Assessment of the Size of the Representative Volume Element of Random Heterogeneous Materials

Paweł Hołobut [1]

[1] Institute of Fundamental Technological Research, Polish Academy of Sciences

pholob@ippt.pan.pl

One of the main concepts in micromechanics of materials and homogenization theory is the Representative Volume Element (RVE). There exist several imprecise definitions of the RVE, describing in qualitative terms a similar entity. The RVE is generally understood as a cubic volume of material which is macroscopically small, but simultaneously large enough to well represent the material's microstructural features in the statistical sense. In particular, the RVE's overall physical properties should be effective in the sense of being invariant with respect to choosing a different sample or further increasing the size of the cube. The RVE size depends on the physical property being considered (elastic or plastic properties, thermal conductivity, etc.), geometry of the microstructure (number of phases and their spatial arrangement), and the differences between the properties of individual phases [1]. The RVE is generally assumed to be large compared with the size of inhomogeneities; however, effective properties can sometimes be estimated using cubes of relatively small sizes [2].

In periodic materials, the RVE can be selected as the elementary periodic cell. In heterogeneous materials with random morphology, however, determination of the size of the RVE is not straightforward. Due to random spatial variations of the microstructure, two samples of the same random material may differ in their average response beyond an acceptable tolerance limit. This problem is frequently investigated using the notion of the Stochastic Volume Element (SVE) [3]. The SVE is a mesoscale volume element, whose size varies between the average size of inhomogeneities and the size of the RVE. The overall properties of the SVE, called apparent properties, are functionals over random fields describing the microstructure and may have significantly different values for different samples. As the size of the SVE increases, more and more inhomogeneities tend to be enclosed in the cube and the laws of large numbers cause that apparent properties tend to effective ones with increasing certainty. In this way, the SVE approaches the RVE.

The present work is devoted to probabilistic analysis of the size of the cube at which the transition from the SVE to the RVE can be assumed to occur. An attempt will be made to provide a precise, quantitative stochastic definition of the RVE, denoting an SVE of such dimensions that the probability of obtaining two samples with apparent properties differing by more than a given error threshold is sufficiently small. This is a slight modification and formalization of the approach proposed in [1]. Fundamental stochastic characteristics such as mean values and covariance matrices of apparent mechanical properties of SVEs will be presented for several example morphologies. Using the proposed definition of the RVE and the computed stochastic characteristics, approximate RVE sizes will be obtained and validated against results of numerical homogenization.

References

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