Development of flow and heat transfer on a wedge with a magnetic field

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THE DEVELOPMENT of the flow and heat transfer of an incompressible laminar viscous electrically conducting fluid on a stationary infinite wedge with an applied magnetic field has been studied when the fluid in the external stream is set into motion impulsively and at the same time, the surface temperature is suddenly raised from its ambient temperature. The effects of the induced magnetic field, viscous dissipation and Ohmic heating have been taken into account. The mathematical problem has been formulated in such a way that at time $t = 0$, it reduces to Rayleigh type of equation and as time $t \to \infty$, it tends to Falkner–Skan type of equation. The scale of time has been chosen such that the traditional infinite region of integration becomes finite which considerably reduces the computational time. The singular parabolic partial differential equations governing the flow have been solved numerically using an implicit finite difference scheme. There is a smooth transition from the Rayleigh solution at $\xi = 0$ ($t^* = 0$) to Falkner–Skan type of solution at $\xi = 1$ ($t^* \to \infty$ when the steady state is reached). The surface shear stress and the surface heat transfer increase or decrease with time when the pressure gradient parameter is greater or less than a certain value. However, the x component of the induced magnetic field at the surface decreases as time increases.

1. Introduction

FLUID DYNAMIC PHENOMENA involving unsteady boundary layers are of great theoretical and practical interest. Much of the work that has been done in this area is related to external aerodynamics. However, there are also several applications in biofluid dynamics, hydronautics and manufacturing. Excellent review papers on the unsteady boundary layers have been contributed by STUART [1], RILEY [2], TELIONIS [3] and MCCROSKEY [4].

When the external stream is impulsively set into motion at time $t = 0$ with uniform velocity along the plane of symmetry of the stationary infinite wedge, the inviscid flow over the wedge is developed instantaneously. But the viscous flow within the boundary layer develops slowly and it reaches a steady flow only after a certain period of time. The development of the boundary layer with time takes place in two stages. For small time, the flow is dominated by the viscous and pressure gradient forces and the unsteady acceleration. The convective acceleration plays only a minor role in the flow development. On the other hand, for large time the flow is dominated by the viscous forces, the pressure gradient and the convective acceleration. During this phase the unsteady acceleration plays only a minor role in the flow development. For $t = 0$, the flow

is governed by the Rayleigh equation and for $t \to \infty$ it is governed by the Falkner–Skan equation. This change in the character of the flow manifests itself mathematically as a change in character of the equations which describe the fluid motion.

STEWARTSON [5] first studied the impulsive motion over a flat plate and found that for $t > 1$, the flow undergoes a transition from Rayleigh flow to Blasius flow. He [5] noted certain difficulties in the mathematical formulation of the problem (i.e., the transition from the Rayleigh flow to Blasius flow is not smooth) and related it to the physics of the flow. Since then several authors [6–11] have studied this problem using different methods. SMITH [12] has studied the analogous wedge problem and encountered the same difficulties which arise in the case of the flat plate. In order to overcome this difficulty, WILLIAMS and RHYNE [13] have formulated the problem of impulsive motion over a wedge in a new set of scaled coordinates which includes both the short time solution (Rayleigh solution) and long time solution (Falkner–Skan solution) and there is a smooth transition from Rayleigh solution to Falkner–Skan solution. In the above studies, the effect of the magnetic field was not considered. INGHAM [14] has studied the effect of the magnetic field on the flow past an impulsively started semi-infinite plate.

The present investigation considers the development of boundary layer flow and heat transfer with time of an electrically conducting fluid over a stationary infinite wedge with a magnetic field when the fluid in the external stream is set into motion impulsively and at the same time the temperature of the wall is suddenly raised from that of the surrounding fluid. The effects of the induced magnetic field, viscous dissipation and Ohmic heating have been included in the analysis. The mathematical problem has been formulated in such a way that for time $t = 0$, it reduces to Rayleigh type of equations and for $t \to \infty$ it reduces to Falkner–Skan type of equation. The scale of time has been selected such that the traditional infinite region of integration becomes finite which considerably reduces the computational time. The singular parabolic partial differential equations governing the flow have been solved numerically using an implicit finite difference scheme. The particular cases of the present results have been compared with those of HALL [6], DENNIS [7], WATKINS [9], INGHAM [10, 14], TADROS and KIRKHOPE [11], WILLIAMS and RHYNE [13], NATH [16] and WATANABE [17].

2. Problem formulation

We assume that for $t < 0$, an infinite wedge lies in the (x, y) plane with the leading edge at $x = y = 0$ in the ambient fluid. The wall T_w is assumed to have the same temperature as that of the surrounding fluid (i.e., $T_w = T_\infty$) which is electrically conducting. A magnetic field H_0 is applied in the x direction at large distance from the surface of the wedge. At time $t = 0$, the external stream
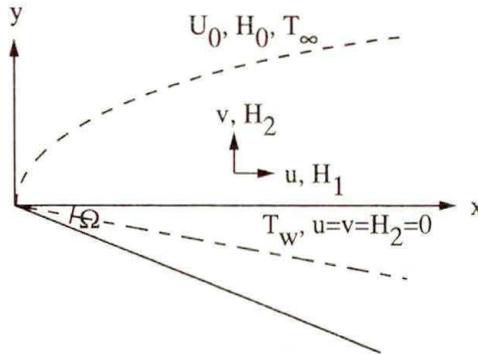
FIG. 1. Coordinate system.

away from the wedge is impulsively set into motion with velocity U_0 parallel to the surface of the wedge (Fig. 1). At the same time the temperature of the wall is raised to T_w from T_∞, the temperature of the surrounding fluid. The effects of the induced magnetic field, viscous dissipation and Ohmic heating have been included in the analysis but the Hall effect has been neglected. It is assumed that there is no applied voltage which implies the absence of an electric field ($E = 0$). The electrical currents flowing in the fluid give rise to an induced magnetic field which would exist if the fluid were an electrical insulator. It has been assumed that the normal component of the induced magnetic field H_2 vanishes at the wall and the parallel component H_1 approaches its given value H_0 at the edge of the boundary layer [15]. The free stream temperature is constant. The solution for small time is described by the Rayleigh's type of equation. For $t \to \infty$, the steady-state equations as given by GRIBBEN [15] and NATH [16] are obtained. Under the above assumptions, the boundary layer and Maxwell's equations governing the unsteady flow can be expressed as [14−17]

$$(2.1) \qquad u_x + v_y = 0,$$

$$(2.2) \qquad (H_1)_x + (H_2)_y = 0,$$

$$(2.3) \qquad u_t + uu_x + vu_y = -\varrho^{-1}(p + \mu_0 H_0^2/2)_x + \nu u_{yy}$$
$$+ (\mu_0/\varrho)\left[H_1(H_1)_x + H_2(H_1)_y\right],$$

$$(2.4) \qquad (H_1)_t + u(H_1)_x + v(H_1)_y - H_1 u_x - H_2 u_y = \alpha_1 (H_1)_{yy},$$

$$(2.5) \qquad T_t + uT_x + vT_y = \nu \mathrm{Pr}^{-1} T_{yy} + (\nu/c_p)u_y^2 + (\varrho c_p \sigma)^{-1}\left[(H_1)_y\right]^2,$$

where

$$(2.6) \qquad -\varrho^{-1}(p + \mu_0 H_0^2/2)_x = U_0(U_0)_x - (\mu_0/\varrho)H_0(H_0)_x,$$
$$U_0 = Ux^m, \qquad H_0 = Hx^m, \qquad \mathrm{Pr} = \nu/\alpha.$$

The boundary conditions for $t \geq 0$ are given by

$$u(x,0,t) = v(x,0,t) = H_2(x,0,t) = 0,$$

(2.7) $$\partial H_1(x,0,t)/\partial y = 0, \qquad T(x,0,t) = T_w,$$

$$u(x,\infty,t_0) = U_0(x), \quad H_1(x,\infty,t) = H_0(x), \quad T(x,\infty,t) = T_\infty.$$

The initial conditions at $t = t_0 \; (< 0)$ are expressed in the form

(2.8) $$u(x,y,t_0) = 0, \qquad H_1(x,y,t_0) = H_0, \qquad T(x,y,t_0) = T_\infty.$$

Here x and y are the distances along and perpendicular to the surface, respectively; u and v are the velocity components along the x and y directions, respectively; H_1 and H_2 are the components of the induced magnetic field along the x and y directions, respectively; U_0 and H_0 are the velocity and the applied magnetic field in the x direction, respectively; p is the pressure; Pr is the Prandtl number; T is the temperature; ϱ and ν are the density and kinematic viscosity, respectively; μ_0 is the magnetic permeability; α and α_1 are, respectively, the thermal diffusivity and magnetic diffusivity; U and H are the velocity and magnetic field in the free stream, respectively; m is the index in the power-law variation of velocity, wall temperature and applied magnetic field; the subscripts t, x and y denote derivatives with respect to t, x and y, respectively; and the subscripts w and ∞ denote conditions at the wall and in the free stream, respectively.

In order to reduce the number of independent variables from three to two in Eqs. (2.1)–(2.5) and to reduce these equations to dimensionless form, we apply the following transformations:

$$\eta = (U/\nu)^{1/2} x^{(m-1)/2} \xi^{-1/2} y, \qquad \xi = 1 - \exp(-U_0 t/x),$$

$$t^* = U_0 t/x, \quad u = \psi_y, \quad v = -\psi_x, \quad H_1 = \phi_y, \quad H_2 = -\phi_x,$$

$$\psi(x,y,t) = (U\nu)^{1/2} x^{(m+1)/2} \xi^{1/2} f(\xi,\eta), \quad \text{Ec} = U^2/[c_p(T_{w0} - T_\infty)],$$

(2.9) $$\phi(x,y,t) = (H^2\nu/U)^{1/2} x^{(m+1)/2} \xi^{1/2} g(\xi,\eta),$$

$$T(x,y,t) = T_\infty + (T_w - T_\infty)\theta(\xi,\eta), \quad S = \mu_0 H^2/(\varrho U^2),$$

$$T_w - T_\infty = (T_{w0} - T_\infty)x^{2m}, \qquad \lambda = \nu/\alpha_1, \qquad \alpha_1 = (\mu_0\sigma)^{-1},$$

$$u = U x^m f'(\xi,\eta), \qquad H_1 = H x^m g'(\xi,\eta),$$

$$\beta = 2m/(m+1), \qquad m > -1,$$

to Eqs. (2.1)–(2.5) and we find that Eqs. (2.1) and (2.2) are identically satisfied and Eqs. (2.3)–(2.5) reduce to

(2.10) $$f''' + 2^{-1}[(m+1)\xi + (1-m)(1-\xi)\ln(1-\xi)]\,ff''$$

$$+ m\xi(1-f'^2) + 2^{-1}\eta(1-\xi)f'' + (1-m)\xi(1-\xi)\ln(1-\xi)f''(\partial f/\partial\xi)$$

$$- \xi m S(1-g'^2) - 2^{-1}[(m+1)\xi + (1-m)(1-\xi)\ln(1-\xi)]Sgg''$$

$$= \xi(1-\xi)\left[1 + (1-m)\ln(1-\xi)f'\right](\partial f'/\partial\xi)$$

$$+ S(1-m)\xi(1-\xi)\ln(1-\xi)g'(\partial g'/\partial\xi),$$

$$(2.11) \quad \lambda^{-1}g''' + 2^{-1}(m+1)\xi(fg'' - f''g) + 2^{-1}(1-m)(1-\xi)\ln(1-\xi)(fg'' - f''g)$$
$$+ 2^{-1}(1-\xi)\eta g'' + (1-m)\xi(1-\xi)\ln(1-\xi)g''(\partial f/\partial\xi)$$
$$- (1-m)\xi(1-\xi)\ln(1-\xi)f''(\partial g/\partial\xi)$$
$$= \xi(1-\xi)\left[1+(1-m)\ln(1-\xi)f'\right]\partial g'/\partial\xi$$
$$- (1-m)\xi(1-\xi)\ln(1-\xi)g'(\partial f'/\partial\xi),$$

$$(2.12) \quad \mathrm{Pr}^{-1}\theta'' + 2^{-1}\left[(m+1)\xi + (1-m)(1-\xi)\ln(1-\xi)\right]f\theta'$$
$$- 2mSf'\theta + 2^{-1}(1-\xi)\eta\theta' + (1-m)\xi(1-\xi)\ln(1-\xi)\theta'(\partial f/\partial\xi)$$
$$+ \mathrm{Ec}\left[(f'')^2 + (S/\lambda)(g'')^2\right] = \xi(1-\xi)\left[1 + (1-m)\ln(1-\xi)f'\right](\partial\theta/\partial\xi).$$

The boundary conditions are given by

$$(2.13) \quad f(\xi,0) = f'(\xi,0) = g(\xi,0) = g''(\xi,0) = 0, \qquad \theta(\xi,0) = 1,$$
$$f'(\xi,\infty) = g'(\xi,\infty) = 1, \qquad \theta(\xi,\infty) = 0.$$

Here ξ and η are the transformed and dimensionless independent variables; t^* is the dimensionless time; ψ and ϕ are the dimensional fluid and magnetic stream functions, respectively; f' is the dimensionless velocity; g' is the dimensionless x component of the induced magnetic field; f and g are the dimensionless fluid and magnetic stream functions, respectively; θ is the dimensionless temperature; S is the dimensionless magnetic parameter; β is the pressure gradient parameter; Ec is the Eckert number; σ is the electrical conductivity; λ is the magnetic Prandtl number; T_{w0} is the value of T_w at $x = 0$; and prime denotes derivative with respect to η.

Equations (2.11)–(2.13) are partial differential equations, but for $\xi = 0$ and $\xi = 1$ they reduce to ordinary differential equations. For $\xi = 0$, the equations are

$$(2.14) \qquad f''' + 2^{-1}\eta f'' = 0,$$
$$(2.15) \qquad \lambda^{-1}g''' + 2^{-1}\eta g'' = 0,$$
$$(2.16) \qquad \mathrm{Pr}^{-1}\theta'' + 2^{-1}\eta\theta' + \mathrm{Ec}\left[(f'')^2 + (S/\lambda)(g'')^2\right] = 0.$$

For $\xi = 1$, the equations are given by

$$(2.17) \quad f''' + 2^{-1}(m+1)ff'' + m(1 - f'^2) - mS(1 - g'^2)$$
$$- 2^{-1}(m+1)Sgg'' = 0,$$
$$(2.18) \quad \lambda^{-1}g''' + 2^{-1}(m+1)(fg'' - f''g) = 0,$$
$$(2.19) \quad \mathrm{Pr}^{-1}\theta'' + 2^{-1}(m+1)f\theta' - 2mf'\theta + \mathrm{Ec}\left[(f'')^2 + (S/\lambda)(g'')^2\right] = 0.$$

The boundary conditions for (2.14)–(2.19) are expressed as

$$(2.20) \quad f(0) = f'(0) = g(0) = g''(0) = 0, \qquad \theta(0) = 1,$$
$$f'(\infty) = g'(\infty) = 1, \qquad \theta(\infty) = 0.$$

It may be remarked that Eq. (2.10) for $S = 0$ reduces to that of WILLIAMS and RHYNE [13]. Equations (2.10) and (2.11) for $m = 0$ are essentially the same as those of INGHAM [14]. Also, Eq. (2.10) for $m = S = 0$ is the same as that of HALL [6], DENNIS [7], WATKINS [9], INGHAM [10] and TADROS and KIRKHOPE [11]. When $\xi = 1$, the self-similar equations (2.17) and (2.18) are the same as those of NATH [16] if we apply the following transformations

$$(2.21) \qquad \begin{aligned} \eta &= (2 - \beta)^{1/2}\eta_1\,, \qquad f(\eta) = (2 - \beta)^{1/2}f_1(\eta_1), \qquad \beta < 2, \\ g(\eta) &= (2 - \beta)^{1/2}g_1(\eta_1), \qquad \beta = 2m/(m+1). \end{aligned}$$

Also, for $S = \mathrm{Ec} = 0$ and for constant wall temperature case (the term $2mf'\theta = 0$), Eqs. (2.17) and (2.19) reduce to those of WATANABE [17] if we apply the transformations

$$(2.22) \qquad \begin{aligned} \eta &= [2/(m+1)]^{1/2}\eta_1\,, \qquad f(\eta) = [2/(m+1)]^{1/2}f_1(\eta_1), \\ \theta(\eta) &= \theta_1(\eta_1). \end{aligned}$$

3. Analytical solution

Equations (2.14)–(2.16) under boundary conditions (2.20) admit closed-form solution. The solution of Eq. (2.14) under conditions (2.20) is expressed as

$$(3.1) \qquad f = \eta \operatorname{erf}(\eta/2) + (\pi)^{-1/2}\left[\exp(-\eta^2/4) - 1\right],$$

hence

$$(3.5) \qquad f' = \operatorname{erf}(\eta/2), \qquad f'' = (\pi)^{-1/2}\exp(-\eta^2/4), \qquad f''(0) = (\pi)^{-1/2}.$$

Equation (2.15) is integrated once to yield the equation

$$(3.3) \qquad \lambda^{-1}g'' + 2^{-1}\eta g' - 2^{-1}g = 0,$$

where the constant of integration is zero by virtue of the conditions $g(0) = g''(0) = 0$. Equation (2.22) under conditions (2.20) has the solution of the form [14]:

$$(3.4) \qquad g = \eta\,.$$

Equation (2.16) under conditions (2.20) for $\mathrm{Pr} = 1$ yields the following solution:

$$(3.5) \qquad \theta = \operatorname{erfc}(\eta/2) + (\mathrm{Ec}/2)\operatorname{erf}(\eta/2)\operatorname{erfc}(\eta/2).$$

Hence

(3.6) $\theta' = -\text{Ec}\,(\pi)^{-1/2}\exp(-\eta^2/4)\,\text{erf}\,(\eta/2) + (\pi)^{-1/2}(\text{Ec}-1)\exp(-\eta^2/4),$

(3.7) $$\theta'(0) = (2^{-1}\text{Ec}-1)(\pi)^{-1/2}.$$

Also, Eq. (2.16) under conditions (2.20) for $\text{Ec}=0$ has the solution of the form (3.7)

(3.8) $$\theta = \text{erfc}\,(\text{Pr}^{1/2}\eta/2).$$

Hence

(3.9)   ████████████████████████ $\theta'(0) = -(\text{Pr}/\pi)^{1/2}.$

4. Numerical solution

It may be noted that Eqs. (2.10) and (2.11) are coupled nonlinear partial differential equations of parabolic type, whereas Eq. (2.12) is an uncoupled linear parabolic partial differential equation. Equations (2.10) – (2.12) under boundary conditions (2.13) and initial conditions (2.14) – (2.16) can be solved numerically. Equations (2.10) – (2.12) can be rewritten as

(4.1) $$\frac{\partial^2 W_1}{\partial \eta^2} + a_1\frac{\partial W_1}{\partial \eta} + a_2 W_1 + a_3\frac{\partial W_2}{\partial \eta} + a_4 W_2 + a_5 = a_6\frac{\partial W_1}{\partial \xi} + a_7\frac{\partial W_2}{\partial \xi},$$

(4.2)   ██

(4.3)   ██

where

$$W_1 = f' = \partial f/\partial \eta, \qquad W_2 = g' = \partial g/\partial \eta, \qquad W_3 = \theta,$$

$$a_1 = \left[2^{-1}(m+1)\xi + 2^{-1}(1-m)(1-\xi)\ln(1-\xi)\right]f$$
$$+2^{-1}\eta(1-\xi) + (1-m)\xi(1-\xi)\ln(1-\xi)(\partial f/\partial \xi),$$

$$a_2 = -m\xi f',$$

$$a_3 = -\left[2^{-1}(1+m)\xi + 2^{-1}(1-m)(1-\xi)\ln(1-\xi)\right]g,$$

$$a_4 = \xi m S g', \qquad a_5 = m\xi(1-\xi),$$

(4.4) $$a_6 = \xi(1-\xi)\left[1 + (1-m)\ln(1-\xi)f'\right],$$

$$a_7 = S(1-m)\xi(1-\xi)\ln(1-\xi)g',$$

$$a_8 = -\left[2^{-1}(m+1)\xi + 2^{-1}(1-m)(1-\xi)\ln(1-\xi)\right]g$$
$$-(1-m)\xi(1-\xi)\ln(1-\xi)(\partial g/\partial \xi),$$

$$a_9 = -(1-m)\xi(1-\xi)\ln(1-\xi)g',$$

$$a_{10} = -2mSf', \qquad a_{11} = \text{Ec}\,f'', \qquad a_{12} = \text{Ec}\,(S/\lambda)g''.$$

The boundary conditions (2.20) can be expressed as

(4.5)
$$W_1(\xi,0) = f(\xi,0) = W_2'(\xi,0) = g(\xi,0) = 0, \qquad W_3(\xi,0) = 1,$$
$$W_1(\xi,\infty) = W_2(\xi,\infty) = 1, \qquad W_3(\xi,\infty) = 0.$$

It may be remarked that a_6 which is the coefficient of $\partial W_i/\partial\xi$, $i = 1,2,3$ in Eqs. (4.1)–(4.3) will be positive when $\xi < 1 - \exp[(m-1)^{-1}]$ as $f' > 0$ $(0 < f' \le 1)$ in $(0 < \eta \le \eta_\infty)$. However a_6 becomes negative for some η when $\xi > 1 - \exp[(m-1)^{-1}]$. When a_6 is positive, Eqs. (4.1)–(4.3) are parabolic partial differential equations and well-posed. Equations (4.1)–(4.3) under initial conditions given by Eqs. (2.14)–(2.16) and boundary conditions (4.5) can be solved by using an implicit finite-difference scheme. When $a_6 < 0$, the problem is no longer well-posed and the forward integration method fails [13]. Such equations are called singular parabolic partial differential equations [10]. Physically, the change in the sign of a_6 is attributed to the change in the direction of flow of information as explained in [13].

In order to overcome the difficulty mentioned above, in the finite-difference scheme we have used either forward or backward differences in ξ direction consistent with the direction of the flow of information. In the η direction, we have used the central difference scheme. This solution technique is based on the technique used by CARTER [18] and the detailed description of this technique is given by WILLIAMS and RHYNE [13]. Hence it is not presented here. Figure 2 shows the computational region and the behaviour of the coefficient a_6.
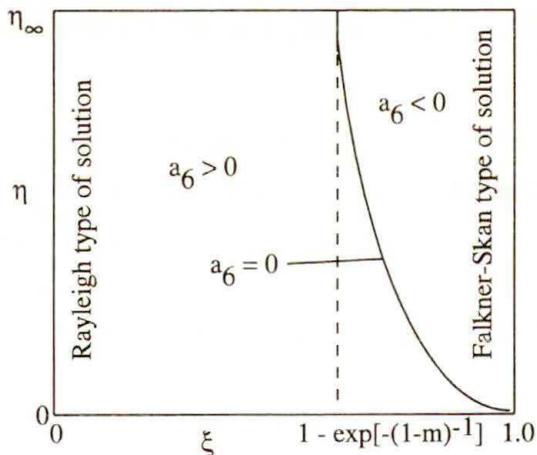
FIG. 2. Schematic representation of computational region.

We have also studied the effect of step sizes $\Delta\eta$ and $\Delta\xi$ and the edge of the boundary layer represented by η_∞ on the solution in order to optimize them. Consequently, the step sizes $\Delta\eta = 0.05$, $\Delta\xi = 0.01$ and $\eta_\infty = 10$ have been used for the computation.

5. Results and discussion

In order to verify the analysis and to check the accuracy of the present method, we have compared our dimensionless surface shear stress parameter $(f''(\xi, 0))$ for $S = 0$ (no magnetic field) with that of WILLIAMS and RHYNE [13] and for $S = m = 0$ (flat plate case without magnetic field) with that of HALL [6], DENNIS [7], WATKINS [9], INGHAM [10], TADROS and KIRKHOPE [11] and WILLIAMS and RHYNE [13]. Also, we have compared our dimensionless surface shear stress parameter $(f''(t^*, 0))$ and the x-component of the induced magnetic field on the surface $(g'(t^*, 0))$ for $m = 0$ (flat plate case) with those of INGHAM [14]. For $\xi = 1$, we have compared the surface shear stress $(f''(0))$ and the x-component of the induced magnetic field $(g'(0))$ with those of NATH [16]. For $\xi = 1$, $S = 0$, $\mathrm{Pr} = 0.73$ we have compared the surface shear stress $(f''(0))$ and the surface heat transfer $(-\theta'(0))$ with those of WATANABE [17]. In all the cases the results are found to be in excellent agreement. Hence for the sake of brevity, the comparison is not shown here. It may be noted that for direct comparison with NATH [16] we have to multiply the shear stress parameter $f''(0)$ by $(2 - \beta)^{1/2}$ where $\beta = 2m/(m + 1)$ and with WATANABE [17], we have to multiply $f''(0)$ and $\theta'(0)$ by $[2/(m + 1)]^{1/2}$.

We have obtained the solution of $(4.1)-(4.3)$ for the pressure gradient parameter $m(\beta)$ in the range $m_1 \le m \le 1$ $(\beta_1 \le \beta \le 1)$ and for several values of the magnetic parameter S $(0.125 \le S \le 0.75)$.

The solution for $m = 1$ $(\beta = 1)$ is of interest because for this case a_6, which is the coefficient of $\partial W_i/\partial \xi$, $i = 1, 2, 3$, in $(4.1)-(4.3)$, is positive for all $\xi < 1$. In this case, it takes an infinite time for a signal from the line $x = 0$ to reach any point x downstream. The flow develops under the influence of the unsteady acceleration, the viscous forces and magnetic field and the imposed pressure gradient. This type of flow has been discussed in detail by STEWARTSON [19].

For $m_1 \le m < 1$ $(m_1 < 0)$, a_6 changes sign between $\xi = 0$ and $\xi = 1$ (Fig. 2). The region where $a_6 < 0$ represents the region where the flow at a given x station is affected by conditions at $x = 0$. The case $m = m_1$ $(m_1 < 0)$ represents the unsteady development of the incipient separation profile [13]. $m_1 = -0.0842$, -0.0773, -0.0667 and -0.0508 for $S = 0.125$, 0.25, 0.50 and 0.75.

The variation of the surface shear stress $(f''(\xi, 0))$, the x component of the induced magnetic field at the surface $(g'(\xi, 0))$ and the heat transfer at the surface $(-\theta'(\xi, 0))$ with the dimensionless time ξ for various values of the pressure gradient parameter m and the magnetic parameter S are shown in Figs. 3–8. From the results it is evident that there is a smooth transition from the Rayleigh solution at $\xi = 0$ (i.e., at $t^* = 0$) to the Falkner–Skan type of solution at $\xi = 1$ (i.e., as $t^* \to \infty$ when the steady-state is reached). For the pressure gradient parameter $m > m_0$, which depends on the magnetic parameter S, the surface shear stress $(f''(\xi, 0))$ increases when ξ increases from zero to 1, but for $m < m_0$ it decreases. On the other hand, the x component of the induced magnetic field at
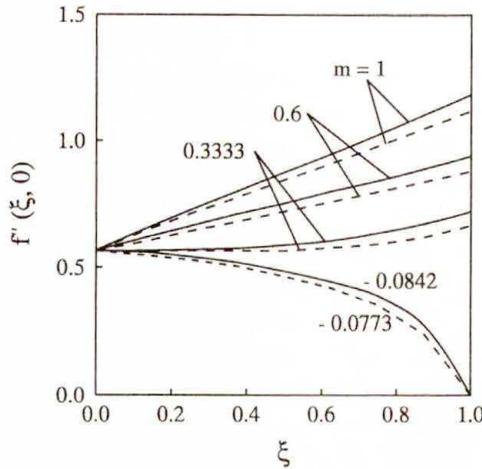
FIG. 3. Variation of the surface shear stress $f''(\xi, 0)$ with ξ for $S = 0.125$ and 0.25.
———, $S = 0.125$; – – – –, $S = 0.25$.
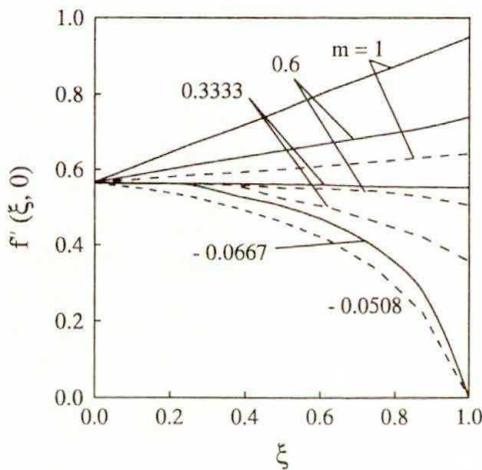
FIG. 4. Variation of the surface shear stress $f''(\xi, 0)$ with ξ for $S = 0.5$ and 0.75.
———, $S = 0.5$; – – – –, $S = 0.75$.

the wall ($g'(\xi, 0)$) and the surface heat transfer ($-\theta'(\xi, 0)$) decrease for all values of m and S when ξ increases from zero to 1 except for $m = 1$ and $S \leq 0.125$ when $-\theta'(\xi, 0)$ slightly increases with ξ. For $m = 1$, $S = 0.5$, $\mathrm{Pr} = 0.73$ $f''(\xi, 0)$ increases by about 68% when ξ increases from zero to 1, but $g'(\xi, 0)$ and $-\theta'(\xi, 0)$ decrease by about 28% and 21%, respectively. For $m = 0.3333$ $f''(\xi, 0)$, $g'(\xi, 0)$ and $-\theta'(\xi, 0)$ decrease, respectively, by about 2.3%, 41% and 40% as ξ increases from zero to 1. Also for all ξ, the shear stress, the x component of the induced magnetic field and the heat transfer ($f''(\xi, 0)$, $g'(\xi, 0)$, $-\theta'(\xi, 0)$) decreases as the magnetic parameter S increases. For example, when $\xi = 0.5$, $m = 0.3333$,
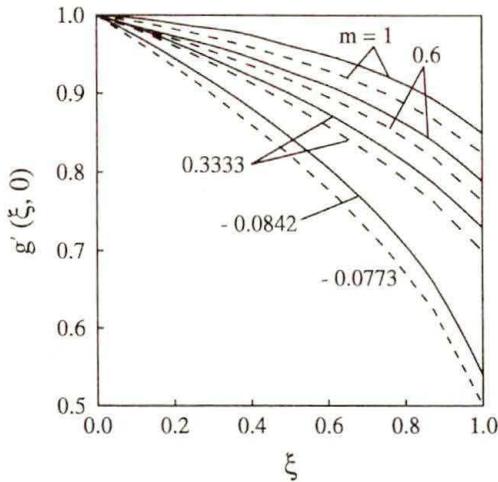
FIG. 5. Variation of the x-component of the induced magnetic field $g'(\xi, 0)$ with ξ for $S = 0.125$ and 0.25. ——, $S = 0.125$; $---$, $S = 0.25$.
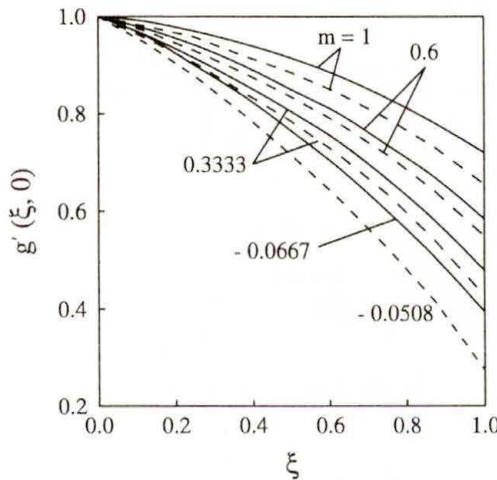
FIG. 6. Variation of the x-component of the induced magnetic field $g'(\xi, 0)$ with ξ for $S = 0.5$ and 0.75. ——, $S = 0.5$; $----$, $S = 0.75$.

$Pr = 0.73$, $f''(\xi, 0)$, $g'(\xi, 0)$ and $-\theta'(\xi, 0)$ decrease by about 29%, 17% and 22%, respectively, as S increases from 0.125 to 0.75. For a given value of ξ ($\xi > 0$), S and Pr, $f''(\xi, 0)$, $g'(\xi, 0)$ and $-\theta'(\xi, 0)$ decreases as the pressure gradient parameter m decreases from 1 to -0.0408. The percentage reduction in $f''(\xi, 0)$, $g'(\xi, 0)$ and $-\theta'(\xi, 0)$ for $\xi = S = 0.5$, $Pr = 0.73$ is about 54, 14 and 21, respectively, as m decreases from 1 to -0.0408. Finally it may be remarked that the effect of variation of m or S on $f''(\xi, 0)$, $g'(\xi, 0)$ and $-\theta'(\xi, 0)$ is most pronounced for $\xi = 1$ (i.e., when the steady state is attained).
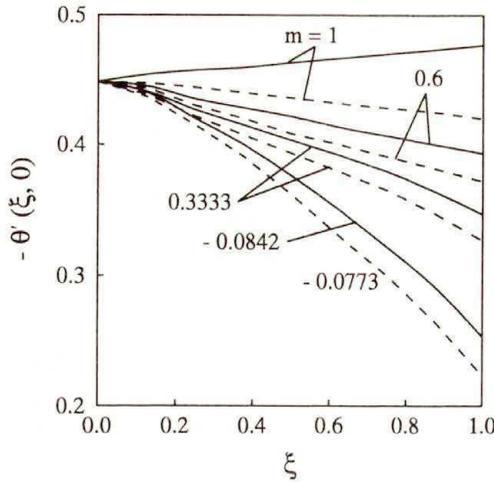
FIG. 7. Variation of the heat transfer parameter at the surface $-\theta'(\xi,0)$ with ξ for $S = 0.125$ and 0.25, $\mathrm{Pr} = 0.73$, $\mathrm{Ec} = 0.1$. ———, $S = 0.125$; – – – –, $S = 0.25$.
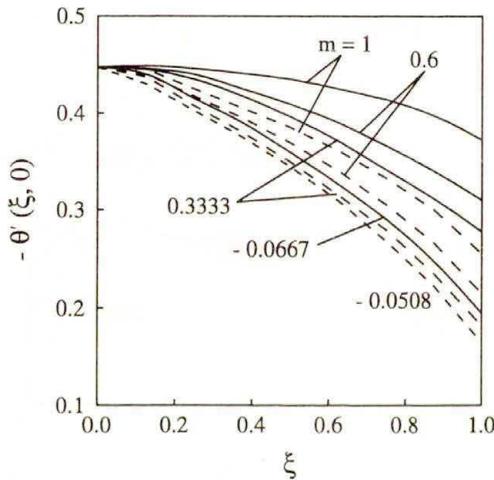
FIG. 8. Variation of the heat transfer parameter at the surface $-\theta'(\xi,0)$ with ξ for $S = 0.5$ and 0.75, $\mathrm{Pr} = 0.73$, $\mathrm{Ec} = 0.1$. ———, $S = 0.5$; – – – –, $S = 0.75$.

The physical problem considered here depends on the magnetic field H, electrical conductivity σ and thermal diffusivity α. These parameters enter the dimensionless equations (2.10) – (2.12) as magnetic parameter α (which is the square of the ratio of the Alfven speed to the free stream velocity), magnetic Prandtl number α (which is the ratio of the viscous to magnetic diffusivities), and the fluid Prandtl number Pr (which is the ratio of the kinematic viscosity to the thermal diffusivity), respectively. Here, we qualitatively discuss the effects of these parameters $(S, \lambda, \mathrm{Pr})$ on our problem.

At the start of the motion (i.e., at $\xi = 0$), the flow is independent of the magnetic parameter S and the magnetic Prandtl number λ, and the effect of these parameters increases with ξ. For fixed values of λ and Pr, the viscous, magnetic and thermal boundary layers continue to thicken and the surface shear stress $(f''(\xi, 0))$, x-component of the induced magnetic field on the surface $(g'(\xi, 0))$ and the surface heat transfer $(-\theta'(\xi, 0))$ decrease as the magnetic parameter S increases until at $S = 1$ the entire flow is plugged (i.e., f, g, θ all tend to zero as $S \to 1$). This is due to the induced current which produces a magnetic counter-field that annuls the entire flow field. Similar trend has been observed by GLAUERT [20], TAN and WANG [21] and DAS [22] for the flat plate case.

The effect of the magnetic Prandtl number λ on the flow field is significant. For zero electrical conductivity $\lambda = 0$ and the problem reduces to the classical boundary layer case. For infinite electrical conductivity, $\lambda \to \infty$. For this case the magnetic lines of forces are frozen into the fluid and no interaction between the magnetic field and flow field takes place. For small λ, the viscous boundary layer is much thinner than the magnetic boundary layer, and for large λ it is the other way around. The surface shear stress $(f''(\xi, 0))$, x-component of the induced magnetic field $(g'(\xi, 0))$ at the surface and the surface heat transfer $(-\theta'(\xi, 0))$ decrease with increasing magnetic Prandtl number λ.

The fluid Prandtl number Pr affects only the thermal field. For small Pr (Pr < 1), the thermal boundary layer is thicker than the viscous boundary layer, and for large Pr (Pr $\gg$ 1) the thermal boundary layer is much thinner than the viscous boundary layer, consequently, the surface heat transfer $(-\theta'(\xi, 0))$ increases with Pr.

6. Conclusions

It is evident from the results that there is a smooth transition from the Rayleigh solution at $\xi = 0$ $(t^* = 0)$ to the Falkner–Skan type of solution at $\xi = 1$ $(t^* \to \infty$ when the steady state is reached). The surface shear stress and the surface heat transfer parameters increase or decrease with time when the pressure gradient parameter is greater or less than a certain value. However, the x-component of the induced magnetic field at the surface decreases as time increases whatever may be the value of the pressure gradient parameter. The surface shear stress, the x-component of the induced magnetic field at the surface and the surface heat transfer decrease as the pressure gradient parameter decreases or the magnetic parameter or the magnetic Prandtl number increases. However, the effect is more pronounced for large times.

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