Basic mathematics in engineering science

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The aim of this cycle of lectures and laboratory exercises is to recollect and, if necessary, supplement elementary knowledge of mathematics, which the PhD students have received during their university courses, with the special stress laid on the practical applications of presented mathematical concepts in engineering science.

Main topics:

1. Elements of mathematical logic, propositional calculus, inference rules.
2. Set theory: algebra of sets, Cartesian product of sets, universal/existential quantifier.
3. Equivalence/order relations, Kuratowski-Zorn lemma, function as a relation.
4. Cardinality of sets, countable/uncountable sets, cardinal arithmetics, scale of alephs/betas.
5. Russell's antinomy, axiomatic theories, and Zermelo-Fraenkel (ZF) set theory as a foundation system for mathematics: axioms of ZF theory, continuum hypothesis, axiom of choice (AC) and ZFC theory.
6. Elements of probability theory: sample space, set of events, probability space, Kolmogorov axioms, classical/statistical definitions of probability, law of large numbers.
7. Elements of combinatorics: permutations/combinations/variations with/without repetitions, inclusion-exclusion principle.
8. Conditional probability, Bayes' theorem, „a priori”/„a posteriori” probabilities, examples of their usage in medicine (drug testing), Monty Hall paradox.
9. Random variable, discrete/continuous probability distributions, Riemann-Stieltjes integral, cumulative distribution function, probability density function, central limit theorem.
10. Elements of statistics: descriptive statistics (measures of central tendency/dispersion/shape), statistical inference (hypotheses testing, type I/II errors), estimation statistics (confidence interval, sampling methods,sample size determination, precision of the estimate).
11. Elements of differential calculus: elementary/special functions, limits and continuity of functions, asymptotes, differentiation and derivatives, differentials, rules of computation.
12. Elements of integral calculus: indefinite/definite integral, integration by parts, interpretation of integrals, formal definitions: Riemann/Darboux/Stieltjes/Lebesgue/ /Radon integrals.

13. Vector and multivariable calculus: scalar/vector fields, vector algebra, differential operators (gradient, divergence, curl, Laplacian), integral theorems, functions of several variables, partial differentiation, Jacobian matrix, directional derivatives, multiple integration.
14. Ordinary differential equations: first order ODEs (with separable variables, homogeneous, linear, Bernoulli's equation, integrating factor, Lagrange's/Clairaut's equation, theorem for existence and uniqueness of the solution), higher order homogeneous/nonhomogeneous linear ODEs (method of undetermined coefficients/variation of parameters)
15. Elements of differential geometry: theory of plane and space curves and surfaces, moving/reference frames, tangent vectors, curvature/torsion of a curve, Gaussian/mean curvature of a surface, surfaces of revolution, minimal surfaces, metric tensor, first/second fundamental forms, geodesic curves on a surface, surfaces of constant curvature.

The total number of lecture hours: 60, laboratory exercises: 60 hours, self-teaching: 90 hours, direct tutoring and consultations: 30 hours.

ECTS Points: 8.