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Bending of a symmetric piezothermoelastic laminated plate with a through crack

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FOLLOWING the theory of linear piezoelectricity, we consider the response of a cracked composite plate with attached piezoelectric polyvinylidene fluoride layers subjected to mechanical, thermal and electric field loading. Piezoelectric layers are added to the upper and lower surfaces. Classical lamination theory is extended to include piezothermoelastic effects, and the bending problem of a symmetric piezoelectric laminated plate with a through crack is considered. Fourier transforms are used to reduce the problem to the solution of a pair of dual integral equations. The integral equations are solved exactly and the moment intensity factor is expressed in closed form.

Notations

c	half of the crack length,
E_0	intensity of uniform electric field,
E_i	x_i axis elastic modulus,
E_z	z component of electric field vector,
d_{kl}	piezoelectric compliance coefficients,
D_{ij}	bending composite plate stiffnesses,
G_{ij}	i-j plane shear modulus,
mm2	mm, piezoelectric material has planes of symmetry in the x_1- and x_2-axes; 2 denotes that the x_3-axis is a two-fold rotational axis,
M_0	intensity of uniform moment,
M_{xx}, M_{yy}, M_{xy}	moment resultants,
$M^E_{xx}, M^E_{yy}, M^E_{xy}$	electric moment resultants,
$M^\theta_{xx}, M^\theta_{yy}, M^\theta_{xy}$	thermal moment resultants,
h	half of the total thickness,
h_k	thickness of the k-th layer,
$J_0(\ )$	zero-order Bessel function of the first kind,
K_I	moment intensity factor,
Q_x, Q_y	vertical shear forces,
T	absolute temperature,
$T_0, -T_0$	temperature rises at bottom and top surfaces, respectively,
T_R	stress-free reference temperature,
u_x, u_y, u_z	rectangular displacement components,
V_y	equivalent shear,
w	middle surface displacement,
x, y, z	coordinate axes of laminate,
x_1, x_2, x_3	coordinate axes of lamina [for PVDF, x_1: rolling direction, x_3: poling direction],
α_i	x_i axis coefficient of thermal expansion,

$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}$ components of strain tensor,
θ $= T - T_R$, temperature rise,
θ_1 angle between the lamina x axis and lamina principal x_1 axis,
ν_{ij} i-j plane Poisson's ratio,
$\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ components of stress tensor.

Superscripts

E electrically induced component,
θ thermally induced component.

Subscript

k k-th layer.

1. Introduction

PIEZOELECTRIC materials and composites are an important branch of modern engineering materials, with wide applications in actuators and sensors in smart materials and structures [1]. Investigations on such smart materials and structures include the works of LEE and JIANG [2], who presented a state space approach for exact analysis of three-dimensional piezoelectric lamina, with the aim at developing an efficient analytical methodology for laminated piezoelectric structures, and BATRA *et al.* [3], who performed an analysis of a simply supported rectangular elastic plate forced into bending vibrations by the application of time harmonic voltages to piezoelectric actuators attached to its bottom and top surfaces. However, it is reported experimentally that flaws or defects produced during their manufacturing process in piezoelectric materials can adversely influence the performance of piezoelectric devices [4]. When piezoelectric materials are subjected to mechanical, thermal and electrical stresses in service, the propagation of defects such as cracks may result in premature failure of these materials. To prevent failure during service and to secure the structural integrity of piezoelectric devices, understanding of fracture behaviour of piezoelectric materials and composites is of great importance [5, 6].

In this investigation, the linear electro-thermoelastic analysis of a symmetric piezoelectric laminated plate with a through crack under a uniform electric field is discussed. The cracked composite plate with piezoelectric polyvinylidene fluoride layers attached to its bottom and top surfaces is loaded by mechanical and thermal bending moments. The electric field and the poling direction are perpendicular to the plate surfaces, and classical lamination theory including piezothermoelastic effects is applied. Fourier transforms are used to reduce the problem to the solution of a pair of dual integral equations. The integral equations are solved exactly and the moment intensity factor is expressed in closed form.

2. Problem statement and basic equations

Consider a symmetric piezothermoelastic laminated plate containing a through crack of length $2c$ constructed of N layers of materials that exhibit the symmetry of an orthorhombic crystal of class mm2 with respect to axes x_1, x_2, x_3 as shown in Fig. 1. Let the coordinate axes x and y be such that they are in the middle plane of the hybrid laminate and the $z = x_3$ axis is perpendicular to this plane.

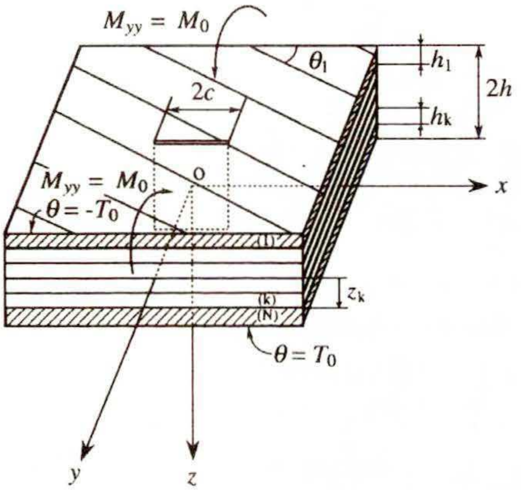

FIG. 1. A symmetric piezothermoelastic laminated plate with a through crack.

The crack is located on the line $y = 0$, $-c < x < c$. The total thickness is $2h$ and the k-th layer has thickness $h_k = z_k - z_{k-1}$ ($k = 1, ..., N$), where $z_0 = -h$ and $z_N = h$. For the present investigation, in which a large uniform electric potential is applied to one or more layers of the cracked laminate, it is assumed that the electric field resulting from variations in stress and temperature (the so-called direct piezoelectric effect) is insignificant compared with the applied electric field [1]. The cracked composite plate is deformed by mechanical and thermal bending moments. If the midplane is a plane of material symmetry, it may be seen that the membrane and bending solutions of the problem would be fully uncoupled.

By employing the usual assumptions of classical lamination theory [7], the rectangular displacement components u_x, u_y, u_z may be expressed as follows:

$$u_x = -zw_{,x}, \qquad u_y = -zw_{,y}, \qquad u_z = w(x, y), \tag{2.1}$$

where a comma denotes partial differentiation with respect to the coordinate and $w(x, y)$ represents the deflection of the middle plane of the composite plate. The strain variations within the laminate are related to the middle surface displacement $w(x, y)$ by the expressions

$$\varepsilon_{xx} = -zw_{,xx}, \qquad \varepsilon_{yy} = -zw_{,yy}, \qquad \varepsilon_{xy} = -zw_{,xy}. \tag{2.2}$$

The constitutive relations for a typical layer k ($k = 1, \ldots, N$), referred to arbitrary plate axes x, y and z, become

$$\tag{2.3} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_k = -z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & \overline{e}_{31} \\ 0 & 0 & \overline{e}_{32} \\ 0 & 0 & \overline{e}_{36} \end{bmatrix}_k \begin{Bmatrix} 0 \\ 0 \\ E_z \end{Bmatrix}_k - \begin{Bmatrix} \overline{\lambda}_1 \\ \overline{\lambda}_2 \\ \overline{\lambda}_6 \end{Bmatrix}_k \theta,$$

where $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ are the components of stress tensor, E_z is the z component of electric field vector, and $\theta = T - T_R$ is the temperature rise from the stress-free reference temperature T_R. For $\overline{Q}_{ij}$, $\overline{e}_{ij}$ and $\overline{\lambda}_i$ we have

$$\tag{2.4} \begin{aligned} \overline{Q}_{11} &= Q_{11}\cos^4\theta_1 + 2(Q_{12}+2Q_{66})\sin^2\theta_1\cos^2\theta_1 + Q_{22}\sin^4\theta_1, \\ \overline{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta_1\cos^2\theta_1 + Q_{12}(\sin^4\theta_1+\cos^4\theta_1), \\ \overline{Q}_{16} &= (Q_{11}-Q_{12}-2Q_{66})\sin\theta_1\cos^3\theta_1 + (Q_{12}-Q_{22}+2Q_{66})\sin^3\theta_1\cos\theta_1, \\ \overline{Q}_{22} &= Q_{11}\sin^4\theta_1 + 2(Q_{12}+2Q_{66})\sin^2\theta_1\cos^2\theta_1 + Q_{22}\cos^4\theta_1, \\ \overline{Q}_{26} &= (Q_{11}-Q_{12}-2Q_{66})\sin^3\theta_1\cos\theta_1 + (Q_{12}-Q_{22}+2Q_{66})\sin\theta_1\cos^3\theta_1, \\ \overline{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta_1\cos^2\theta_1 + Q_{66}(\sin^4\theta_1+\cos^4\theta_1); \end{aligned}$$

$$\tag{2.5} \begin{aligned} Q_{11} &= \frac{E_1}{1-\nu_{12}\nu_{21}}, \\ Q_{12} &= \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \\ Q_{22} &= \frac{E_2}{1-\nu_{12}\nu_{21}}, \\ Q_{66} &= G_{12}; \end{aligned}$$

$$\tag{2.6} \begin{aligned} \overline{e}_{31} &= (Q_{11}d_{31} + Q_{12}d_{32})\cos^2\theta_1 + (Q_{12}d_{31} + Q_{22}d_{32})\sin^2\theta_1, \\ \overline{e}_{32} &= (Q_{11}d_{31} + Q_{12}d_{32})\sin^2\theta_1 + (Q_{12}d_{31} + Q_{22}d_{32})\cos^2\theta_1, \\ \overline{e}_{36} &= [(Q_{11} - Q_{12})d_{31} + (Q_{12} - Q_{22})d_{32}]\sin\theta_1\cos\theta_1; \end{aligned}$$

$$\tag{2.7} \begin{aligned} \overline{\lambda}_1 &= (Q_{11}\alpha_1 + Q_{12}\alpha_2)\cos^2\theta_1 + (Q_{12}\alpha_1 + Q_{22}\alpha_2)\sin^2\theta_1, \\ \overline{\lambda}_2 &= (Q_{11}\alpha_1 + Q_{12}\alpha_2)\sin^2\theta_1 + (Q_{12}\alpha_1 + Q_{22}\alpha_2)\cos^2\theta_1, \\ \overline{\lambda}_6 &= [(Q_{11} - Q_{12})\alpha_1 + (Q_{12} - Q_{22})\alpha_2]\sin\theta_1\cos\theta_1; \end{aligned}$$

in which E_i is x_i axis elastic modulus, ν_{ij} is the i-j plane Poisson's ratio, G_{ij} is the i-j plane shear modulus, d_{kl} are the piezoelectric compliance coefficients, α_i is the x_i axis coefficient of thermal expansion, and θ_1 is the angle between the lamina x axis and lamina principal x_1 axis. Additionally, the elastic moduli and Poisson's ratios are related by

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2}. \tag{2.8}$$

Integrating the constitutive relations of Eq. (2.3) through the composite plate thickness leads to the structure material stiffness relationships. The bending composite plate stiffnesses are given as

$$D_{ij} = \sum_{k=1}^{N} \int\limits_{z_{k-1}}^{z_k} (\overline{Q}_{ij})_k z^2\, dz \qquad (i,j = 1,2,6). \tag{2.9}$$

Electric and thermal moment resultants are given by

$$\begin{Bmatrix} M^E_{xx} \\ M^E_{yy} \\ M^E_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int\limits_{z_{k-1}}^{z_k} \begin{Bmatrix} \overline{e}_{31} \\ \overline{e}_{32} \\ \overline{e}_{36} \end{Bmatrix}_k (E_z)_k z\, dz, \tag{2.10}$$

$$\begin{Bmatrix} M^\theta_{xx} \\ M^\theta_{yy} \\ M^\theta_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int\limits_{z_{k-1}}^{z_k} \begin{Bmatrix} \overline{\lambda}_1 \\ \overline{\lambda}_2 \\ \overline{\lambda}_6 \end{Bmatrix}_k \theta z\, dz. \tag{2.11}$$

Combining the results of Eqs. (2.9)–(2.11), the moment resultants (M_{xx}, M_{yy}, M_{xy}) can be written as

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = -\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} - \begin{Bmatrix} M^E_{xx} \\ M^E_{yy} \\ M^E_{xy} \end{Bmatrix} - \begin{Bmatrix} M^\theta_{xx} \\ M^\theta_{yy} \\ M^\theta_{xy} \end{Bmatrix}. \tag{2.12}$$

Note that M^E_{xy}, M^θ_{xy} and D_{16}, D_{26} are identically zero for the cross-ply construction, since the coefficients $\overline{e}_{36}$, $\overline{\lambda}_6$ in Eqs. (2.10), (2.11) and $\overline{Q}_{16}$, $\overline{Q}_{26}$ in Eq. (2.9) are zero for ply angles of 0° or 90°.

The usual plate equilibrium conditions in the case of zero mechanical loading are

$$\begin{aligned} M_{xx,x} + M_{xy,y} - Q_x &= 0, \\ M_{yx,x} + M_{yy,y} - Q_y &= 0, \end{aligned} \tag{2.13}$$

$$Q_{x,x} + Q_{y,y} = 0, \tag{2.14}$$

where Q_x and Q_y are the vertical shear forces. Substituting Eq. (2.12) into Eqs. (2.13), (2.14) yields

$$Q_x = -\left[D_{11}w_{,xxx} + D_{26}w_{,yyy} + (D_{12} + 2D_{66})w_{,xyy} + 3D_{16}w_{,xxy}\right] - \left[M^E_{xx,x} + M^E_{xy,y} + M^\theta_{xx,x} + M^\theta_{xy,y}\right], \tag{2.15}$$

$$Q_y = -\left[D_{16}w_{,xxx} + D_{22}w_{,yyy} + 3D_{26}w_{,xyy} + (D_{12} + 2D_{66})w_{,xxy}\right] - \left[M^E_{xy,x} + M^E_{yy,y} + M^\theta_{xy,x} + M^\theta_{yy,y}\right], \tag{2.16}$$

$$D_{11}w_{,xxxx} + 2D_{12}w_{,xxyy} + 4D_{16}w_{,xxxy} + D_{22}w_{,yyyy} + 4D_{26}w_{,xyyy} + 4D_{66}w_{,xxyy} + (M^E_{xx,xx} + M^E_{yy,yy} + 2M^E_{xy,xy} + M^\theta_{xx,xx} + M^\theta_{yy,yy} + 2M^\theta_{xy,xy}) = 0. \tag{2.17}$$

Assuming a symmetric cross-ply panel having angles of 0° or 90°, the governing equation (2.17) simplifies to

$$D_{11}w_{,xxxx} + 2D_{12}w_{,xxyy} + D_{22}w_{,yyyy} + 4D_{66}w_{,xxyy} + M^E_{xx,xx} + M^E_{yy,yy} + M^\theta_{xx,xx} + M^\theta_{yy,yy} = 0. \tag{2.18}$$

The hybrid laminate with a through crack is bent by uniform moments of intensity M_0 at infinity and is subjected to an applied uniform electric field $E_z = E_0$ in addition to the upper and lower surface temperatures $\theta = -T_0$, T_0. The plate is subjected to the linear temperature variation

$$\theta(z) = \frac{T_0}{h}z\,. \tag{2.19}$$

Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0 \leq x < \infty$, $0 \leq y < \infty$ only. The boundary conditions can be written as

$$V_y = M_{xy,x} + Q_y = 0 \qquad (y = 0, \quad 0 \leq x < \infty), \tag{2.20}$$

$$\begin{aligned} M_{yy} &= 0 \qquad (y = 0, \quad 0 \leq x < c), \\ u_y &= 0 \qquad (y = 0, \quad c \leq x < \infty), \end{aligned} \tag{2.21}$$

where V_y is the equivalent shear.

3. Solution procedure

We assume that the solution w is of the form

$$w = \frac{D_{12} - D_{22}}{2(D_{11}D_{22} - D_{12}^2)}M_0x^2 + \frac{D_{12} - D_{11}}{2(D_{11}D_{22} - D_{12}^2)}M_0y^2 + \frac{2}{\pi}\int_0^\infty \left[A_1(s)e^{-s\gamma_1 y} + A_2(s)e^{-s\gamma_2 y}\right]\cos(sx)\,ds, \tag{3.1}$$

where $A_1(s)$ and $A_2(s)$ are the unknown functions to be determined later, and γ_1 and γ_2 are

$$\gamma_1 = \left\{ \frac{(D_{12} + 2D_{66}) + (D_{12}^2 + 4D_{12}D_{66} + 4D_{66} - D_{11}D_{22})^{1/2}}{D_{22}} \right\}^{1/2}, \tag{3.2}$$

$$\gamma_1 = \left\{ \frac{(D_{12} + 2D_{66}) - (D_{12}^2 + 4D_{12}D_{66} + 4D_{66} - D_{11}D_{22})^{1/2}}{D_{22}} \right\}^{1/2}. \tag{3.3}$$

The boundary condition of Eq. (2.20) leads to the following relation between unknown functions:

$$\gamma_1 \left[D_{12} + 4D_{66} - D_{22}\gamma_1^2 \right] A_1(s) + \gamma_2 \left[D_{12} + 4D_{66} - D_{22}\gamma_2^2 \right] A_2(s) = 0. \tag{3.4}$$

Application of the boundary conditions (2.21) gives rise to a pair of dual integral equations:

$$\begin{aligned} C \int_0^\infty sA(s)\cos(sx)\,ds &= \frac{\pi}{2}(M_0 - M_{yy}^E - M_{yy}^\theta) \qquad (0 \le x < c), \\ \int_0^\infty A(s)\cos(sx)\,ds &= 0 \qquad (c \le x < \infty), \end{aligned} \tag{3.5}$$

in which C, M_{yy}^E and M_{yy}^θ are known as

$$\begin{aligned} C = (D_{22}\gamma_1^2 - D_{12}) & \frac{4D_{66} + D_{12} - D_{22}\gamma_2^2}{\gamma_1 D_{22}(\gamma_1^2 - \gamma_2^2)} \\ & - (D_{22}\gamma_2^2 - D_{12}) \frac{4D_{66} + D_{12} - D_{22}\gamma_1^2}{\gamma_2 D_{22}(\gamma_1^2 - \gamma_2^2)}, \end{aligned} \tag{3.6}$$

$$M_{yy}^E = \sum_{k=1}^N \frac{(\overline{e}_{32})_k}{2} E_0(z_k^2 - z_{k-1}^2), \tag{3.7}$$

$$M_{yy}^\theta = \sum_{k=1}^N \frac{(\overline{\lambda}_2)_k}{3h} T_0(z_k^3 - z_{k-1}^3). \tag{3.8}$$

The unknown $A(s)$ is related to $A_j(s)$ $(j = 1, 2)$ as follows:

$$A(s) = s\left[\gamma_1 A_1(s) + \gamma_2 A_2(s)\right]. \tag{3.9}$$

The set of dual integral equations (3.5) may be solved by using a new function $\Phi(\xi)$ defined by

$$A(s) = \frac{\pi}{2}\frac{c^2}{C} \int_0^1 \xi^{1/2}\Phi(\xi) J_0(cs\xi)\,d\xi, \tag{3.10}$$

where $J_0(\ )$ is the zero-order Bessel function of the first kind. Having satisfied Eq. (3.5) for $c \le x < \infty$, the remaining condition for $0 \le x < c$ leads to an Abel integral equation for $\Phi(\xi)$. The solution for $\Phi(\xi)$ is expressed by

$$\Phi(\xi) = \left(M_0 - M_{yy}^E - M_{yy}^\theta\right)\xi^{1/2}. \tag{3.11}$$

The moment intensity factor is obtained as

$$K_I = \lim_{x \to c^+} \{2\pi(x-c)\}^{1/2} M_{yy}(x,0) = M_0(\pi c)^{1/2}\left(1 - \frac{M_{yy}^E + M_{yy}^\theta}{M_0}\right). \tag{3.12}$$

4. Numerical results and discussion

The thermoelastic response of a cracked piezoelectric laminated plate subjected to mechanical, thermal and electric field loading is considered. The hybrid laminate chosen is a graphite/epoxy composite with a symmetric construction of $[0°/90°/0°/90°]_s$ where $[\ ]_s$ denotes symmetry about the middle surface. Each graphite/epoxy lamina is of constant thickness. Two double thick layers of polyvinylidene fluoride (PVDF), piezoelectric polymers poled in $\mp z$-direction, are added to the upper and lower surfaces to make a ten-layer hybrid composite structure. Material and geometric properties for the graphite/epoxy lamina and the PVDF layer are given in Table 1.

Table 1. Properties of graphite/epoxy and PVDF.

Graphite/epoxy
$E_1 = 181\,\text{GPa}, \ E_2 = 10.3\,\text{GPa}$
$G_{12} = 7.17\,\text{GPa}$
$\nu_{12} = 0.28$
$\alpha_1 = 0.02 \times 10^{-6}\,1/\text{K}, \ \alpha_2 = 22.5 \times 10^{-6}\,1/\text{K}$
$h_{GE} = h_k = 1.25 \times 10^{-4}\,\text{m} \ \ (k = 2, ..., 9)$
$\varrho_{GE} = \varrho_k = 1580\,\text{kg/m}^3 \ \ (k = 2, ..., 9)$
Polyvinylidene fluoride (PVDF)
$E_1 = E_2 = 2\,\text{GPa}, \ G_{12} = 0.752\,\text{GPa}, \ \nu_{12} = 0.33$
$\alpha_1 = \alpha_2 = 120 \times 10^{-6}\,1/\text{K}$
$d_{31} = d_{32} = 23 \times 10^{-12}\,\text{m/V}$
$h_P = h_1 = h_{10} = 2.5 \times 10^{-4}\,\text{m}$
$\varrho_P = \varrho_1 = \varrho_{10} = 1800\,\text{kg/m}^3$

Figure 2 exhibits the variation of the normalized moment intensity factor $|K_I/M_0(\pi c)^{1/2}|$ against the electric field E_0 for $M_0 = 5\,\text{Nm/m}$ and $T_0 = 40°\,\text{C}$.

The existence of the electric field $E_z = E_0$ produces smaller values of the moment intensity factor.

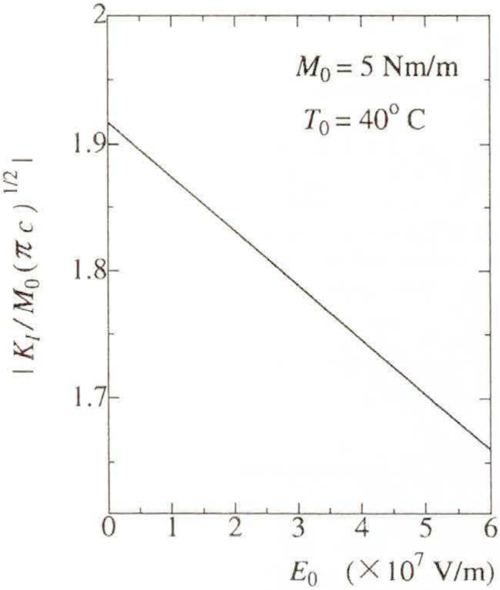

FIG. 2. Moment intensity factor $|K_I/M_0(\pi c)^{1/2}|$ versus E_0.

Only the converse piezoelectric effect has been considered here, whereby an electric field is applied to piezoelectric layers in order to suppress the structure's overall deformation and singular moment near the crack tip. However, advantage can also be taken of the direct piezoelectric effect by employing another piezoelectric layer as a sensor. By coupling the two effects with appropriate feedback control, a smart structure can be achieved [1]. Work in this area is currently being pursued.

5. Conclusions

The response of a cracked composite plate with attached piezoelectric polyvinylidene fluoride layers under mechanical, thermal, and electrical field loading has been analyzed theoretically. Classical lamination theory including piezothermoelastic effects is applied and the results are expressed in terms of the moment intensity factor. The moment intensity factor decreases with the increase of the electric field. The results presented demonstrate the feasibility of suppressing thermomechanically induced flexure and singular moment near the crack tip via the piezothermoelastic effects.

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