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Dedicated to Professor Franz Ziegler on the occasion of his 70th birthday

On nonlinear stochastic vibratory systems with stiffness degradation

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Summary. In the paper a coupled response–degradation problem for a nonlinear vibrating system is analyzed. The analysis allows to account for the effect of stiffness degradation (during the vibration process) on the response and, in the same time, gives the actual stress values for estimation of damage accumulating in the system. The paper constitutes an extension of the approach (a sequential characterization of the degradation random process) presented in [4] to vibrating systems with nonlinear restoring term.

1 Introduction

As is well known, dynamic excitation of engineering systems (including randomly varying excitation) causes variable stress amplitudes generated in mechanical/structural members and, in the consequence, irreversible changes in the material structure. These changes, known as damage accumulation, may have different physical content. But despite the diversity of underlying physical/mechanical phenomena, it is useful to describe them jointly within a single model relating the rate of damage evolution over time with applied stress. Models of this type operate with a certain damage measure $D(t)$, which characterizes a damage state at time t. It is usually assumed that $D(t)$ is on the interval $[0, D^*]$, where D^* denotes a critical damage, and that is a nondecreasing function of time. In some situations (e.g., in the case of fatigue accumulation) external actions and generated stresses can be conveniently related to discrete values of time (e.g., by N, the number of cycles).

Since the variable stress causing damage (and, in the consequence, stiffness degradation) are generated by a vibratory system it is natural to formulate jointly the system dynamics and damage accumulation. Such an analysis allows to account the effect of stiffness degradation during the vibration process on the response and, at the same time, gives the actual stress values for estimation of damage.

A coupled analysis of random vibration and fatigue accumulation (fatigue crack growth) has been treated by Grigoriu for linear random oscillators in [1] (cf. also [2], [3]). The authors of the present paper in [4] provided for such a linear systems a sequential approach to the coupled response–degradation problem. In [5] the analysis of nonlinear systems with fatigue degradation (fatigue crack growth), but in

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the linear elastic component of the system is performed. In this paper, the sequential approach presented in [4] is extended to nonlinear systems with a wide class of nonlinearities; it includes, e.g., the degradation due to fatigue in nonlinear elastic components of the system.

2 General formulation

For a wide class of nonlinear vibratory systems with random excitation (both, external or parametric) the coupled response–degradation problem can be formulated in the following form:

$$\ddot{Y}(t) + F[\dot{Y}(t), Y(t), D(t), X(t,\gamma)] = 0, \tag{1}$$

$$Q[\dot{D}(t), D(t), Y(t), \dot{Y}(t)] = 0, \tag{2}$$

$$Y(t_0) = Y_0 \dot{Y}(t_0) = Y_{1,0} D(t_0) = D_0, \tag{3}$$

where $Y(t)$ is an unknown response process, $D(t)$ is a degradation process, $F[.]$ is the given function of indicated variables satisfying the appropriate conditions for the existence and uniqueness of the solution, $X(t, \gamma)$ is the given stochastic process characterizing the excitation; $\gamma \in \Gamma$, and Γ is the space of elementary events in the basic scheme (Γ, B, P) of probability theory, $Q[.]$ symbolizes the relationship between degradation and response process; its specific mathematical form depends on the particular situation; $Y_0, Y_{1,0}, D_0$ are given initial values of the response and degradation, respectively.

An important special class of the response–degradation problems is obtained if relationship (2) takes the form of a differential equation, that is, Eqs. (1), (2) are

$$\ddot{Y}(t) + F[\dot{Y}(t), Y(t), D(t), X(t,\gamma)] = 0, \tag{4}$$

$$\dot{D}(t) = G[D(t), Y(t), \dot{Y}(t)], \tag{5}$$

where G is the appropriate function specifying the evolution of degradation; its mathematical form is inferred from the elaboration of empirical data, or it is derived from the analysis of the physics of the process. In Eq. (5) dependence on $Y(t)$ and $\dot{Y}(t)$ is regarded here in a more relaxed sense than usual. The degradation rate $\dot{D}(t)$ may depend on the actual values of $Y(t), \dot{Y}(t)$, but it can also depend on some functionals of $Y(t), \dot{Y}(t)$; for example, on the integral of $Y(\tau)$, $\tau \in [t_0, t]$. In fatigue degradation problem the degradation $D(t)$ can be interpreted as a "normalized" crack size (cf. [4]) calculated according to the Paris law, or as a "normalized" damage calculated according to the Palmgren–Miner rule, when $Y(t)$ is not included itself, but the stress range $\Delta S = S_{\max} - S_{\min}$, i.e., a quantity related to the response amplitude $H = (Y_{\max} - Y_{\min})/2$.

The special class of problems characterized generally by Eqs. (1) and (2) is obtained if the functional relationship (2) does not include $\dot{D}(t)$, and $D(t)$ depends on some statistical characteristics of the response process $Y(t)$; a good example could be a vibrating system in which a degradation process depends on the time which the response $Y(t)$ spends above some critical level y^*. This is the case of an elasto-plastic oscillatory system with $D(t)$ interpreted as accumulated plastic deformation governed by the plastic excursions of the response $Y(t)$ into the plastic domain (in this situation y^* may be regarded as the yield limit of the material in question). Formally, the situations indicated above can be characterized by the equations

$$\ddot{Y}(t) + F[\dot{Y}(t), Y(t), D(t), X(t,\gamma)] = 0, \tag{6}$$

$$D(t) = D_0 + \sum_{i=1}^{N(t)} d_i(\gamma), \tag{7}$$

where $d_i(\gamma) = \Delta D_i(\gamma)$ are random variables characterizing the elementary degradations associated with the specific degradation process; the magnitude of $d_i(\gamma)$ depends on characteristics of the process $Y(t)$ above a fixed level y^*. The process $N(t)$ is a stochastic counting process characterizing a number of degrading events in the interval $[t_0, t]$.

3 Nonlinear random vibration with stiffness degradation

A nonlinear symmetric relationship between stress S and strain ε in uniaxial tension–compression can be represented as

$$\begin{aligned} S(\varepsilon) &= E_0 \sum_{k=1}^{M} v_k \varepsilon^k = E_0 g(\varepsilon; v_1, \ldots, v_M), \quad -\varepsilon^* < \varepsilon < \varepsilon^*, \\ S(-\varepsilon) &= -S(-\varepsilon), \end{aligned} \tag{8}$$

where (cf. [6])

$$g(\varepsilon; v_1, \ldots, v_M) = E_0 \sum_{k=1}^{M} v_k \varepsilon^k, \tag{9}$$

where E_0 is an empirical modulus of elasticity and ε^* is a certain fixed strain satisfying the condition $\varepsilon^* \ll \varepsilon_{\text{yield}}(\varepsilon_{\text{yield}}$ – yield strain). The parameters $v_k (k = 1, \ldots, M)$ are nondimensional empirical parameters characterizing the physical nonlinearity of the elastic element in the vibratory system. The function $g(\varepsilon)$ is nondecreasing function on the interval $[-\varepsilon^*, \varepsilon^*]$ and satisfying the condition $g(-\varepsilon) = -g(-\varepsilon)$.

From the above the relation between nonlinear force F and displacement x takes the following form:

$$F(x) = k_0 g(x; \eta_1, \ldots, \eta_M), g(-x) = -g(-x), \quad -x^* < x < x^*, \tag{10}$$

where

$$g(x; \eta_1, \ldots, \eta_M) = k_0 \sum_{k=1}^{M} \eta_k x^k, \quad k_0 = \frac{E_0 s_l}{l_0}, \quad \eta_k = \frac{v_k}{l_0^{k-1}}, \quad x^* = \varepsilon^* l_0. \tag{11}$$

The parameters η_k are dimensional parameters of nonlinearity of the restoring force, k_0 is the stiffness of the linear elastic element with initial length l_0 and cross-sectional area s_l, respectively.

The analysis of random vibration problems coupled with degradation creates serious difficulties. Due to this fact it is not easy to construct an analytical and general method for the solution of the system of equations presented in the previous section. The appropriate simplifications and approximations have to be introduced.

Let us consider vibrations of the oscillator with nonlinear restoring force $g(x; \eta_1, \ldots, \eta_M)$ subjected to Gaussian white noise excitation. Assume that during the vibration process a fatigue damage develops in the component which affects the stiffness (or natural frequency) of the vibrating system under consideration (cf. Fig. 1, for illustration). Let us denote by $k(D)$ the stiffness dependence on the degradation measure D. In addition we assume that the static displacement y_0 (equilibrium state, which in general is also the function of degradation measure D, i.e., $y_0(D)$) due to the gravity force acting on the mass M is close to zero and is very small in relation to the vibration amplitude of the system. In such a case $y_0(D) \cong 0$ and the governing equation can be written in the form

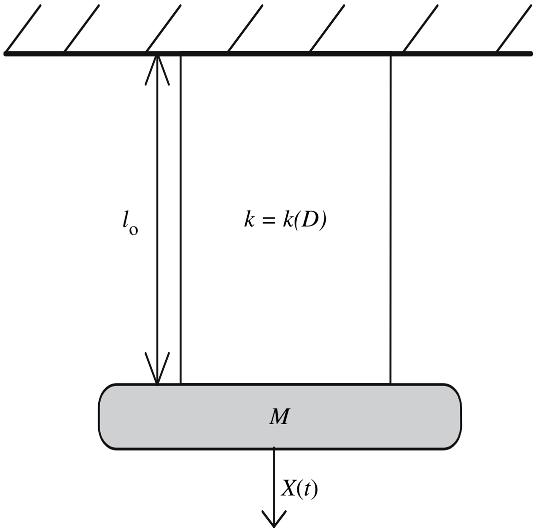

Fig. 1. Diagram of vibratory system with stiffness degradation

$$M\ddot{y}(t) + c\dot{y}(t) + k(D)g(y;\eta_1,\ldots,\eta_M) = \xi(t,\gamma), \tag{12}$$

where $\xi\,(t,\gamma)$ is a random process assumed to be stationary Gaussian white noise. The response process $y(t)$ characterizes the displacement and M, c are the mass and damping coefficients, respectively. Dividing, both sides of Eq. (12) by M and then introducing new variables $Y = y/\sigma_y$, $\tau = \omega_0 t$, where σ_y denotes a standard deviation of the stationary response of the linear system $g(y) = y$ without degradation (i.e., when $\omega_0^2 = k(D_0)/M, D_0 = 0$), we obtain a dimensionless form of Eq. (12)

$$\ddot{Y}(\tau) + 2\zeta\dot{Y}(\tau) + q(D)g(Y;\beta_1,\ldots,\beta_M) = \xi_1(\tau,\gamma), \quad \beta_k = \frac{v_k\sigma_y^{k-1}}{l_0^{k-1}}, \tag{13}$$

where $\omega_0^2\ q(D) = k(D)/M$ and $q(D)$ is a monotonically decreasing function of the degradation measure D satisfying the condition $q(D_0) = 1$; $\xi_1(\tau,\gamma) = \xi(\tau/\omega_0,\gamma)/k_0\sigma_y$ is the stationary Gaussian white noise with correlation function $K_{\xi_1}(\tau_2 - \tau_1) = 4\zeta\delta(\tau_2 - \tau_1)$ where $\delta(.)$ is the Dirac delta function. When $q\,(D_0) = 1$ and $g(y) = y$ the Eq. (13) has a property that $\sigma_Y = \sigma_{\dot{Y}} = 1$, where $\sigma_{\dot{Y}}$ is the standard deviation of the velocity.

Generally, the function q representing the stiffness dependence on the degradation measure $D(\tau)$ can be taken in the form of a polynomial (cf. [4])

$$q(D) = 1 - \sum_{i=1}^{K}\theta_i D^i, \quad q(D=0) = 1. \tag{14}$$

However, it can be also approximated by the exponential function (cf. [7])

$$q(D) = \alpha_1 + \alpha_2\exp(-\alpha_3 D^{\alpha_4}), \tag{15}$$

where α_1, α_2, α_3, α_4 are positive constants, such that $\alpha_1 + \alpha_2 = 1$ (to have $q\ (D_0 = 0) = 1$). The values of the empirical parameters α_i, θ_i should be identified from the experiment.

4 Coupled analysis of the response–degradation problem

In order to include into the analysis the explicit dependence of the degradation on the response amplitude of the system with nonlinear restoring force we assume that we deal with the cumulative model of degradation measure D. When $2\,\zeta \ll 1$ (in Eq. (13)) the response process $Y(\tau)$ is a narrow band process generating in each response cycle N the stress range ΔS_N. The increment ΔD_N of the degradation D during the cycle N can be represented as

$$\Delta D_N = C(\Delta S_N)^m, \tag{16}$$

where C and m are empirical constants and ΔS is the stress range. For example, this is the case of fatigue accumulation; the Paris–Erdogan equation for fatigue crack growth (in one cycle) in elastic materials can be transformed to (16) (cf. [4]).

$D_N(\gamma)$ characterizes the state of the degradation random process after N cycles. In order to account for the cumulative nature of the degradation process and its randomness let us represent $D_N(\gamma)$ in the form of a sequence of random variables $\Delta D_N(\gamma)$, $N = 0,\ 1, \ldots,\ N^*$. Therefore

$$D_N(\gamma) = \sum_{i=1}^{N} \Delta D_i(\gamma), \quad \Delta D_i(\gamma) = D_i(\gamma) - D_{i-1}(\gamma). \tag{17}$$

The coupled computational response–degradation model has the form

$$\ddot{Y}(\tau) + 2\zeta\dot{Y}(\tau) + q[D_{N-1}(\gamma)]g(Y;\beta_1,\ldots,\beta_M) = \xi_1(\tau,\gamma), \tag{18}$$

$$D_N(\gamma) = D_{N-1}(\gamma) + \Delta D_N(\gamma), \tag{19}$$

where $\Delta D_N(\gamma)$ denotes the increment of the degradation process during N-th cycle. It is defined by formula (16) in which ΔS_N is the stress range in N-th cycle.

It is assumed that the degradation starts when the response $Y(t)$ is in its stationary state and that the response is a narrow-band process ($2\,\zeta \ll 1$). Because of the assumption that the equilibrium state $y_0(D) \cong 0$ we approximate the range $\Delta Y_i = Y_{\max,i} - Y_{\min,i}$ by the amplitude H_i of the $Y(t)$. Because of the nonlinear relationship (9), the stress range ΔS_i in the i-th cycle is

$$\Delta S_i = l_0^{-1} E_0 \sigma_y g(H_i;\beta_1,\ldots,\beta_M). \tag{20}$$

Finally, the increment ΔD_N of the degradation process occurring in Eq. (16) has the form

$$\Delta D_N(\gamma) = C_1 g^m(H_N(\gamma);\beta_1,\ldots,\beta_M), \tag{21}$$

where the constant C_1 is obtained during the transformation from a dimensional to a nondimensional system.

Equations (18) and (19) along with (16) and (21) constitute a complete sequential model for the characterization of the response–degradation process $Y(t)$, $D(t)$ in discretized time instants (cycles) $N = 0, 1, \ldots, N^*$. Because the degradation process is slow in comparison to the response itself and the degradation process D starts when the system (18) reached its stationary state for initial stiffness $q(D_{N=0})$ generated by a deterministic or random value of the initial damage measure $D_{N=0} = D_0$, we take the distribution of the amplitude H_N given D_{N-1}. In this model the response after N cycles is affected by the stiffness degradation state after $N-1$ cycles, whereas the degradation process after N cycles depends on the response amplitude H_N at cycle N, given D_{N-1}.

The probabilistic characteristics of the response–degradation process Y_N, D_N (where $Y_N = H_N$ and H_N is the amplitude of the process Y at cycle N) can be obtained via conditioning. Let us denote by $H_N|D_{N-1}$ the conditional amplitude of the process $Y(t)$ at the N-th cycle given a fixed value of the stiffness in cycle N (specified by the degradation level at $(N-1)$-st cycle). In the case of the considered nonlinear system the conditional probability distribution (density) of H_N, i.e., $\hat{f}_{H_N}(h|D_{N-1})$ has the following form [3]

$$\hat{f}_{H_N}(h|D_{N-1}) = q(D_{N-1})g(h;\beta_1,\ldots,\beta_M)\exp\left(-q(D_{N-1})\int_0^h g(z;\beta_1,\ldots,\beta_M)dz\right). \tag{22}$$

Because the random variable describing the increment ΔD_N in Eq. (21) is a nonlinear transformation of the random amplitude $H_N(\gamma)$, the conditional probability density of the increment ΔD_N at given degradation D_{N-1} has the form

$$\hat{f}_{\Delta D_N|D_{N-1}}(x|D_{N-1}) = \left|\frac{d\varphi(x)}{dx}\right|\hat{f}_{H_N}(\varphi(x)|D_{N-1}), \tag{23}$$

where $\varphi(x)$ is the inverse function of $g^m(h;\ \beta_1,\ldots,\beta_M)$. The joint distribution of D_{N-1} and ΔD_N needed to evaluate the probability distribution of the degradation $D_N(\gamma)$ at cycle N is represented by

$$f_{\Delta D_N, D_{N-1}}(x,y) = \hat{f}_{\Delta D_N|D_{N-1}}(x|y)f_{D_{N-1}}(y). \tag{24}$$

The density function of the random variable $D_N(\gamma)$ defined as the sum of $\Delta D_N(\gamma)$ and $D_{N-1}(\gamma)$ is given as following convolution:

$$f_{D_N}(z) = \int_0^z f_{\Delta D_N, D_{N-1}}(z-y,y)dy = \int_0^z \hat{f}_{\Delta D_N|D_{N-1}}(z-y|y)f_{D_{N-1}}(y)dy, \tag{25}$$

where $\hat{f}_{\Delta D_N|D_{N-1}}$ is given by formula (24).

Therefore, the probability density of the degradation process at the N-th cycle is expressed by the conditional density $\hat{f}_{\Delta D_N|D_{N-1}}$ given by the explicit formula (23) and by the density of the degradation process at the cycle $N-1$. This integral recursive formula (25) can serve as base for calculations. The probability distribution of the response amplitude at cycle N, given the degradation at cycle $N-1$, is expressed by formula (22).

The above procedure can be easily extended to the more general class of following nonlinear systems with stiffness degradation:

$$\ddot{Y}(\tau) + F[\dot{Y}(\tau), Y(\tau), D_{N-1}(\gamma)] = \xi_1(\tau,\gamma), \tag{26}$$

$$D_N(\gamma) = D_{N-1}(\gamma) + \Delta D_N(\gamma), \tag{27}$$

where $\Delta D_N(\gamma)$ is given by (16). Under the assumptions that the response of the degrading system is a narrow band process (i.e., small damping restriction) and degradation starts when the stationary state of the system is reached, the conditional probability density function of $H_N|D_{N-1}$ is the conditional probability density function of local maxima of $Y_N|D_{N-1}$ and can be written down in the known form (i.e., [2])

$$\hat{f}_{H_N}(h|D_{N-1}) = -\frac{1}{v_0^+(D_{N-1})}\frac{\bar{d}v_h^+(D_{N-1})}{\bar{d}h}, \tag{28}$$

where $v_x^+(D_{N-1})$ is the mean value of the x-upcrossing rate of $Y_N|D_{N-1}$ given by

$$v_x^+(D_{N-1}) = \int_0^\infty u f_{Y\dot{Y}}(x,u|D_{N-1})du. \tag{29}$$

In all situations when processes $Y_N|D_{N-1}$ and $\dot{Y}_N|D_{N-1}$ can be regarded as statistically independent, the formula (28) reduces to the form

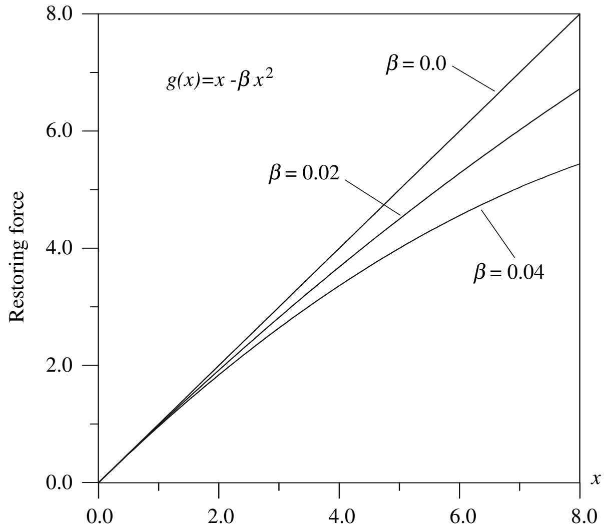

Fig. 2. Example of the nonlinear (softening) restoring force of an elastic element in a vibratory system

$$\hat{f}_{H_N}(h|D_{N-1}) = -\frac{1}{f_{Y|D_{N-1}}(0)} \frac{df_{Y|D_{N-1}}(y)}{dy}\bigg|_{y=h}, \tag{30}$$

which is an extension of the formula (22) which-holds-for related to system (18). Indeed, for the nonlinear oscillator (18) we have the following joint conditional probability density function (cf. [8]) in the stationary state f_Y:

$$(y_1, y_2|D_{N-1}) = f_{Y|D_{N-1}}(y_1) f_{\dot{Y}|D_{N-1}}(y_2) = C_2 \exp[-q(D_{N-1})\Psi(y_1)] \exp[-y_2^2/2], \tag{31}$$

where $\Psi(y_1) = \int_0^{y_1} g(z; \beta_1, \ldots, \beta_M) dz$. After appropriate calculation according to Eq. (30) we obtain Eq. (22).

For a wide class of nonlinear oscillators such as Caughey oscillators or the oscillator given in (18) the stationary response distribution has a closed analytical form and can be directly adopted to the proposed above analysis of nonlinear systems with stiffness degradation. In other cases the approximation methods such as the maximum entropy method (cf. [8]) or path integration method (cf. [9]) could be used to approximate the probability density function $f_{Y\dot{Y}}(x, u|D_{N-1})$.

5 Numerical example

The effectiveness of the method described above is verified for the response–degradation nonlinear vibratory stochastic system (18) with restoring force of type (11) (see Fig. 2)

$$g(Y) = Y - \beta Y^2 \mathrm{sgn}(Y), \quad -\varepsilon^* l_0/\sigma_y < Y < \varepsilon^* l_0/\sigma_y, \tag{32}$$

where $\mathrm{sgn}(y)$ denotes the signum function (i.e., $\mathrm{sgn}(y) = 1$ for $y > 0$ or and $\mathrm{sgn}(y) = -1$ for $y < 0$). To show the effect of the stiffness degradation on the response of the system, the degradation function defined in Eq. (14) is taken in the form

$$q(D) = 1 - D^2, \quad \langle D \rangle \in [\langle D_0 \rangle, \langle D^* \rangle], \tag{33}$$

where $\langle \cdot \rangle$ denotes the mean value. In the calculations the following values of parameters are assumed both for the linear and the nonlinear system: $\langle D^* \rangle = 0.5$, the damping coefficient on the system (18) is

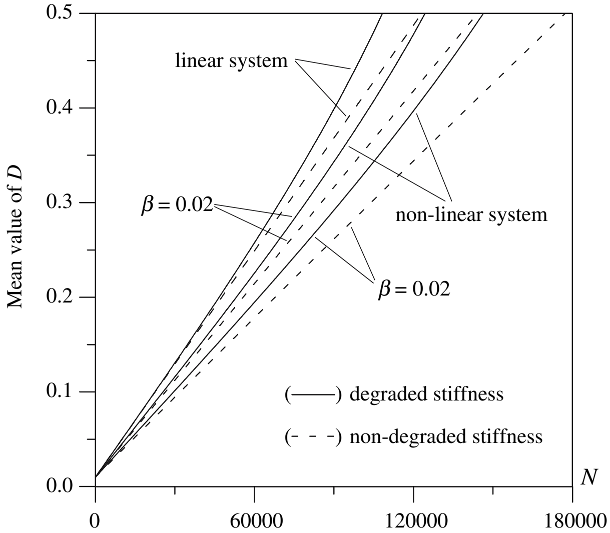

Fig. 3. Mean value of the degradation measure D versus number of cycles

$\zeta = 0.01$ and the values of coefficients in Eq. (21) are as follows: $C_1 = 1.06 \times 10^{-6}$, $m = 3$. The initial degradation measure is assumed to be a Gaussian random variable $D_0(\gamma)$ with mean value $\langle D_0 \rangle = 0.01$ and standard deviation $\sigma_D = 10^{-3}$. The value of the parameter C_1 is the same for the linear and the nonlinear system to show the difference between damage accumulation in these systems.

To show the effect of the stiffness nonlinearity on the degradation process two cases were considered. First, the response of the nondegraded (uncoupled system with constant initial stiffness) and degraded linear system was calculated and then the system with nonlinear (softening) restoring force (32) was evaluated numerically according to the method described in Sect. 4. Figures 3 and 4 illustrate the evolution of the mean values and standard deviations of the degradation measure D for the nondegraded and the degraded system with linear and nonlinear stiffness, respectively. In Fig. 3 we see that for the fixed number of cycles the mean value of the degradation measure D for the degraded system with nonlinear stiffness is smaller than the mean value of the degraded system with linear stiffness. This is because of the softening stress–strain relation for the nonlinear system. In this figure we see that the mean value of D for the system with stiffness degradation has nonlinear behavior, whereas it is linear in the case of the system where stiffness degradation is not taken into account. This figure also shows that the difference between the mean value of D for the nonlinear system with stiffness degradation and without stiffness degradation is dependent on the parameter of nonlinearity β, respectively. Figure 4 visualizes the comparison between standard deviations of the degradation measure for the considered systems. A significant growth of the standard deviation of the degradation measure D in degraded systems is observed. Figure 5 shows the probability density functions of the degradation measure for different numbers of response cycles N. This figure indicates also that stiffness degradation should play an important role in the reliability analysis of the system. For example, for fixed level $D^* = 0.5$ and $N = 100{,}000$ of cycles we have a probability of failure $P_F = 1 - P(D < D^*) \approx 0.025$ for the nondegraded nonlinear system (see dashed probability density) and $P_F = 1 - P(D < D^*) \approx 0.27$ for the degraded nonlinear system (see continuous probability density). The similar situation is observed for the linear system. The nondegraded system is understood here as the system whose stiffness degradation is not taken into account.

Figures 6 and 7 shows the mean and standard deviation of the stiffness degradation $q(D)$ versus the number of cycles of the response process Y. Figures 8 and 9 illustrate probabilistic characteristics of the response process $Y(\tau)$. More specifically, they show the mean and standard

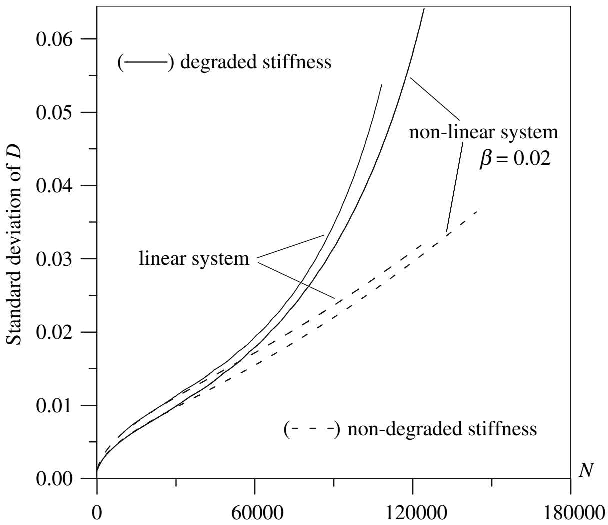

Fig. 4. Standard deviation of the degradation measure D versus number of cycles

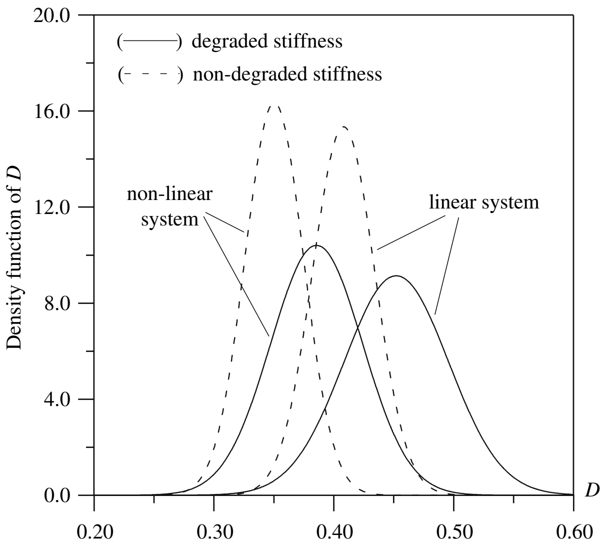

Fig. 5. Probability densities of the degradation measure D for number of cycles $N = 100{,}000$

deviation of the response amplitude both in the case of the system with nondegraded stiffness and the system with degraded stiffness. Again, the distortion of the results due to stiffness degradation and degree of stiffness nonlinearity is clearly visible for larger numbers of cycles.

The presented results show that the behavior of dynamical stochastic systems with stiffness degradation should be investigated by the use of coupled equations describing simultaneously the response and the degradation processes.

6 Conclusions

In this paper the general formulation and analysis of the response–degradation problems for randomly vibrating systems are presented. Such a coupled formulation makes it possible to account for the effect of stiffness degradation (during the vibration process) on the response and,

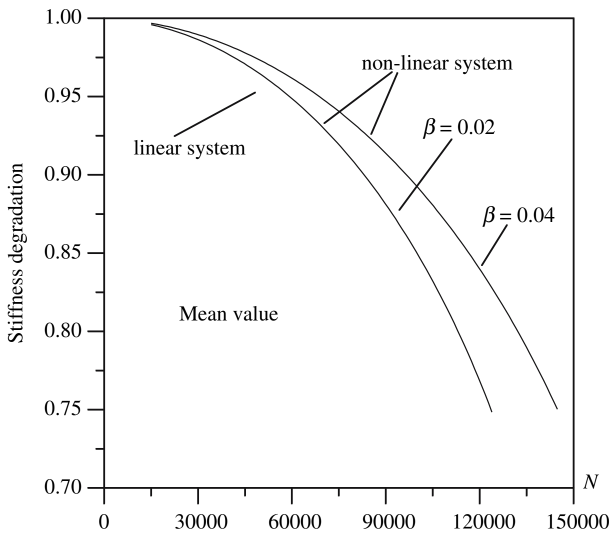

Fig. 6. Mean value of the stiffness degradation function $q(D)$ vs. the number of cycles

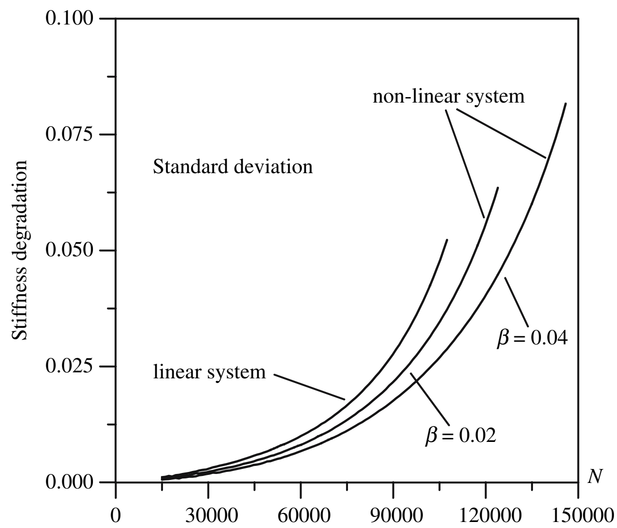

Fig. 7. Standard deviation of the stiffness degradation function $q(D)$ vs. the number of cycles

simultaneously, it yields the actual stress values for characterization of the accumulation of degradation.

It has been shown in the paper how such a coupled, vibration–fatigue degradation problem, can be treated effectively. The method presented in Sect. 4 yields the statistical characteristics of the process $[Y(\tau),\ D(\tau)]$ as functions of a number of cycles of the response process. Numerical calculations provide a quantitative and graphical information on the response-degradation process. Calculations provide a quantitative and graphical information on the response process. This kind of information may be used in the reliability estimation of real vibrating systems. The results are obtained for fixed values of the intensity of external noise and the specific stiffness–degradation function (33). Of course, the form of the function $q(D)$, which usually comes from empirical data (and depends on the material properties of the vibrating component) may have a significant effect on the response characteristics. Some preliminary results presented in this paper have been announced earlier in [10].

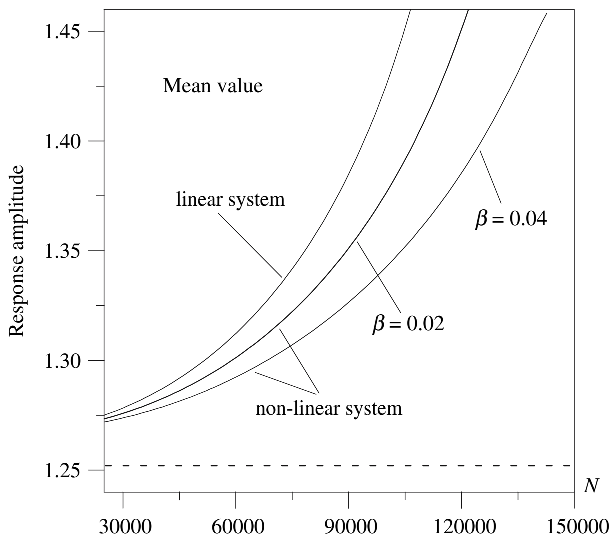

Fig. 8. Mean value of the response amplitude of linear system with non-degraded stiffness (*dashed line*) and systems with degraded stiffness (*continuous line*)

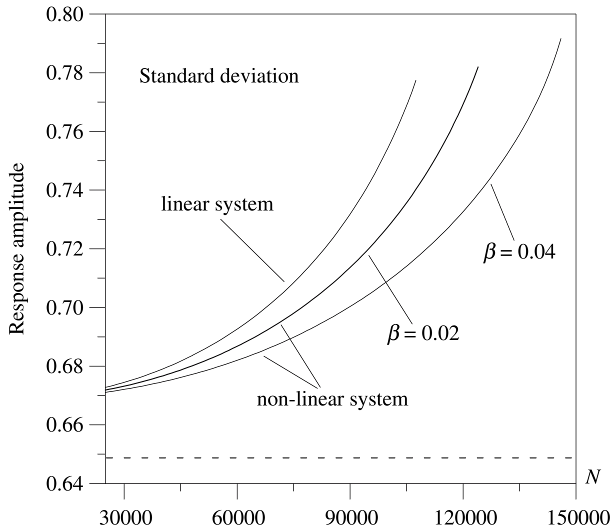

Fig. 9. Standard deviation of the response amplitude of linear system with nondegraded stiffness (*dashed line*) and systems with degraded stiffness (*continuous line*)

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