p(q)}^{\top}(s), \qquad \overline{\mathbf{M}_{p(q)}(it)} = \mathbf{M}_{p(q)}^{\top}(it), \qquad t \in \mathbb{R},$$

$$\det \mathbf{M}_{p(q)}(it) > 0, \qquad m_{pkk}(it) > 0, \qquad m_{qkk}(it) < 0, \qquad t \in \mathbb{R}_+.$$

Besides, for any exterior boundary conditions ($\mathcal{J}^{\pm} = 1 - 4$) asymptotic expansions near the infinity point hold true ($|\Im s| \to \infty$):

$$\mathbf{M}_p(s) = -\frac{1}{2s\mu_{m_+}^+} \left[ (1 - 2\nu_{m_+}^+)\mathbf{E}_2 - 2(1 - \nu_{m_+}^+)\mathrm{tg}(\phi_{m_+}^+ s)\mathbf{I} \right] + O\left( P_3(s)e^{-2|\Im s|\phi_{m_+}^+} \right),$$

$$\mathbf{M}_q(s) = -\frac{1}{2s\mu_1^-} \left[ (1 - 2\nu_1^-)\mathbf{E}_2 + 2(1 - \nu_1^-)\mathrm{tg}(\phi_1^- s)\mathbf{I} \right] + O\left( P_4(s)e^{-2|\Im s|\phi_1^-} \right),$$

but in the neighbourhood of the zero point ($s \to 0$), they depend on the exterior boundary conditions along the wedge surfaces:

$\mathcal{J}^{\pm} = 1$ :

$$\mathbf{M}_p(s), \ \mathbf{M}_q(s) = \mathcal{O}(1), \qquad \mathbf{m}_p(s), \ \mathbf{m}_q(s) = \mathcal{O}(1),$$

$\mathcal{J}^{\pm} = 2$ :

$$\mathbf{M}_p(s) \sim s^{-2}\mathbf{T}_2^+, \qquad \mathbf{M}_q(s) \sim s^{-2}\mathbf{T}_2^-,$$

$$\mathbf{m}_p(s) \sim -s^{-2}\mathbf{T}_2^+ \left( \Xi_+ + \mathbf{S}(-\theta_0^+)\Delta\mathbf{p}_0(0) \right),$$

$$\mathbf{m}_q(s) \sim -s^{-2}\mathbf{T}_2^- \left( \Xi_- - \mathbf{S}(-\theta_{m_-}^-)\Delta\mathbf{q}_{m_-}(0) \right),$$

$\mathcal{J}^{\pm} = 3$ :

$$\mathbf{M}_p(s) \sim c_3^p s^{-2}\mathbf{T}_3(\theta_0^+), \qquad \mathbf{M}_q(s) \sim c_3^q s^{-2}\mathbf{T}_3(\theta_{m_-}^-),$$

$$\mathbf{m}_p(s) \sim -2c_3^p s^{-2} \left( \xi_1^+ \sin\theta_0^+ + \xi_2^+ \cos\theta_0^+ + \Delta q_0^{(2)}(0) \right) \begin{pmatrix} \sin\theta_0^+ \\ \cos\theta_0^+ \end{pmatrix},$$

$$\mathbf{m}_q(s) \sim -2c_3^q s^{-2} \left( \xi_1^- \sin\theta_{m_-}^- + \xi_2^- \cos\theta_{m_-}^- - \Delta q_{m_-}^{(2)}(0) \right) \begin{pmatrix} \sin\theta_{m_-}^- \\ \cos\theta_{m_-}^- \end{pmatrix},$$

$\mathcal{J}^{\pm} = 4$ :

$$\mathbf{M}_p(s) \sim c_4^p s^{-2}\mathbf{T}_4(\theta_0^+), \qquad \mathbf{M}_q(s) \sim c_4^q s^{-2}\mathbf{T}_4(\theta_{m_-}^-),$$

$$\mathbf{m}_p(s) \sim -2c_4^p s^{-2} \left( \xi_1^+ \cos\theta_0^+ - \xi_2^+ \sin\theta_0^+ + \Delta q_0^{(1)}(0) \right) \begin{pmatrix} \cos\theta_0^+ \\ -\sin\theta_0^+ \end{pmatrix},$$

$$\mathbf{m}_q(s) \sim -2c_4^q s^{-2} \left( \xi_1^- \cos\theta_{m_-}^- - \xi_2^- \sin\theta_{m_-}^- - \Delta q_{m_-}^{(1)}(0) \right) \begin{pmatrix} \cos\theta_{m_-}^- \\ -\sin\theta_{m_-}^- \end{pmatrix},$$

where the next terms in these relations are estimated as being of $\mathcal{O}(s^{-1})$ as $s \to 0$. Besides, near point $s = 1$ the following equations hold true:

$\mathcal{J}^{\pm} = 1, 3:$

$$\mathbf{M}_p(s), \ \mathbf{M}_q(s) = \mathcal{O}(1), \qquad \mathbf{m}_p(s), \ \mathbf{m}_q(s) = \mathcal{O}(1), \qquad s \to 1,$$

$\mathcal{J}^{\pm} = 2:$

$$\mathbf{M}_{p(q)}(s) \sim \frac{1}{s-1} \begin{pmatrix} d_{12}^{p(q)} & 0 \\ d_{22}^{p(q)} & 0 \end{pmatrix}, \qquad \mathbf{m}_{p(q)}(s) \sim -\frac{\Upsilon_{p(q)}}{s-1} \begin{pmatrix} d_{12}^{p(q)} \\ d_{22}^{p(q)} \end{pmatrix},$$

$\mathcal{J}^{\pm} = 4:$

$$\mathbf{M}_{p(q)}(s) \sim \frac{1}{s-1} \begin{pmatrix} d_4^{p(q)} & 0 \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{m}_{p(q)}(s) \sim -\frac{\Upsilon_{p(q)}}{s-1} \begin{pmatrix} d_4^{p(q)} \\ 0 \end{pmatrix}.$$

Here $P_3(s)$, $P_4(s)$ are some polynomials of the second degree, matrix-functions $\mathbf{T}_3(\phi)$, $\mathbf{T}_4(\phi)$ are of the form:

$$\mathbf{T}_3(\phi) = \begin{pmatrix} 2\sin^2\phi & \sin 2\phi \\ \sin 2\phi & 2\cos^2\phi \end{pmatrix}, \qquad \mathbf{T}_4(\phi) = \begin{pmatrix} 2\cos^2\phi & -\sin 2\phi \\ -\sin 2\phi & 2\sin^2\phi \end{pmatrix},$$

but $\Delta p_0^{(j)}(0)$, $\Delta q_{m_-}^{(j)}(0)$ $(j = 1, 2)$ are the components of vectors $\Delta \mathbf{p}_0(0)$, $\Delta \mathbf{q}_{m_-}(0)$ defined in (4.12). Vectors $\mathbf{\Xi}_{\pm}$, and constants $\Upsilon_p$, $\Upsilon_q$ are defined like this:

$$\mathbf{\Xi}_+ = [\xi_1^+, \xi_2^+]^\top = \sum_{j=1}^{m_+ - 1} \mathbf{S}(\theta_{m_+}^+ - \theta_j^+)\Delta\mathbf{p}_j(0) - \sum_{j=1}^{m_+} \int_{\theta_{j-1}^+}^{\theta_j^+} \mathbf{S}(\theta_{m_+}^+ - \phi)\tilde{\mathbf{X}}^j(0,\phi)d\phi,$$

$$\mathbf{\Xi}_- = [\xi_1^-, \xi_2^-]^\top = -\sum_{j=1}^{m_- - 1} \mathbf{S}(\theta_0^- - \theta_j^-)\Delta\mathbf{q}_j(0) + \sum_{j=1}^{m_-} \int_{\theta_{j-1}^-}^{\theta_j^-} \mathbf{S}(\theta_0^- - \phi)\tilde{\mathbf{X}}^j(0,\phi)d\phi,$$

$$\Upsilon_p = [1,0] \left\{ \sum_{j=0}^{m_+ - 1} \Delta\mathbf{p}_j(1) - \sum_{j=1}^{m_+} \int_{\theta_{j-1}^+}^{\theta_j^+} \tilde{\mathbf{X}}^j(1,\phi)d\phi \right\},$$

$$\Upsilon_q = [1,0] \left\{ \sum_{j=1}^{m_-} \int_{\theta_{j-1}^-}^{\theta_j^-} \tilde{\mathbf{X}}^j(1,\phi)d\phi - \sum_{j=1}^{m_+} \Delta\mathbf{q}_j(1) \right\}.$$

Constants $c_3^p = -a_+^{m_+}$, $c_4^p = a_-^{m_+}$, $c_3^q = b_+^1$, $c_4^q = -b_-^1$ are calculated by recurrent relations:

$$a_{\pm}^{j+1} = a_{\pm}^j \left\{ 1 + \frac{2\mu_{j+1}^+ a_{\pm}^j}{1 - \nu_{j+1}^+} \left[ \phi_{j+1}^+ \pm \sin\phi_{j+1}^+ \cos\left(\phi_{j+1}^+ + 2\sum_{k=1}^j \phi_k^+\right) \right] \right\}^{-1},$$

$$b_{\pm}^{j-1} = b_{\pm}^{j}\left\{1 + \frac{2\mu_{j-1}^{-}a_{\pm}^{j}}{1-\nu_{j-1}^{-}}\left[\phi_{j-1}^{-} \pm \sin\phi_{j-1}^{-}\cos\left(\phi_{j-1}^{-} + 2\sum_{k=j}^{m_{-}}\phi_{k}^{-}\right)\right]\right\}^{-1},$$

$$a_{\pm}^{1} = \frac{1-\nu_{1}^{+}}{\mu_{1}^{+}[2\phi_{1}^{+} \pm \sin 2\phi_{1}^{+}]}, \qquad b_{\pm}^{m_{-}} = \frac{1-\nu_{m_{-}}^{-}}{\mu_{m_{-}}^{-}[2\phi_{m_{-}}^{-} \pm \sin 2\phi_{m_{-}}^{-}]},$$

but matrices $\mathbf{T}_2^{\pm}$ are defined by the following recurrent matrix equations:

$$\mathbf{T}_2^{+} = \frac{2(1-\nu_{m_{+}}^{+})}{\mu_{m_{+}}^{+}}\mathbf{B}_{m_{+}}^{+}\mathbf{D}_{m_{+}}^{+}, \qquad \mathbf{T}_2^{-} = \frac{2(1-\nu_{1}^{-})}{\mu_{1}^{-}}\mathbf{B}_{1}^{-}\mathbf{D}_{1}^{-},$$

$$\mathbf{B}_{j}^{+} = \mathbf{I} - \left\{\mathbf{I} + \frac{\mu_{j}^{+}(1-\nu_{j-1}^{+})}{\mu_{j-1}^{+}(1-\nu_{j}^{+})}\mathbf{S}(\phi_{j}^{+})\mathbf{B}_{j-1}^{+}\mathbf{D}_{j-1}^{+}\mathbf{E}_1\left(\mathbf{D}_{j}^{+}\right)^{-1}\mathbf{E}_1\mathbf{S}^{-1}(\phi_{j}^{+})\right\}^{-1},$$

$$\mathbf{B}_{j}^{-} = \mathbf{I} - \left\{\mathbf{I} + \frac{\mu_{j}^{-}(1-\nu_{j+1}^{-})}{\mu_{j+1}^{-}(1-\nu_{j}^{-})}\mathbf{S}^{-1}(\phi_{j}^{-})\mathbf{B}_{j+1}^{-}\mathbf{D}_{j+1}^{-}\mathbf{E}_1\left(\mathbf{D}_{j}^{-}\right)^{-1}\mathbf{E}_1\mathbf{S}(\phi_{j}^{-})\right\}^{-1},$$

$$\mathbf{B}_{1}^{+}, \ \mathbf{B}_{m_{-}}^{-} = \mathbf{I}, \qquad \mathbf{D}_{j}^{\pm} = \frac{\pm 1}{4[(\phi_{j}^{\pm})^2 - \sin^2\phi_{j}^{\pm}]}\begin{pmatrix} 2\phi_{j}^{\pm} + \sin 2\phi_{j}^{\pm} & \mp 2\sin^2\phi_{j}^{\pm} \\ \mp 2\sin^2\phi_{j}^{\pm} & 2\phi_{j}^{\pm} - \sin 2\phi_{j}^{\pm} \end{pmatrix}.$$

Taking into account the volume of the paper, we do not present here the corresponding recurrent relations for the constants $d_{ij}^{p(q)}$ in the asymptotics near point $s = 1$, because they are not directly used in the analysis of systems of integral equations.

COROLLARY 2. In the process of proving Lemma 2 it can be also shown that matrix-functions $\mathbf{M}_p(s)$, $\mathbf{M}_q(s)$ can be represented in the following form:

$$\mathbf{M}_{p(q)}(s) = \mathbf{M}_{p(q)}^{+}(s) + \mathbf{M}_{p(q)}^{-}(s),$$

where matrix-functions $\mathbf{M}_{p(q)}^{\pm}(s)$ satisfy the relations:

$$\mathbf{M}_{p(q)}^{\pm}(s) = \pm\mathbf{M}_{p(q)}^{\pm}(-s), \qquad \mathbf{M}_{p(q)}^{+}(s) = [\mathbf{M}_{p(q)}^{+}(s)]^{\top}, \qquad \mathbf{M}_{p(q)}^{-}(s) = -[\mathbf{M}_{p(q)}^{-}(s)]^{\top}.$$

COROLLARY 3. Let us assume that if the following statements hold true:

1. Domain $\Omega$ is symmetric with respect to $0X_2$ axis ($m_+ = m_-$, $\theta_j^+ + \theta_{m_+-j}^- + \pi = 0$),

2. The constants in Eqs. (2.2) and in interior boundary conditions (2.5), (2.6) for the corresponding wedges are identical ($\mu_j^+ = \mu_{m_+-j+1}^-$, $\nu_j^+ = \nu_{m_+-j+1}^-$, $\tau_j^+ = \tau_{m_+-j}^-$),

3. The types of the boundary conditions on both of the exterior wedge surfaces are identical ($\mathcal{J}^+ = \mathcal{J}^-$);

then it can be shown that:

$$\mathbf{M}_p(s) + \mathbf{M}_q(s) = \begin{pmatrix} 0 & f_1(s) \\ f_1(-s) & 0 \end{pmatrix}, \qquad \mathbf{M}_p(s) - \mathbf{M}_q(s) = \begin{pmatrix} f_2(s) & 0 \\ 0 & f_3(s) \end{pmatrix},$$

where $f_j(s)$ $(j = 1-3)$ are such functions that $f_j(-it) = \overline{f_j(it)}$, and $s f_1(s)$, $f_2(s)$, $f_3(s)$ are even real functions.

REMARK 4. The following relations should be true if the tractions are prescribed along the external wedge boundaries $\Gamma_0^+$, $\Gamma_{m_-}^-$:

$$
(4.21) \quad
\begin{aligned}
\mathcal{J}^+ = 2 : & \qquad \mathbf{p}_t^{m+}(0) = \Xi_+ + \mathbf{S}(-\theta_0^+)\Delta\mathbf{p}_0(0), \\
\mathcal{J}^- = 2 : & \qquad \mathbf{p}_b^1(0) = \Xi_- - \mathbf{S}(-\theta_{m_-}^-)\Delta\mathbf{p}_{m_-}(0), \\
\mathcal{J}^+ = 3 : & \qquad [0,1]\{\Delta\mathbf{p}_0(0) + \mathbf{S}(\theta_0^+)[\Xi_+ - \mathbf{p}_t^{m+}(0)]\} = 0, \\
\mathcal{J}^- = 3 : & \qquad [0,1]\{\Delta\mathbf{q}_{m_-}(0) - \mathbf{S}(\theta_{m_-}^-)[\Xi_- - \mathbf{q}_b^1(0)]\} = 0, \\
\mathcal{J}^+ = 4 : & \qquad [1,0]\{\Delta\mathbf{p}_0(0) + \mathbf{S}(\theta_0^+)[\Xi_+ - \mathbf{p}_t^{m+}(0)]\} = 0, \\
\mathcal{J}^- = 4 : & \qquad [1,0]\{\Delta\mathbf{q}_{m_-}(0) - \mathbf{S}(\theta_{m_-}^-)[\Xi_- - \mathbf{q}_b^1(0)]\} = 0.
\end{aligned}
$$

They follow from relations (4.18), (4.19), because their left-hand sides can have a simple pole only at point $s = 0$, but the right-hand sides have a second degree pole, in general.

REMARK 5. Let us note that $\widetilde{\mathbf{z}}^+(1) = \pi\overline{\mathbf{z}}^+(0)$. Then, using the first relations from (4.13), (4.16) for $\mathbf{q}_b^1$, $\mathbf{p}_t^{m+}$ we obtain:

$$
(4.22) \quad
\begin{aligned}
(2,2,\mathcal{J}) : & \qquad 2\pi\mathbf{z}_*^+ - \Xi_W = 0, \\
(2,3,\mathcal{J}) : & \quad [0,1]\mathbf{S}(\theta_{m_-}^-)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0, \\
(2,4,\mathcal{J}) : & \quad [1,0]\mathbf{S}(\theta_{m_-}^-)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0, \qquad \mathcal{J} = 1-5, \\
(3,2,\mathcal{J}) : & \quad [0,1]\mathbf{S}(\theta_0^+)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0, \\
(4,2,\mathcal{J}) : & \quad [1,0]\mathbf{S}(\theta_0^+)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0.
\end{aligned}
$$

Here we have introduced the following notation:

$$
(4.23) \qquad \Xi_W = \Xi_+ - \Xi_- + \Delta\mathbf{p}_*(0) + \Delta\mathbf{q}_*(0) + \mathbf{S}(-\theta_0^+)\mathbf{p}_0(0) + \mathbf{S}(-\theta_{m_-}^-)\mathbf{q}_{m_-}(0),
$$

where $\Xi_W$ is the principal vector of all forces and tractions acting on the wedge-shaped part of body $\Omega^+ \cup \Omega^-$. This Remark allows us to find the values of the constant vector $\mathbf{z}_*^+ = \overline{\mathbf{z}}^+(0)$ (or one of its components) for the corresponding problems.

The proofs of the Remarks follow from the fact that functions $\mathbf{v}_t^{m+}(s)$, $\mathbf{w}_b^1(s)$ can have only a simple pole at point $s = 0$. Besides, for certain values of the exterior wedge angles $\theta_0^+$, $\theta_{m_-}^-$ we can also conclude that for any $\mathcal{J} = 1-5$

$$
(4.24) \quad
\begin{aligned}
(3,3,\mathcal{J}) : & \quad [1,0]\mathbf{S}(\theta_0^+)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0, & \quad \theta_0^+ = \theta_{n_-}^-; \\
(4,4,\mathcal{J}) : & \quad [0,1]\mathbf{S}(\theta_0^+)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0, & \\
(3,4,\mathcal{J}) : & \quad [0,1]\mathbf{S}(\theta_0^+)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0, & \\
(4,3,\mathcal{J}) : & \quad [1,0]\mathbf{S}(\theta_0^+)\{2\pi\mathbf{z}_*^+ - \Xi_W\} = 0, & \quad \theta_0^+ - \theta_{m_-}^- = \pi/2.
\end{aligned}
$$

Let us note that for an arbitrary problem $(\mathcal{J}^+, \mathcal{J}^-, \mathcal{J})$ there are exactly two additional equations for constant vectors $\mathbf{u}_*$ or $\mathbf{z}_*^+$ (or their components) in (3.27), (3.29), (4.22) and (4.24).

REMARK 6. The following additional relations should be true if the normal component of tractions are prescribed along the external wedge boundaries $\Gamma_0^+$, $\Gamma_{m_-}^-$ and $\gamma_1, \gamma_2 > 1$ (see (4.7)):

$$(4.25) \qquad \mathcal{J}^+ = 2, 4: \quad [1, 0]\mathbf{p}_t^{m+}(1) = \Upsilon_p, \qquad \mathcal{J}^- = 2, 4: \quad [1, 0]\mathbf{q}_b^1(1) = \Upsilon_q.$$

They are the usual torque balance equations, and they follow from relations (4.18), (4.19), because their right-hand sides can have a simple pole at point $s = 1$, but the left-hand sides are analytic in this point.

Besides, in problems $(2, 2, \mathcal{J})$, $(2, 4, \mathcal{J})$, $(4, 2, \mathcal{J})$, $(4, 4, \mathcal{J})$ we can additionally obtain from (4.18) and (4.25) that:

$$(4.26) \qquad -2\pi i[1, 0]\mathbf{z}_*^- = [1, 0]\{\Delta\mathbf{p}_*(1) - \Delta\mathbf{q}_*(1)\} + \Upsilon_p + \Upsilon_q,$$

where $\mathbf{z}_*^-$ is defined in (3.27). To this end, the identity

$$\tilde{\mathbf{z}}_-(2) = \int_0^\infty r\mathbf{z}_-(r)\, dr = -i\pi \left( \frac{2i}{2\pi} \int_0^\infty \mathbf{z}_-(r) \sin r\lambda\, dr \right)' \Bigg|_{\lambda=0} = -i\pi\mathbf{z}_*^-,$$

is used.

## 5. Conclusions

So, we have investigated the solutions of the problems of both (layered and wedge-shaped) parts of the domain. All interfacial conditions along the regions of similar geometry (2.4) ("layer – layer") and (2.5), (2.6) ("wedge – wedge") have been satisfied. Now, it is necessary to take into account interfacial conditions (2.7), (2.8) along the "layer – wedges" interfaces.

Let us note here that each of the relations (3.25), (4.18) and (4.19) as well as Lemma 1 and Lemma 2 are important. They are necessary to solve the boundary value problems for layered and wedge regions, separately. Namely, if we have arbitrary boundary conditions on the boundary $\Gamma_0$ of the types $1 - 5$ (2.11), then we can find the respective integral transform of the corresponding solution in a closed form. Moreover, if the boundary conditions are of a general form (contact with the other body and so on), then the respective relation (3.25), (4.18) and (4.19) makes it possible to investigate the corresponding problem along boundary $\Gamma_0$ only. Then the information on the behaviour of the auxiliary matrix-functions and vector-functions (Lemma 1 and Lemma 2) will play an important role (for example, to reduce the problem to integral equations).

## Acknowledgment

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