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# Magnetohydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface

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THE PROBLEM of the boundary layer flow and heat transfer on a continuous moving wavy surface in a quiescent electrically conducting fluid with a constant transverse magnetic field is formulated. The resulting parabolic differential equations are solved numerically using the Keller-box scheme. Detailed results for the velocity and temperature fields are presented, and also the results for the skin-friction coefficient and the local Nusselt number. These results are given for different values of the amplitude of the wavy surface and magnetic parameter when the Prandtl number equals 0.7. It is shown that the flow and heat transfer characteristics are substantially altered by both the magnetic parameter and the amplitude of the wavy surface.

## 1. Introduction

THE INTERACTION between an electrically conducting fluid and an applied magnetic field is an important practical problem which has been studied very often in relation to the magnetohydrodynamic (MHD) power generator and boundary layer flow control. Hydrodynamic behaviour of boundary layers along a flat plate in the presence of a constant transverse magnetic field was first analysed by ROSSOW [1], who assumed that magnetic Reynolds number was so small that the induced magnetic field could be ignored. This problem has been further investigated by many researchers, including LEWIS [2], KATAGIRI [3], LIRON and WILHELM [4], CHUANG [5], INGHAM [6], PATHAK and CHOUDHARY [7], SOUNDALGEKAR *et al.* [8], WATANABE [9], and WATANABE and POP [10], among others.

The purpose of this paper is to study the MHD boundary layer flow and heat transfer over a continuous moving wavy surface in an electrically conducting fluid at rest, in the presence of a constant transverse magnetic field. The transformed nonsimilar boundary layer equations were solved numerically using the Keller-box method [11] for some values of the amplitude of the wavy surface $a$, and magnetic parameter $M$ with the Prandtl number Pr equal 0.7. We have studied the effect of the parameters $a$ and $M$ on the velocity and temperature fields, as well as on the skin-friction coefficient and the local Nusselt number. We expect that the physical insight gained in this paper will enable the understanding of the complex situations where boundary layer approximation is not made.

It is worth pointing out that the MHD flow and heat transfer over a wavy surface is of importance in several heat transfer collectors where the presence of roughness elements disturbs the flow past surfaces and alters the heat transfer

rate. On the other hand, a continuously moving surface in an electrically conducting fluid permeated by a uniform transverse magnetic field has many practical applications in manufacturing metallurgical processes involving the cooling of continuous strips or filaments by drawing them through a quiescent fluid. Mention may be made of drawing, annealing and tinning of coper wires. In all these cases the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final products of desired characteristics might be achieved. Another interesting application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field.

## 2. Basic equations

Consider a wavy surface at wall temperature $T_w$ moving tangentially from left to right with a constant velocity $U$ through a stagnant electrically conducting fluid of temperature $T_\infty$, where $T_w > T_\infty$. The wavy surface is electrically insulated and a constant magnetic field $B_0$ normal to the surface is imposed. The geometry and the coordinate system, which is fixed in space, are illustrated in Fig. 1. The wavy surface is described by

$$\bar{y} = \bar{S}(\bar{x}) = \bar{a}\sin(\pi\bar{x}/l), \tag{2.1}$$

where $\bar{a}$ is the amplitude of the wavy surface and $l$ is the characteristic length scale associated with the waves. In the present analysis the magnetic Reynolds number is assumed to be small and therefore, the induced magnetic field will be very small and can be neglected compared to the applied field. Under this approximation,

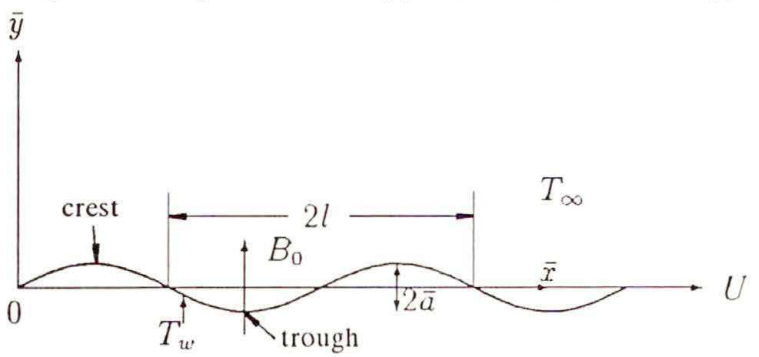


FIG. 1. Physical model and coordinate system.

the basic equations governing the steady flow of a viscous incompressible and electrically conducting fluid in presence of a uniform transverse magnetic field

are

$$
\begin{aligned}
&\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0,\\
&\overline{u}\frac{\partial \overline{u}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} = -\frac{1}{\varrho}\frac{\partial \overline{p}}{\partial \overline{x}} + \nu\nabla^2\overline{u} - \frac{\sigma_0 B_0^2}{\varrho}\overline{u},\\
&\overline{u}\frac{\partial \overline{v}}{\partial \overline{x}} + \overline{v}\frac{\partial \overline{v}}{\partial \overline{y}} = -\frac{1}{\varrho}\frac{\partial \overline{p}}{\partial \overline{y}} + \nu\nabla^2\overline{v},\\
&\overline{u}\frac{\partial T}{\partial \overline{x}} + \overline{v}\frac{\partial T}{\partial \overline{y}} = \frac{\nu}{\mathrm{Pr}}\nabla^2 T,
\end{aligned}
\tag{2.2}
$$

where $\overline{u}$ and $\overline{v}$ are the components of velocity along the $\overline{x}$- and $\overline{y}$-directions, respectively, $T$ is the temperature, $\overline{p}$ is the pressure, $\varrho$, $\nu$ and $\sigma_0$ are the density, kinematic viscosity and electric conductivity of the fluid, and $\nabla^2$ is the Laplacian expressed in Cartesian coordinates.

The appropriate boundary conditions for the above equations are

$$
\begin{aligned}
&\overline{y} = \overline{S}(\overline{x}): \quad \overline{u}t_{\overline{y}} - \overline{v}t_{\overline{x}} = 0, \quad \overline{u}t_{\overline{x}} + \overline{v}t_{\overline{y}} = U, \quad T = T_w, \quad \text{all} \quad \overline{x} > 0,\\
&\overline{y} \to \infty: \quad \overline{u} = \overline{v} = 0, \quad \overline{p} = p_\infty, \quad T = T_\infty, \quad \text{all} \quad \overline{x} \geq 0,\\
&\overline{x} = 0: \quad \overline{p} = p_\infty, \quad T = T_\infty, \quad \text{all} \quad \overline{y} \neq 0,
\end{aligned}
\tag{2.3}
$$

where $t_{\overline{x}}$ and $t_{\overline{y}}$ are the components of the unit vector tangent to the wavy surface along $(\overline{x}, \overline{y})$-directions.

Equations (2.2) may now be nondimensionalized by using the following variables

$$
\begin{aligned}
&x = \overline{x}/l, \quad y = \overline{y}/l, \quad u = \overline{u}/U, \quad v = \overline{v}/U,\\
&p = (\overline{p} - p_\infty)/\varrho U^2, \quad \theta = (T - T_\infty)/\Delta T, \quad a = \overline{a}/l, \quad S(x) = \overline{S}(\overline{x})/l,
\end{aligned}
\tag{2.4}
$$

where $\Delta T = T_w - T_\infty$. Using these variables and introducing the nondimensional stream function $\psi$ defined as

$$
u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x},
\tag{2.5}
$$

Eq. (2.5) can then be written as

$$
\begin{aligned}
&\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\nabla^2\left(\frac{\partial \psi}{\partial y}\right) - M\frac{\partial \psi}{\partial y},\\
&\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial x \partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\nabla^2\left(\frac{\partial \psi}{\partial x y}\right),\\
&\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y} = -\frac{1}{\mathrm{Pr}}\frac{1}{\mathrm{Re}}\nabla^2\theta.
\end{aligned}
\tag{2.6}
$$

Also, the boundary conditions (2.3) become

$$
\begin{aligned}
&y = S(x): \quad S_x\frac{\partial\psi}{\partial y} + \frac{\partial\psi}{\partial x} = 0, \quad \frac{\partial\psi}{\partial y} - S_x\frac{\partial\psi}{\partial x} = \sigma, \quad \theta = 1, \quad \text{all} \quad x > 0,\\
&y \to \infty: \quad \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial x} = 0, \quad p = 0, \quad \theta = 0, \quad \text{all} \quad x \geq 0,\\
&x = 0: \quad p = 0, \quad \theta = 0, \quad \text{all} \quad y \neq 0;
\end{aligned} \tag{2.7}
$$

here $\sigma$ is defined according to

$$
\sigma = \left(1 + S_x^2\right)^{1/2} \tag{2.8}
$$

with $S_x = dS/dx$. Here $\mathrm{Re} = Ul/\nu$ is the Reynolds number and $M = \sigma_0 B_0^2 l/\varrho U$ is the magnetic field parameter. We notice that $(t_x, t_y) = (1/\sigma, S_x/\sigma)$ were used in (2.7). It should be noted that the value $\sigma = 1$, i.e. $a = 0$, corresponds to the case of a flat surface. In this case we take for $l$ a characteristic length $L$ along the flat surface.

The effect of the wavy undulations can be transferred from the boundary conditions (2.7) to the governing equations by means of the transformation given by (see REES and POP [12, 13]),

$$
\hat{x} = x, \qquad \hat{y} = y - S(x). \tag{2.9}
$$

Applying (2.9) to Eqs. (2.6) and dropping the hat we get the following equations:

$$
\begin{aligned}
&\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = -\frac{\partial p}{\partial x} - S_x\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}L_1\psi - M\frac{\partial\psi}{\partial y},\\
&\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y} + S_x\left(\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}\right) - S_{xx}\left(\frac{\partial\psi}{\partial y}\right)^2\\
&\qquad = \frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}L_2\psi,\\
&\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{1}{\mathrm{Re}}L_3\theta,
\end{aligned} \tag{2.10}
$$

and the boundary conditions (2.7) become

$$
\begin{aligned}
&y = 0: \quad \psi = 0, \quad \frac{\partial\psi}{\partial y} = 1/\sigma, \quad \theta = 1, \quad \text{ali} \quad x > 0,\\
&y \to \infty: \quad \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial x} = 0, \quad p = 0, \quad \theta = 0, \quad \text{all} \quad x \geq 0,\\
&x = 0: \quad p = 0, \quad \theta = 0, \quad \text{all} \quad y \neq 0,
\end{aligned} \tag{2.11}
$$

where $L_1$, $L_2$ and $L_3$ are the operators defined by

$$
\begin{aligned}
L_1 &= \sigma^2\frac{\partial^3}{\partial y^3} + \frac{\partial^3}{\partial y\partial x^2} - 2S_x\frac{\partial^3}{\partial x\partial y^2} - S_{xx}\frac{\partial^2}{\partial y^2}, \\
L_2 &= -S_x\sigma^2\frac{\partial^3}{\partial y^3} + (1+3S_x^2)\frac{\partial^3}{\partial x\partial y^2} - 3S_x\frac{\partial^3}{\partial y\partial x^2} \\
&\qquad - 3S_{xx}\frac{\partial^2}{\partial x\partial y} + 3S_xS_{xx}\frac{\partial^2}{\partial y^2} - S_{xxx}\frac{\partial}{\partial y} + \frac{\partial^3}{\partial x^3}, \\
L_3 &= \sigma^2\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} - 2S_x\frac{\partial^2}{\partial x\partial y} - S_{xx}\frac{\partial}{\partial y}.
\end{aligned}
\tag{2.12}
$$

Next, we introduce the boundary layer variables

$$
\tilde{x} = x, \quad \tilde{y} = \sqrt{\mathrm{Re}}\,y, \quad \tilde{\psi} = \sqrt{\mathrm{Re}}\,\psi, \quad \tilde{p} = p, \quad \tilde{\theta} = \theta. \tag{2.13}
$$

Substituting (2.13) into (2.10) and formally letting $\mathrm{Re} \to \infty$, we obtain, after dropping the tilde,

$$
\begin{aligned}
&\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = -\frac{\partial p}{\partial x} + \mathrm{Re}^{1/2}S_x\frac{\partial p}{\partial y} + \sigma^2\frac{\partial^3\psi}{\partial y^3} - M\frac{\partial\psi}{\partial y}, \\
&S_x\left(\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}\right) - S_{xx}\left(\frac{\partial\psi}{\partial y}\right)^2 + S_x\sigma^2\frac{\partial^3\psi}{\partial y^3} = \mathrm{Re}^{1/2}\frac{\partial p}{\partial y}, \\
&\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = \frac{1}{\mathrm{Pr}}\sigma^2\frac{\partial^2\theta}{\partial y^2}.
\end{aligned}
\tag{2.14}
$$

Equation $(2.14)_2$ indicates that the pressure gradient in the $y$-direction must be of $O(\mathrm{Re}^{-1/2})$. This implies that the lowest order pressure gradient in the $x$-direction can be determined from the inviscid flow solution. In the present problem, the inviscid flow field is at rest and hence $\partial p/\partial x = 0$. Now, elemination of $\partial p/\partial y$ between $(2.14)_1$ and $(2.14)_2$ results in the following boundary layer equations for the problem under consideration:

$$
\begin{aligned}
&\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} + \frac{\sigma_x}{\sigma}\left(\frac{\partial\psi}{\partial y}\right)^2 = \sigma^2\frac{\partial^3\psi}{\partial y^3} - \frac{M}{\sigma^2}\frac{\partial\psi}{\partial y}, \\
&\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = \frac{1}{\mathrm{Pr}}\sigma^2\frac{\partial^2\theta}{\partial y^2},
\end{aligned}
\tag{2.15}
$$

subject to the corresponding boundary conditions

$$
\begin{aligned}
&y = 0 : \quad \psi = 0, \quad \frac{\partial\psi}{\partial y} = 1/\sigma, \quad \theta = 1, \quad \text{all } x > 0, \\
&y \to \infty : \quad \frac{\partial\psi}{\partial y} = \frac{\partial\psi}{\partial x} = 0, \quad \theta = 0, \quad \text{all } x \geq 0, \\
&x = 0 : \quad \theta = 0, \quad \text{all } y \neq 0.
\end{aligned}
\tag{2.16}
$$

To solve Eqs. (2.15) along with the boundary conditions (2.16), we introduce the following group of transformations:

$$\psi = \sigma\xi^{1/2} f(\xi,\eta), \qquad \theta = g(\xi,\eta), \tag{2.17}$$

where

$$\eta = \frac{y}{\sigma}\xi^{-1/2}, \qquad \xi = x. \tag{2.18}$$

Equations (2.15) then become

$$\begin{aligned} f''' + \frac{1}{2}ff'' + \frac{\sigma_\xi}{\sigma}\xi(ff'' - f'^2) - \frac{M}{\sigma^2}\xi f' &= \xi\left(f'\frac{\partial f'}{\partial\xi} - f''\frac{\partial f}{\partial\xi}\right), \\ \frac{1}{\mathrm{Pr}}g'' + \frac{1}{2}fg' + \frac{\sigma_\xi}{\sigma}\xi fg' &= \xi\left(f'\frac{\partial g}{\partial\xi} - g'\frac{\partial f}{\partial\xi}\right) \end{aligned} \tag{2.19}$$

subject to the boundary conditions

$$\begin{aligned} f(\xi,0) &= 0, \qquad f'(\xi,0) = 1/\sigma, \qquad g(\xi,0) = 1, \\ f'(\xi,\infty) &= h(\xi,\infty) = 0, \qquad g(\xi,\infty) = 0, \end{aligned} \tag{2.20}$$

where primes denote partial differentiation with respect to $\eta$. We notice that Eqs. (2.19) reduce to those derived by REES and POP [14] when there is no applied magnetic field ($M = 0$) in the flow field.

The physical quantities of interest are the skin-friction coefficient and the local Nusselt number defined as

$$C_f = \frac{\overline{\tau}_w}{\varrho U^2}, \qquad \mathrm{Nu}_x = \frac{\overline{x}\,\overline{q}_w}{k\Delta T}, \tag{2.21}$$

where the skin-friction $\overline{\tau}_w$ and the heat flux $\overline{q}_w$ at the wall are given by

$$\overline{\tau}_w = \mu\left(\frac{\partial\overline{u}}{\partial\overline{y}} + \frac{\partial\overline{v}}{\partial\overline{x}}\right)_{\overline{y}=0}, \qquad \overline{q}_w = -k\mathbf{n}\cdot\nabla T. \tag{2.22}$$

Here $\mu$ and $k$ are the viscosity and thermal conductivity of the fluid, and

$$\mathbf{n} = \left(-\frac{S_x}{\sigma}, \frac{1}{\sigma}\right) \tag{2.23}$$

is the unit vector normal to the wavy surface. Using (2.4), (2.9), (2.13) and (2.17), we get the skin-friction coefficient and the local Nusselt number from the following expressions:

$$C_f\mathrm{Re}_x^{1/2} = \frac{1}{\sigma}f''(\xi,0), \qquad \mathrm{Nu}_x/\mathrm{Re}_x^{1/2} = -g'(\xi,0), \tag{2.24}$$

where $\mathrm{Re}_x = U\overline{x}/\nu$ is the local Reynolds number.

## 3. Results and discussion

An implicit finite-difference method together with the Keller-box elimination technique [11] have been used to solve the parabolic differential equations (2.19) along with the boundary condition (2.20). Since a good description of this method is available in [15–17], it will not be repeated here. The accuracy of the predicted

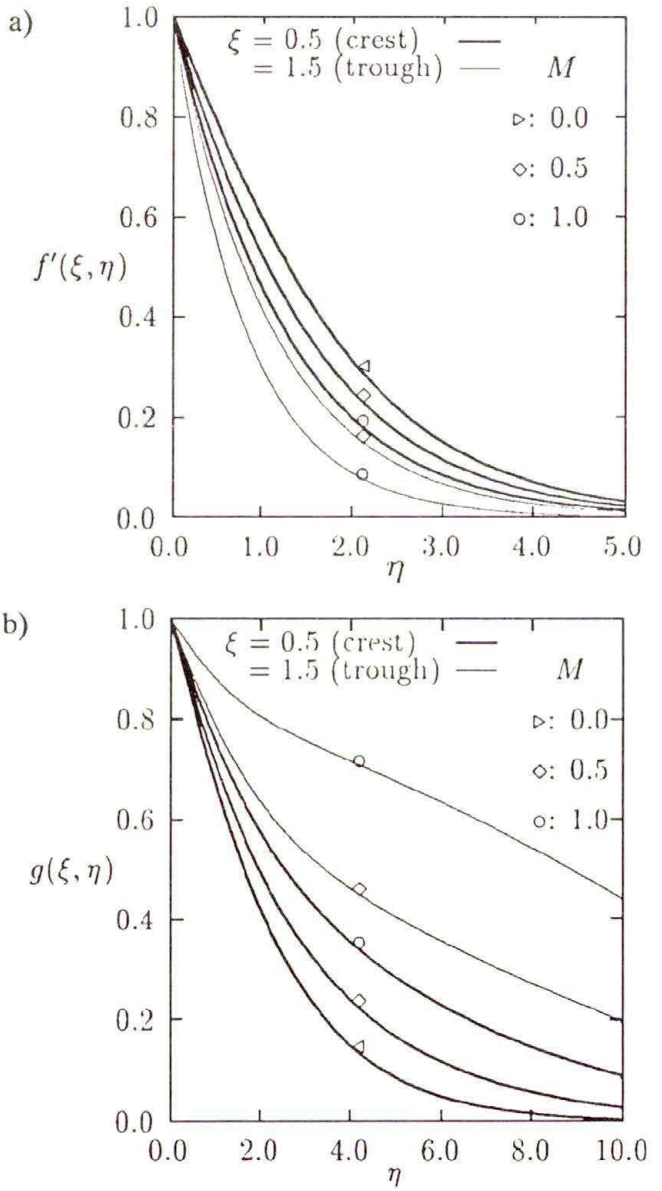


FIG. 2. a) Velocity profiles against $\eta$ for different $M$ with $a = 0.1$; b) temperature profiles against $\eta$ for different $M$ with $a = 0.1$ and Pr = 0.7.

results has been established by comparison with known results for the skin-friction coefficient and the local Nusselt number of a continuously moving flat plate ($a = 0$) in a viscous electrically non-conducting fluid with $M = 0$. Thus, REES and POP [14] found $C_f \mathrm{Re}_x^{1/2} = -0.4438$ and $\mathrm{Nu}_x/\mathrm{Re}_x^{1/2} = -0.3492$ for $\mathrm{Pr} = 0.7$, while the present calculations give $C_f \mathrm{Re}_x^{1/2} = -0.4439$ and $\mathrm{Nu}_x/\mathrm{Re}_x^{1/2} = -0.3509$. It is seen that these results are in excellent agreement and therefore we are confident that our present solution is very accurate.

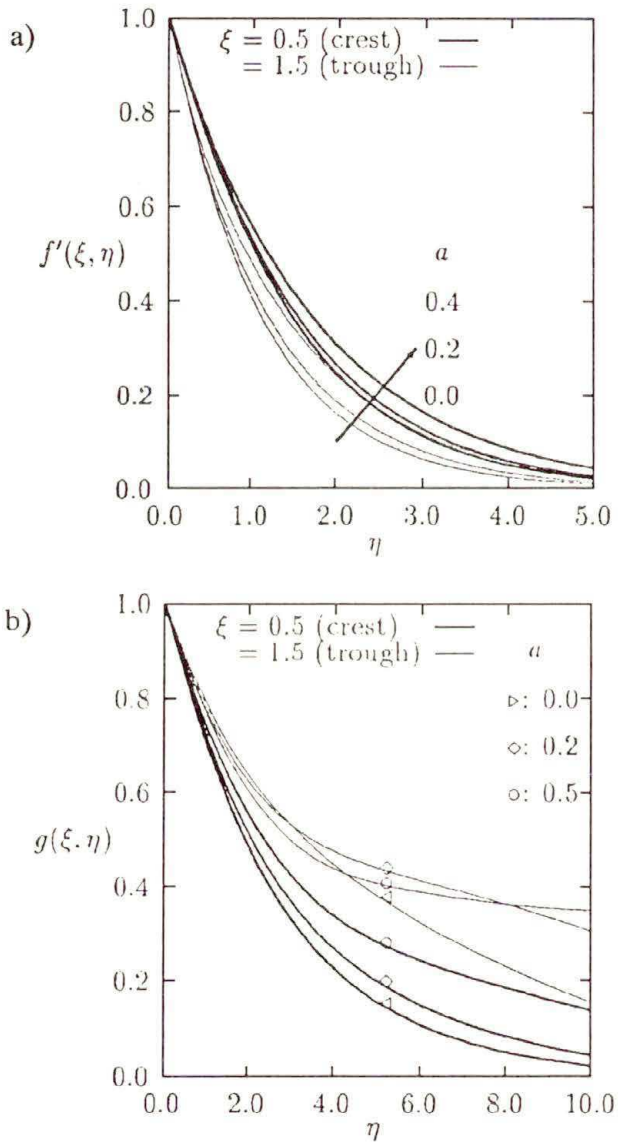


FIG. 3. a) Velocity profiles against $\eta$ for different $a$ with $M = 0.5$; b) temperature profiles against $\eta$ for different $a$ with $M = 0.5$ and $\mathrm{Pr} = 0.7$.

Representative velocity and temperature profiles are shown in Figs. 2 and 3 exhibiting the effects of the wave amplitude $a$ and of the magnetic field parameter $M$. Results are given for Pr = 0.7 only. Then, since behaviour of these profiles at crest and trough positions is very similar, the case of $\xi = 0.5$ (crest) and $\xi = 1.5$ (trough) are only presented in this paper. Figures 2 and 3 show clearly that both the velocity and temperature profiles increase with the increase of $M$. However, Fig. 2 indicates that for $a = 0.1$ and $M = 0$ (non-magnetic field) at both the through and crest positions, the velocity and temperature profiles are almost identical due to which the differences between the thick and thin curves are not observable. But, at a larger value of $a$ (0.5, say), there is a considerable difference at these two positions (trough and crest) in the velocity and temperature profiles for $M = 0$. On the other hand, the velocity profiles decrease, while the temperature profiles increase owing to the increase of the amplitude of the wavy surface.

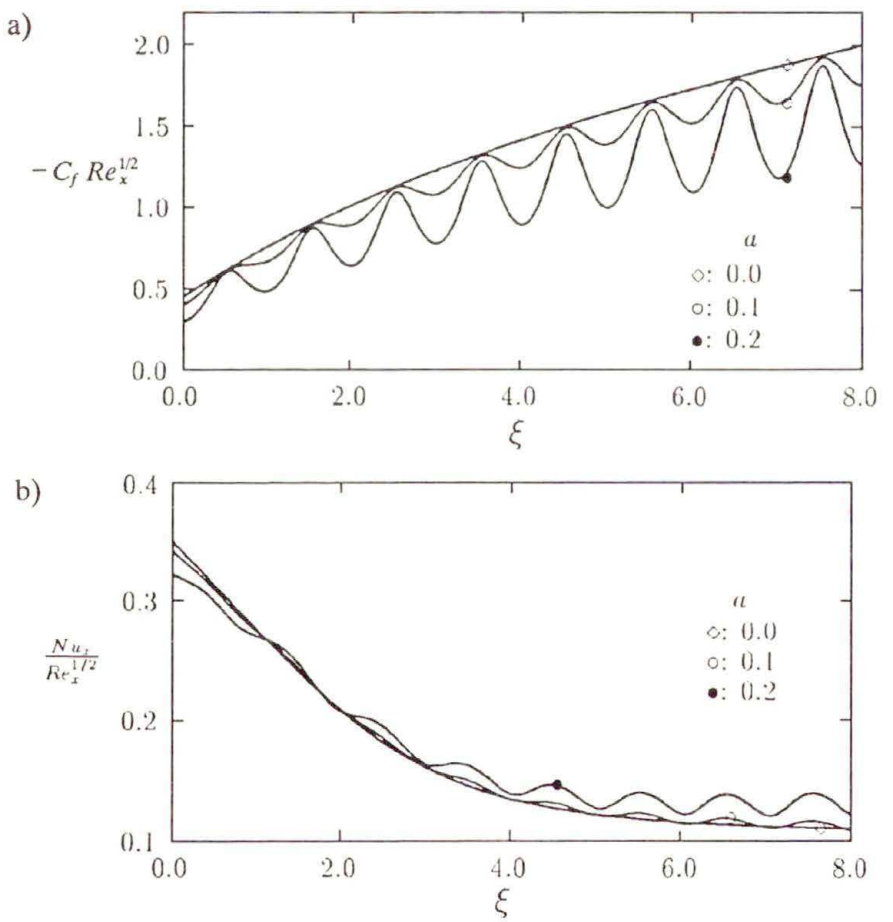


FIG. 4. a) Skin-friction coefficient for different $a$ with $M = 0.5$; b) local Nusselt number for different $a$ with $M = 0.5$ and Pr = 0.7.

In Figs. 4 and 5 the variation with $a$, $M$ and $\xi$ of the skin-friction coefficient and the local Nusselt number is illustrated. It is observed that these quantities vary periodically in the direction of $\xi$ when $a \neq 0$ (wavy surface), while they vary smoothly for $a = 0$ (flat plate). Further, Fig. 4 a shows that the skin-friction coefficient is less than or equal to that corresponding to a flat surface ($a = 0$); this is due to the effect of centrifugal forces, the third term of Eq. $(2.19)_1$.

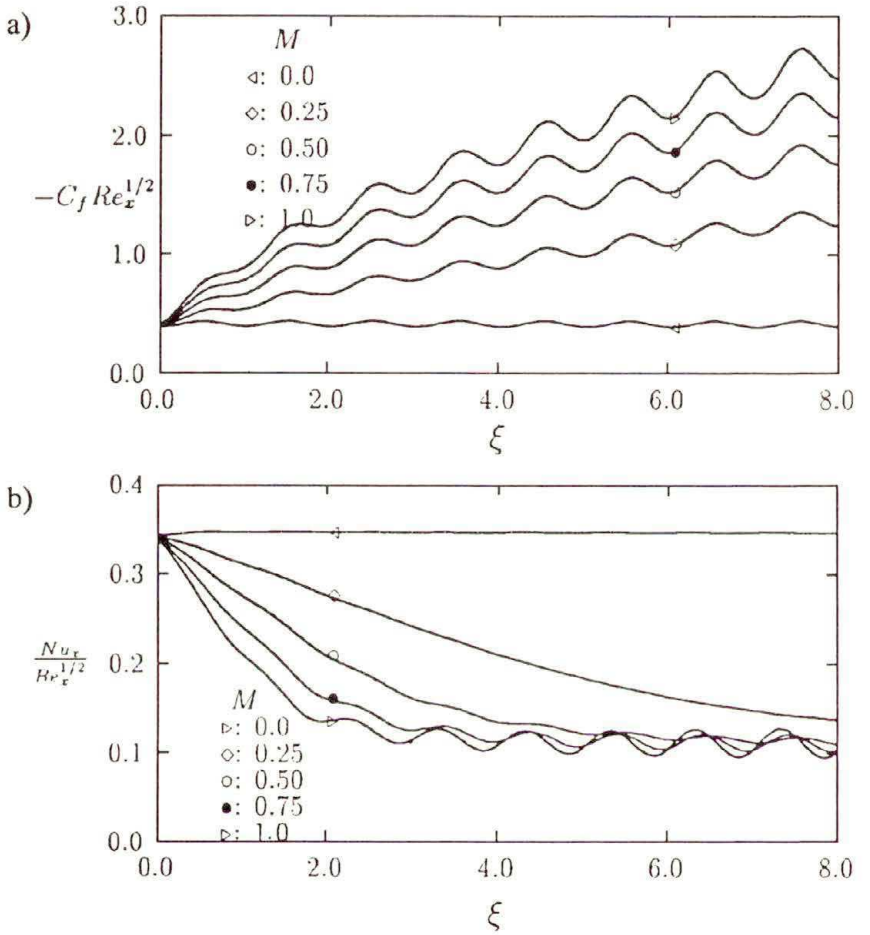


FIG. 5. a) Skin-friction coefficient for different $M$ with $a = 0.1$; b) local Nusselt number for different $M$ with $a = 0.1$ and $\mathrm{Pr} = 0.7$.

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