The deformable discrete element method - formulation and application

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ABSTRACT

The deformable discrete element method (DDEM) is a novel method proposed by Rojek et al. [1] in order to enhance the capabilities of standard discrete element method (DEM) while preserving its efficiency and general framework. The so called soft contact approach used in DEM treats particles as pseudo-rigid and allows a small overlap between them assuming it to be equivalent to particle deformation at the contact. The known drawback of the DEM method is the inaccurate representation of macroscopic properties. For instance, maximum Poisson's ratio that can be obtained with bonded disc (2D) DEM model is 0.33 [2], which inhibits accurate simulation of elastic deformation and elastic waves. Additionally, the contacts in DEM are independent which is justifiable for rigid particles with negligible deformation but leads to incorrect behaviour otherwise. The DDEM cures these limitations by introducing the concept of global mode of particle deformation resulting from the particle stress which in turn is induced by the contact forces. The global particle deformation leads to the change of overlap in local deformation zone at the contact (cf. Figure 1) and formation of new contacts simultaneously, which affects the macroscopic response of the particle assembly. In particular it widens the range of Poisson's ratio that can be reproduced in DEM, which is a key parameter in problems such as wave propagation. The main purpose of this work is to present the DDEM formulation and show how it improves the modelling capabilities of the standard DEM. An important application of DEM model namely, wave propagation problem will be considered for this purpose. A numerical example will be presented to illustrate the wave propagation phenomenon in an elastic solid discretized with discs (2D elements).

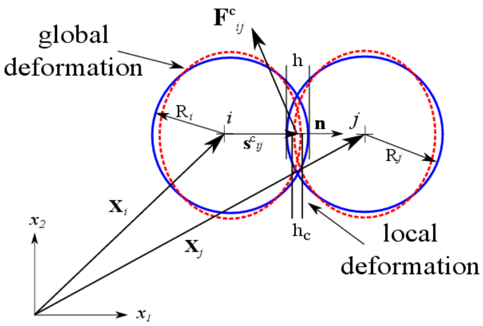

Figure 1: The idea of the deformable discrete element method (DDEM)

The idea of DDEM is shown in Figure 1. Assuming uniform stress, the internal particle stress is determined from the contact forces as a volume average stress using the relationship [3]:

$$\boldsymbol{\sigma}_p = \frac{1}{\mathrm{V}_p}\sum_{c=1}^{n_{pc}} \mathbf{s}^c \otimes \mathbf{F}^c \tag{1}$$

where V_p – particle volume, n_{pc} – no. of elements in contact, $\mathbf{s}^c$ – vector connecting particle center with contact point, $\mathbf{F}^c$ – contact force and symbol $\otimes$ is outer tensor product. The uniform particle strains are obtained from stresses employing inverse constitutive relationship:

$$\boldsymbol{\epsilon}_p = \mathbf{D}\colon\boldsymbol{\sigma}_p \quad (2$$

where **D** is elastic compliance tensor. Consequently, the modified particle overlap is evaluated.

DDEM has been applied to wave propagation phenomenon using a rectangular sample of length (l) ~ 16.5 mm, width (w) ~ 2 mm, discretized with 682 bonded disc elements with non-uniform size (cf. Figure 2). A cohesive linear elastic contact model has been used. Simulations are performed for the ratio of tangential to normal contact stiffness, k_t/k_n ranging between 0.0 to 1.0 by assuming particle density, ρ_P = 2000 kg/m^3 and normal contact stiffness, k_n = 1·10^{10} N/m. Longitudinal and shear wave is triggered in sample by defining initial particle displacements on the left edge in x and y directions repectively as shown in Figure 2. In case of longitudinal wave, particles at the top and bottom edge are unconstrained in x and y directions which allows us to treat particle assembly as a bar. Periodic boundary conditions are applied in case of shear wave on top and bottom edges. Particles on the right edge are fixed in x - direction. Peak to peak method is used on displacement-time curve to evaluate velocity of wave between two nodes with known distance between them (cf. Figure 3). Wave velocity for a given k_t/k_n ratio is determined as an average of wave velocities obtained through displacement-time curve for 5 pairs of node highlighted (in green color) in Figure 2.

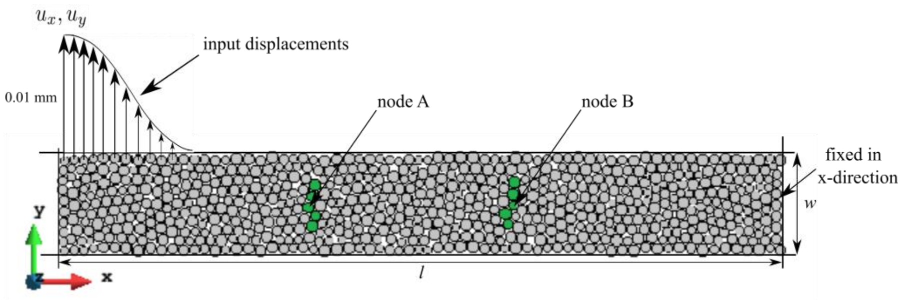

Figure 2: Application of DEM – simulation of wave propagation in elastic solid discretized with disc elements.

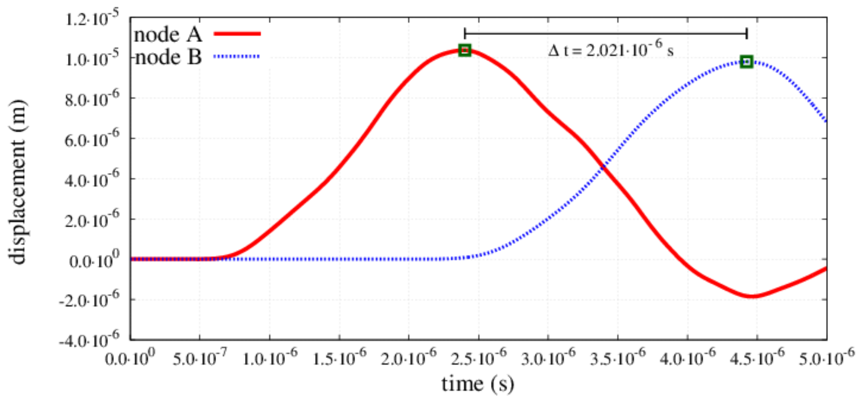

Figure 3: Peak to peak method is used to determine wave velocity for example, between node A and node B.

In conclusion, results showcasing the enhanced modelling capabilities of DDEM will be presented. It will be shown that DDEM introduces flexibility over standard DEM and extends the range of material properties that can be simulated using DEM in problems of geotechnics and civil engineering.

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REFERENCES

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