Double-diffusive convection in compressible fluids with suspended particles in porous medium

R. C. SHARMA (SHIMLA), TRILOK CHAND (NALAGARH) and V. K. BHARDWAJ (SHIMLA)

THE DOUBLE-DIFFUSIVE convection in compressible fluids with suspended particles in porous medium is considered. The suspended particles are found to have destabilizing effect whereas stable solute gradient, rotation and compressibility have stabilizing effect on the system. The medium permeability has a destabilizing effect in the absence of rotation but has both stabilizing and destabilizing effects in the presence of rotation. The stable solute gradient and rotation are found to introduce oscillatory modes in the system which are non-existent in their absence.

1. Introduction

THE PROBLEM of thermosolutal convection in fluids in a porous medium is of importance in geophysics, soil sciences, ground-water hydrology and astrophysics. The development of geothermal power resources holds increased general interest in the study of the properties of convection in porous media. The scientific importance of the field has also increased because hydrothermal circulation is the dominant heat transfer mechanism in the development of young oceanic crust (LISTER [3]). Generally it is accepted that comets consist of a dusty "snowball" of a mixture of frozen gases which, in the process of their journey, changes from solid to gas and *vice-versa*. The physical properties of comets, meteorites and interplanetary dust strongly suggest the importance of porosity in the astrophysical context. A mounting evidence, both theoretical and experimental, suggests that Darcy's equation provides an unsatisfactory description of the hydrodynamic conditions, particularly near the boundaries of a porous medium. BEAVERS *et al.* [10] have experimentally demonstrated the existence of shear within the porous medium near surface, where the porous medium is exposed to a freely flowing fluid, thus forming a zone of shear-induced flow field. The Darcy's equation however, cannot predict the existence of such a boundary zone, since no macroscopic shear term is included in this equation (JOSEPH and TAO [11]). To be mathematically compatible with the Navier – Stokes equations and physically consistent with the experimentally observed boundary shear zone mentioned above, Brinkman proposed the introduction of the term $\frac{\mu}{\varepsilon}\nabla^2\mathbf{V}$ in addition to $-\left(\frac{\mu}{k_1}\right)\mathbf{V}$ in the equations of fluid motion. The elaborate statistical justification of the Brinkman equations has been presented by SAFFMAN [12] and LUNDGREN [13]. STOMMEL and FEDOROV [14] and LINDEN [2] have remarked that the length scales characteristic of double-diffusive convecting layers in the ocean could be sufficiently large for Earth's rotation to

become important in their formation. Moreover, the rotation of the Earth distorts the boundaries of a hexagonal convection cell in a fluid flowing through a porous medium, and the distortion plays an important role in the extraction of energy in the geothermal regions. Brakke [1] explained a double-diffusive instability that occurs when a solution of a slowly diffusing protein is laid over a denser solution of more rapidly diffusing sucrose. Nason et al. [5] found that this instability, which is deleterious to certain biochemical separations, can be suppressed by rotation in the ultracentrifuge. Scanlon and Segel [6] have studied the effect of suspended particles on the onset of thermal convection.

The conditions under which convective motions in double-diffusive convection are important (e.g. in lower parts of the Earth's atmosphere, astrophysics and several geophysical situations) are usually far removed from the consideration of a single component fluid and rigid boundaries and therefore, it is desirable to consider a fluid acted on by solute gradient and free boundaries. The compressibility and suspended particles are important in such situations. Sharma and Sharma [7] and Sharma and Veena Kumari [8] have considered the thermosolutal convection in porous medium under varying assumptions of hydrodynamics and hydromagnetics.

Keeping in mind the importance in geophysics, astrophysics and various applications mentioned above, the thermosolutal convection in compressible fluids with suspended particles in a porous medium, in the absence and presence of a uniform rotation, separately, has been considered in the present paper.

2. Formulation of the problem and perturbation equations

Consider an infinite horizontal, compressible fluid-particle layer of thickness d bounded by the planes $z = 0$ and $z = d$ in a porous medium of porosity ε and permeability k_1. This layer is heated from below and subjected to a stable solute gradient such that steady adverse temperature gradient $\beta(= |dT/dz|)$ and a solute concentration gradient $\beta'(= |dC/dz|)$ are maintained.

Let ϱ, μ, p and $\mathbf{V}(u, v, w)$ denote respectively the density, viscosity, pressure and filter velocity of the pure fluid; $\mathbf{V}_d(\overline{x}, t)$ and $N(\overline{x}, t)$ denote filter velocity and number density of the particles, respectively. If g is acceleration due to gravity, $K = 6\pi\varrho\nu\varepsilon'$ where ε' is the particle radius, $\mathbf{V}_d = (l, r, s)$, $\overline{x} = (x, y, z)$ and $\boldsymbol{\lambda}_1 = (0, 0, 1)$, then the equation of motion and continuity for the fluid are

$$(2.1) \qquad \frac{\varrho}{\varepsilon}\left[\frac{\partial \mathbf{V}}{\partial t} + \frac{1}{\varepsilon}(\mathbf{V}\cdot\nabla)\mathbf{V}\right] = -\nabla p - \varrho g\boldsymbol{\lambda}_1 + \left(\frac{\mu}{\varepsilon}\nabla^2 - \frac{\mu}{k_1}\right)\mathbf{V} + \frac{KN}{\varepsilon}(\mathbf{V}_d - \mathbf{V}),$$

$$(2.2) \qquad \left(\varepsilon\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)\varrho + \varrho\nabla\cdot\mathbf{V} = 0.$$

Since the distances between particles are assumed to be quite large compared with their diameter, the interparticle relations, buoyancy force, Darcian force and

pressure force on the particles are ignored. Therefore the equations of motion and continuity for the particles are

(2.3)

(2.4)

Let c_v, c_p, c_{pt}, T, C and q denote respectively the heat capacity of fluid at constant volume, heat capacity of fluid at constant pressure, heat capacity of particles, temperature, solute concentration and "effective thermal conductivity" of the clean fluid. Let c_v', c_{pt}' and q' denote the analogous solute coefficients. When particles and the fluid are in thermal and solute equilibrium, the equations of heat and solute conduction give

(2.5) $$[\varrho c_v \varepsilon + \varrho_s c_s (1 - \varepsilon)] \frac{\partial T}{\partial t} + \varrho c_v (\mathbf{V} \cdot \nabla) T$$
$$+ mN c_{pt} \left(\varepsilon \frac{\partial}{\partial t} + \mathbf{V}_d \cdot \nabla \right) T = q \nabla^2 T,$$

(2.6) $$[\varrho c_v' \varepsilon + \varrho_s c_s' (1 - \varepsilon)] \frac{\partial C}{\partial t} + \varrho c_v' (\mathbf{V} \cdot \nabla) C$$
$$+ mN c_{pt}' \left(\varepsilon \frac{\partial}{\partial t} + \mathbf{V}_d \cdot \nabla \right) C = q' \nabla^2 C,$$

where ϱ_s, c_s are the density and heat capacity of the solid matrix, respectively.

SPIEGEL and VERONIS [9] have expressed any state variable (pressure, density or temperature), say X, in the form

$$X = X_m + X_0(z) + X'(x, y, z, t),$$

where X_m stands for the constant space distribution of X, X_0 is the variation in X in the absence of motion, and $X'(x, y, z, t)$ stands for the fluctuations in X due to the motion of the fluid. Following SPIEGEL and VERONIS [9], we have

$$T(z) = -\beta z + T_0,$$
$$p(z) = p_m - g \int_0^z (\varrho_m + \varrho_0) \, dz,$$
$$\varrho(x) = \varrho_m \left[1 - \alpha(T - T_m) + \alpha'(C - C_m) + \alpha''(p - p_m) \right],$$
$$\alpha = -\left(\frac{1}{\varrho} \frac{\partial \varrho}{\partial T} \right), \qquad \alpha' = \left(\frac{1}{\varrho} \frac{\partial \varrho}{\partial C} \right), \qquad \alpha'' = \left(\frac{1}{\varrho} \frac{\partial \varrho}{\partial p} \right).$$

Thus p_m, ϱ_m stand for the constant space distribution of p and ϱ and T_0, ϱ_0 stand for the temperature and density of the fluid at the lower boundary (and in the absence of motion).

Since density variations are mainly due to variations in temperature and solute concentration, Eqs. (2.1)–(2.6) must be supplemented by the equation of state

$$(2.6') \qquad \varrho(z) = \varrho_m \left[1 - \alpha(T - T_m) + \alpha'(C - C_m) \right].$$

Let $\delta\varrho$, δp, θ, γ, $\mathbf{V}$, $\mathbf{V}_d$ and N denote the perturbations in fluid density ϱ, pressure p, temperature T, solute concentration C, fluid velocity $(0,0,0)$, particles velocity $(0,0,0)$ and particle number density N_0, respectively. Then the linearized perturbation equations, under the Spiegel and Veronis assumptions, are

$$(2.7)$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} = -\frac{1}{\varrho_m}\nabla\delta p - g\left(\frac{\delta\varrho}{\varrho_m}\right)\boldsymbol{\lambda}_1 + \left(\frac{\nu}{\varepsilon}\nabla^2 - \frac{\nu}{k_1}\right)\mathbf{V} + \frac{KN_0}{\varrho_m\varepsilon}(\mathbf{V}_d - \mathbf{V}),$$

$$\nabla\cdot\mathbf{V} = 0,$$

$$mN_0\frac{\partial \mathbf{V}_d}{\partial t} = KN_0(\mathbf{V} - \mathbf{V}_d),$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla\cdot(N_0\mathbf{V}_d) = 0,$$

$$(E + h\varepsilon)\frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)(w + hs) + \kappa\nabla^2\theta,$$

$$(E' + h'\varepsilon)\frac{\partial \gamma}{\partial t} = \beta'(w + h's) + \kappa\nabla^2\gamma.$$

Here

$$E = \varepsilon + (1 - \varepsilon)\frac{\varrho_s c_s}{\varrho_m c_v}, \qquad E' = \varepsilon + (1 - \varepsilon)\frac{\varrho_s c'_s}{\varrho_m c'_v},$$

$$h = f\frac{c_{pt}}{c_v}, \qquad h' = f\frac{c'_{pt}}{c'_v}, \qquad f = \frac{mN_0}{\varrho_m}, \qquad \kappa = \frac{q}{\varrho_m c_v}, \qquad \kappa' = \frac{q'}{\varrho_m c'_v}$$

and

$$\delta\varrho = -\varrho_m(\alpha\theta - \alpha'\gamma).$$

Using d, d^2/κ, κ/d, $\varrho\nu\kappa/d^2$, βd and $\beta' d$ to denote the length, time, velocity, pressure, temperature and solute concentration scale factors, respectively, the linearized dimensionless perturbation equations become

$$(2.8) \qquad p_1^{-1}\frac{\partial \mathbf{V}^*}{\partial t^*} = -\nabla^*\delta p^* + R\theta^*\boldsymbol{\lambda}_1 - S\gamma^*\boldsymbol{\lambda}_1 + \left(\frac{1}{\varepsilon}\nabla^{*2} - \frac{1}{P}\right)\mathbf{V}^* + \omega(\mathbf{V}_d^* - \mathbf{V}^*),$$

$$(2.9) \qquad \nabla^*\cdot\mathbf{V}^* = 0,$$

$$(2.10) \qquad \left(\tau\frac{\partial}{\partial t^*} + 1\right)\mathbf{V}_d^* = \mathbf{V}^*,$$

$$(2.11) \qquad \left(\frac{\partial M}{\partial t^*} + \nabla^*\cdot\mathbf{V}_d^*\right) = 0,$$

(2.12)
$$(E + h\varepsilon)\frac{\partial \theta^*}{\partial t^*} = \frac{G-1}{G}(w^* + hs^*) + \nabla^{*2}\theta^*,$$

$$(E' + h'\varepsilon)\frac{\partial \gamma^*}{\partial t^*} = (w^* + h's^*) + \frac{1}{\lambda}\nabla^{*2}\gamma^*,$$

where

$$P = \frac{k_1}{d^2}, \qquad G = \frac{c_p\beta}{g}, \qquad p_1 = \frac{\varepsilon\nu}{\kappa}, \qquad R = \frac{g\alpha\beta d^4}{\nu\kappa}, \qquad S = \frac{g\alpha'\beta'd^4}{\nu\kappa'},$$

$$M = \frac{\varepsilon N}{N_0}, \qquad \omega = \frac{KN_0 d^2}{\varrho_m \nu\varepsilon}, \qquad \tau = \frac{m\kappa}{Kd^2}, \qquad f = \frac{mN_0}{\varrho_m} = \tau\omega p \quad \text{and} \quad \lambda = \frac{\kappa}{\kappa'},$$

and starred (∗) quantities are expressed in dimensionless form. Hereafter, we suppress the stars for convenience.

Eliminating V_d from Eq. (2.8) with the help of (2.10) and then eliminating u, v, δp from the three scalar equations of (2.8), and using (2.9), we obtain

(2.13)
$$\left[L_1 - L_2\left(\frac{1}{\varepsilon}\nabla^2 - \frac{1}{p}\right)\right]\nabla^2 w = L_2(R\nabla_1^2\theta - S\nabla_1^2\gamma),$$

$$L_2\left[(E + h\varepsilon)\frac{\partial}{\partial t} - \nabla^2\right]\theta = \left(\frac{G-1}{G}\right)\left(\tau\frac{\partial}{\partial t} + H\right)w,$$

$$L_2\left[(E' + h'\varepsilon)\frac{\partial}{\partial t} - \frac{1}{\lambda}\nabla^2\right]\gamma = \left(\tau\frac{\partial}{\partial t} + H'\right)w,$$

where

$$L_1 = p_1^{-1}\left(\tau\frac{\partial^2}{\partial t^2} + F\frac{\partial}{\partial t}\right), \qquad L_2 = \left(\tau\frac{\partial}{\partial t} + 1\right), \qquad \nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \qquad F = f + 1, \qquad H = h + 1, \qquad H' = h' + 1.$$

Decomposing the perturbations into normal modes by seeking solutions in the form of functions of x, y and t

(2.14) $[w, \theta, \gamma] = [W(z), \Theta(z), \Gamma(z)]\exp(ik_x x + ik_y y + nt),$

where n is, in general, complex, and $k = (k_x^2 + k_y^2)^{1/2}$ is the wave number of disturbance.

Eliminating θ, γ between Eqs. (2.13) and using expression (2.14), we obtain

(2.15) $$\left[L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}(D^2 - k^2)\right]\left[D^2 - k^2 - n(E + h\varepsilon)\right]$$

$$\cdot \left[D^2 - k^2 - \lambda n(E' + h'\varepsilon)\right](D^2 - k^2)W$$

$$= \left(\frac{G-1}{G}\right)(\tau n + H)Rk^2\left[D^2 - k^2 - \lambda n(E' + h'\varepsilon)\right]W$$

$$- \lambda(\tau n + H')Sk^2\left[D^2 - k^2 - n(E + h\varepsilon)\right]W,$$

where

$$L_1 = p_1^{-1}(\tau n^2 + Fn),$$

$$L_2 = \tau n + 1 \quad \text{and} \quad D = \frac{d}{dz}.$$

3. Principle of exchange of stabilities and oscillatory modes

Let

$$(3.1) \qquad U = (D^2 - k^2)W \quad \text{and} \quad X = \left[L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}(D^2 - k^2)\right]U.$$

In terms of X, the equation satisfied by W is

$$(3.2) \qquad \left[D^2 - k^2 - n(E + h\varepsilon)\right]\left[D^2 - k^2 - \lambda n(E' + h'\varepsilon)\right]X$$

$$= k^2 \left(\frac{G-1}{G}\right) R(\tau n + H)\left[D^2 - k^2 - \lambda n(E' + h'\varepsilon)\right]W$$

$$-\lambda k^2 S(\tau n + H')\left[D^2 - k^2 - n(E + h\varepsilon)\right]W.$$

Consider the case of two free surfaces having uniform temperature and solute concentration. The boundary conditions appropriate for the problem are

$$(3.3) \qquad W = D^2W = 0, \qquad \Theta = \Gamma = 0 \quad \text{at} \quad z = 0 \text{ and } 1.$$

Multiplying Eq.(3.2) by X^*, the complex conjugate of X, integrating over the range of z and using the boundary conditions (3.3), we obtain

$$(3.4) \qquad I_1 + n\left[(E + h\varepsilon) + \lambda(E' + h'\varepsilon)\right]I_2 + \lambda n^2(E + h\varepsilon)(E' + h'\varepsilon)I_3$$

$$= k^2\left(\frac{G-1}{G}\right)R(\tau n + H)\left(L_1^* + \frac{L_2^*}{P}\right)[I_4 + \lambda n(E' + h'\varepsilon)I_5]$$

$$-\lambda k^2 S(\tau n + H')\left(L_1^* + \frac{L_2^*}{P}\right)[I_4 + \lambda n(E + h'\varepsilon)I_5]$$

$$+k^2\frac{L_2^*}{\varepsilon}\left[\left(\frac{G-1}{G}\right)R(\tau n + H) - \lambda S(\tau n + H')\right]I_6$$

$$+k^2\lambda n\frac{L_2^*}{\varepsilon}\left[\left(\frac{G-1}{G}\right)R(\tau n + H)(E' + h'\varepsilon) - S(\tau n + H')(E + h\varepsilon)\right]I_7,$$

where

$$I_1 = \int_0^1 (|D^2X|^2 + 2k^2|DX|^2 + k^4|X|^2)dz,$$

$$I_2 = \int_0^1 (|DX|^2 + k^2|X|^2)dz,$$

$$I_3 = \int_0^1 (|X|^2)dz,$$

(3.5)

$$I_4 = \int_0^1 (|U|^2)dz,$$

$$I_5 = \int_0^1 (|DW|^2 + k^2|W|^2)dz,$$

$$I_6 = \int_0^1 (|DU|^2 + k^2|U|^2)dz,$$

$$I_7 = \int_0^1 (|D^2W|^2 + 2k^2|DW|^2 + k^4|W|^2)dz.$$

The integrals $I_1 - I_7$ are all positive definite.

Putting $n = in_0$, where n_0 is real, into Eq. (3.4) and equating imaginary parts, we obtain

(3.6) $\quad n_0^2 = \left\{ [(E + h\varepsilon) + \lambda(E' + h'\varepsilon)] I_2 + k^2 \left[\left(\frac{G-1}{G} \right) R \left(\frac{HF}{p_1} + \frac{\tau h}{P} \right) \right. \right.$

$$-\lambda S \left(\frac{H'F}{p_1} + \frac{\tau h'}{P} \right) \right] I_4 + \lambda k^2 \left[S(E + h\varepsilon)H' - \left(\frac{G-1}{G} \right) R(E' + h'\varepsilon)H \right]$$

$$\cdot \left(\frac{I_5}{P} + \frac{I_7}{\varepsilon} \right) + \frac{\tau k^2}{\varepsilon} \left[\left(\frac{G-1}{G} \right) Rh - \lambda Sh' \right] I_6 \right\}$$

$$\left/ \left\{ \lambda k^2 \left[- \left(\frac{G-1}{G} \right) R(E' + h'\varepsilon) \left\{ \frac{\tau(f-h)}{p_1} + \frac{\tau^2}{P} \right\} \right. \right. \right.$$

$$+ S(E + h\varepsilon) \left\{ \frac{\tau(f-h)}{\cdot \ p_1} + \frac{\tau^2}{P} \right\} \right] I_5 + \frac{k^2 \tau^2}{p_1} \left\{ \left(\frac{G-1}{G} \right) R - \lambda S \right\} I_4$$

$$+ \frac{k^2 \tau^2 \lambda}{\varepsilon} \left[- \left(\frac{G-1}{G} \right) R(E' + h'\varepsilon) + S(E + h\varepsilon) \right] I_7 \right\},$$

or

(3.7) $n_0 = 0.$

In the absence of stable solute gradient, Eqs. (3.6) and (3.7) become

(3.8) $n_0^2 = -\dfrac{\left(\dfrac{G}{G-1}\right)(E + h\varepsilon)I_2 + k^2 R\left\{p_1^{-1}HF + \dfrac{\tau h}{P}\right\}I_4 + \dfrac{k^2\tau}{\varepsilon}RhI_6}{k^2\tau^2 R p_1^{-1} I_4},$

or

(3.9) $n_0 = 0.$

Since the integrals are positive definite and n_0 is real, it follows that $n_0 = 0$ and the principle of exchange of stabilities is satisfied, in the absence of stable solute gradient. In the presence of stable solute gradient, the principle of exchange of stabilities is not satisfied and oscillatory modes come into play. The stable solute gradient, thus, introduces oscillatory modes which were non-existent in its absence.

4. Dispersion relation and discussion

When instability sets in as stationary convection, the marginal state will be characterized by $n = 0$ and Eq. (2.15) reduces to

(4.1) $\left[\dfrac{1}{P} - \dfrac{1}{\varepsilon}(D^2 - k^2)\right](D^2 - k^2)^2 W = \left(\dfrac{G-1}{G}\right)k^2 RHW - \lambda k^2 S H'W.$

Considering the case of two free boundaries, it can be shown that all the even order derivatives of W vanish on the boundaries and hence the proper solution of Eq. (4.1) characterizing the lowest mode is

(4.2) $W = W_0 \sin \pi z,$

where W_0 is a constant. Substituting the solution (4.2) in Eq. (4.1), we obtain

(4.3) $R = \dfrac{\left(\dfrac{G}{G-1}\right)\left[\left(\dfrac{1}{P} + \dfrac{\pi^2 + k^2}{\varepsilon}\right)(\pi^2 + k^2)^2 + \lambda k^2 H'S\right]}{k^2 H}.$

If R_c denotes the critical Rayleigh number in the absence of compressibility and $\overline{R}_c$ stands for the critical Rayleigh number in the presence of compressibility, then we find that

$$\overline{R}_c = \left(\dfrac{G}{G-1}\right)R_c.$$

Since critical Rayleigh number is positive and finite, so $G > 1$ and we obtain a stabilizing effect of compressibility as its result is to postpone the onset of double-diffusive convection in a fluid-particle layer of porous medium.

It is evident from Eq. (4.3) that

(4.4)
$$\frac{dR}{dP} = -\left(\frac{G}{G-1}\right)\frac{(\pi^2 + k^2)^2}{k^2 H P^2},$$

and

(4.5)
$$\frac{dR}{dS} = \lambda\left(\frac{G}{G-1}\right)\frac{H'}{H}.$$

The medium permeability and suspended particles have thus destabilizing effects, whereas the stable solute gradient has a stabilizing effect on the thermosolutal convection in compressible fluids with suspended particles in a porous medium.

5. Effect of rotation

In this section, we consider the same problem as that studied above except that the system is in a state of uniform rotation $\Omega(0,0,\Omega)$. The Coriolis force acting on the particles is also neglected under the assumptions made in the problem. The linearized nondimensional perturbation equations of motion for the fluid are

(5.1)
$$p_1^{-1}\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}\delta p + \omega(l - u) + T_A^{1/2}v + \left(\frac{1}{\varepsilon}\nabla^2 - \frac{1}{P}\right)u,$$
$$p_1^{-1}\frac{\partial v}{\partial t} = -\frac{\partial}{\partial y}\delta p + \omega(r - v) - T_A^{1/2}u + \left(\frac{1}{\varepsilon}\nabla^2 - \frac{1}{P}\right)v,$$
$$p_1^{-1}\frac{\partial w}{\partial t} = -\frac{\partial}{\partial z}\delta p + \omega(s - w) + R\theta - S\gamma + \left(\frac{1}{\varepsilon}\nabla^2 - \frac{1}{P}\right)w,$$

where $T_A = \dfrac{4\Omega^2 d^4}{\varepsilon^2\nu^2}$ is the nondimensional number accounting for rotation, and Eqs. (2.8)–(2.12) remain unaltered.

Eliminating $\mathbf{V}_d(l, r, s)$ with the help of (2.10) and then eliminating u, v, δp between Eqs. (5.1), using (2.9) we obtain

(5.2)
$$\left(L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}\nabla^2\right)^2\nabla^2 w + L_2^2 T_A\frac{\partial^2 w}{\partial z^2}$$
$$= L_2\left(L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}\nabla^2\right)\nabla_1^2(R\theta - \lambda S\gamma).$$

Eliminating θ and γ between Eqs. $(2.13)_{2,3}$ and (5.2) and using expression (2.14), we get

$$
(5.3) \quad \left[D^2 - k^2 - n(E + h\varepsilon)\right]\left[D^2 - k^2 - \lambda n(E' + h'\varepsilon)\right]
$$
$$
\cdot \left[\left\{L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}(D^2 - k^2)\right\}^2 (D^2 - k^2) + L_2^2 T_A D^2\right] W
$$
$$
= \left\{L_1 + \frac{L_2}{P} - \frac{L_2}{\varepsilon}(D^2 - k^2)\right\} k^2 \left[\left(\frac{G-1}{G}\right)\left\{D^2 - k^2 - \lambda n(E' + h'\varepsilon)\right\}\right.
$$
$$
\left.\cdot(\tau n + H)R - \lambda\left\{D^2 - k^2 - n(E + h\varepsilon)\right\}(\tau n + H')S\right]W.
$$

For the stationary convection, $n = 0$ and Eq. (5.3) reduces to

$$
(5.4) \quad (D^2 - k^2)\left[\left\{\frac{1}{P} - \frac{D^2 - k^2}{\varepsilon}\right\}^2 (D^2 - k^2) + T_A D^2\right] W
$$
$$
= k^2 \left\{\frac{1}{P} - \frac{D^2 - k^2}{\varepsilon}\right\}\left[\left(\frac{G-1}{G}\right) RH - \lambda SH'\right] W.
$$

Considering again the case of two free boundaries with constant temperature and solute concentration and using the proper solution (4.2), we obtain from Eq. (5.4)

$$
(5.5) \quad R = \left(\frac{G}{G-1}\right)\left[\frac{\pi^2 + k^2}{k^2 H}\left\{\frac{(\pi^2 + k^2)\left(\frac{1}{P} + \frac{\pi^2 + k^2}{\varepsilon}\right)^2 + \pi^2 T_A}{\left(\frac{1}{P} + \frac{\pi^2 + k^2}{\varepsilon}\right)}\right\} + \lambda S\frac{H'}{H}\right].
$$

It is evident from Eq. (5.5) that

$$
\frac{dR}{dT_A} = \left(\frac{G}{G-1}\right)\frac{\pi^2(\pi^2 + k^2)}{\left(\frac{1}{P} + \frac{\pi^2 + k^2}{\varepsilon}\right)k^2 H},
$$

$$
(5.6) \quad \frac{dR}{dH} = -\left(\frac{G}{G-1}\right)(\pi^2 + k^2)\left[\frac{\left(\frac{1}{P} + \frac{\pi^2 + k^2}{\varepsilon}\right)^2(\pi^2 + k^2) + \pi^2 T_A}{\left(\frac{1}{P} + \frac{\pi^2 + k^2}{\varepsilon}\right)k^2 H^2} + \lambda S\frac{H'}{H^2}\right],
$$

$$
\frac{dR}{dS} = \lambda\left(\frac{G}{G-1}\right)\frac{H'}{H}.
$$

Therefore the suspended particles have a destabilizing effect, whereas the rotation and stable solute gradient have stabilizing effects on the system under consideration.

Equation (5.5) also yields

$$(5.7) \qquad \frac{dR}{dP} = \left(\frac{G}{G-1}\right)\frac{(\pi^2+k^2)}{k^2 H}\left[-\frac{\pi^2+k^2}{P^2} + \frac{\pi^2 T_A}{P^2\left(\dfrac{1}{P}+\dfrac{\pi^2+k^2}{\varepsilon}\right)^2}\right].$$

If

$$T_A > \left(1+\frac{k^2}{\pi^2}\right)\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)^2,$$

then dR/dP is possitive.

If

$$T_A < \left(1+\frac{k^2}{\pi^2}\right)\left(\frac{1}{P}+\frac{\pi^2+k^2}{\varepsilon}\right)^2,$$

then dR/dP is negative.

Thus the medium permeability has both stabilizing and destabilizing effects, depending on the rotation parameter, on the thermosolutal convection in a compressible fluid with suspended particles rotating in a porous medium.

References

1. M.K. BRAKKE, Arch. Biochem. Biophysics, 55, 175, 1955.
2. P.F. LINDEN, Geophys. Fluid Dynamics, 6, 1, 1974.
3. C.R.B. LISTER, Geophys. J. Roy. Astr. Soc., 26, 515, 1972.
4. J.A.M. McDONNEL, Cosmic dust, John Wiley and Sons, Toronto, p. 330, 1978.
5. P. NASON, V. SCHUMAKER, B. HALSALL and J. SCHWEDES, Biopolymers, 7, 241, 1969.
6. J.W. SCANLON and L.A. SEGEL, Phys. Fluids, 16, 1573, 1973.
7. R.C. SHARMA and K.N. SHARMA, J. Math. Phys. Sci., 16, 167, 1982.
8. R.C. SHARMA and V. KUMARI, Japan J. Industrial Appl. Math., 9, 79, 1992.
9. E.A. SPIEGEL and G. VERONIS, Astrophys. J., 131, 442, 1960.
10. G.S. BEAVERS, E.M. SPARROW and R.A. MAGNUSON, J. Basic Engng. Trans. ASME, D92, 843, 1970.
11. D.D. JOSEPH and L.N. TAO, Zeit. Angew. Math. Mech., 44, 361, 1964.
12. P.G. SAFFMAN, Stud. Appl. Math., 50, 93, 1971.
13. T.S. LUNDGREN, J. Fluid Mech., 51, 273, 1972.
14. H. STOMMEL and K.N. FEDOROV, Tellus, 19, 306, 1967.

DEPARTMENT OF MATHEMATICS, GOVT. COLLEGE, NALAGARH,
DEPARTMENT OF MATHEMATICS, ST. BEDES COLLEGE, SHIMLA
and
DEPARTMENT OF MATHEMATICS
HIMACHAL PRADESH UNIVERSITY, SHIMLA, INDIA.