Nondeterministic Cellular Automaton for Modelling Urban Traffic with Self-organizing Control

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Abstract. Controlling flow in networks by means of decentralized strategies have gained a lot of attention in recent years. Typical advantages of such approach – efficiency, scalability, versatility, fault tolerance – make it an interesting alternative to more traditional, global optimization. In the paper it is shown how the continuous, macroscopic, self-organizing control proposed by Lämmer and Helbing [10] can be implemented in the discrete, nondeterministic cellular automaton (CA) model of urban traffic. Using various examples, it is demonstrated that the decentralized approach outperforms the best nonresponsive solution based on fixed cycles. In order to analyse relatively large parameter space, an HPC cluster has been used to run multiple versions of a serial CA simulator. The presented model can serve as a test bed for testing other optimization methods and vehicle routing algorithms realized with the use of CA.

Keywords: Urban traffic · Nondeterministic cellular automaton
Self-organizing control · Decentralized control

1 Introduction

In communication networks, controlling strategies have a profound impact on the overall performance [1,2]. Particularly, optimization in traffic networks is especially important due to a tremendous affection it has on peoples everyday life. In order to address this issue, one has to apply some kind of traffic model and then propose optimization procedures.

There exists a large number of traffic models which generally fall in one of these classes: microscopic where vehicles are represented as particles (e.g., follow-the-leader models); cellular automata (CA) where a vehicle's state corresponds to a cell's state; based on some master equation (e.g., mean field models); macroscopic continuous models (e.g., kinetic waves), and more [3,4]. Obviously, a good traffic model has to reproduce all its properties which are observed in the real world.

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Regarding optimization, one of the most common ways to do it is to choose some pre-calculated schemes, which are aimed at synchronizing green times along main arterials. In principle such methods force the traffic flow to comply with previously designed patterns in order to minimize travel times. However, since traffic demand varies, there is a need for some responsiveness to the current traffic state. In order to improve efficiency of control methods, it is necessary to implement on-line optimization techniques based on real time traffic intensity observations. This can be done in a centralized system, in which there exists a central unit possessing all information concerning current state of the network. Obviously all the measuring devices must be somehow connected to a central unit (which is expensive). Moreover, optimizing globally may be NP-hard [5,6] making it even more difficult to react in real-time. Consequently, there is a recent trend towards decentralized and self-organizing optimization techniques [7–12] which instantly and locally respond to the current traffic state (known, e.g., from vehicle detectors mounted at some distance before an intersection). Naturally, it is desired that such locally defined mechanisms will produce near-optimal global solution. One of the most efficient and versatile decentralized self-controlled strategies has been proposed by Lämmer and Helbing (LH, [10]). The authors have defined the scheme with the use of a model similar to kinematic waves approach [13].

In this paper it is shown in details how this LH controlling mechanism can be implemented in a network of cellular automata with the use of nondeterministic Nagel-Schreckenberg (NS) model [14] (i.e., with the randomization parameter $P > 0$). The efficiency of this solution is analysed by considering three scenarios in regular lattice networks. It is shown that the self-controlled intersections converge to the best possible cycles and phase-shifts for periodic networks, and that they outperform constant cycle (CC) solutions if vehicles are able to randomly change moving directions (e.g., they turn). Lastly, stochastic boundary conditions are applied and it is shown that the LH strategy clears the network significantly more efficient if additional perturbations are allowed.

CA traffic models can be relatively easily parallelized, making it a very useful tool for efficient prediction, analysis and optimization. Moreover, they can be implemented withe use of FPGA [15] or GPGPU [16] further increasing efficiency. The results presented here are calculated with a serial program designed to advance a network of CA's. However, since it was desired to obtain a full study of parameter space, these programs have been run in parallel in an HPC cluster for various initial conditions and control variables. Therefore, meaningful statistics could be calculated in a reasonable time (couple of hours).

2 The Model

The city traffic model is essentially similar to the work presented by Chowdhury and Schadschneider [17], and Brockfeld et al. [18]. There are N^2 nodes (intersections) $I_{i,j}$, $i = 1, \ldots, N$, $j = 1, \ldots, N$, which form a square lattice. Each node has two incoming links (one-lane and one-way streets): one from west-side and one from south-side, and two leaving links: one towards east-side and one towards north-side, Fig. 1. Nodes make a decision which traffic stream should be served. In addition to the previous work, [17,18], here a setup time $\tau = 2$ is specified (the amount of time for which the both streams have "red light" when switching from one stream to the other).

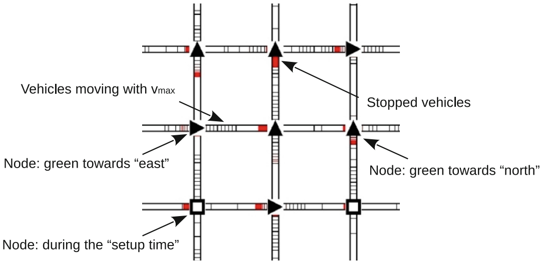

Fig. 1. A sample view of part of a grid-like network with link length $D = 100$.

The links in a network represent a single-lane street which is a one-dimensional cellular automaton with D cells ($D = 100$ is used throughout this paper). An occupied cell n symbolizes a single vehicle, and a discrete, integer variable v_n corresponds to its velocity. Let the maximum allowed velocity be $v_{\max}$ (here $v_{\max} = 5$) and the distance to the next vehicle is d_n, the distance to the next intersection is s_n. In the classical model by [14] with urban-like modifications [18], which take into account traffic light, the four consecutive steps for parallel updating at discrete time steps can be written as:

1. Acceleration: $v_n \leftarrow \min(v_n + 1, v_{\max})$,
2. Breaking:
   - Traffic light at the intersection to which the link is connected is "red" or the intersection is in setup time: $v_n \leftarrow \min(v_n, d_n - 1, s_n - 1)$
   - Traffic light is "green". If two cells behind the intersection are occupied: $v_n \leftarrow \min(v_n, d_n - 1, s_n - 1)$, otherwise $v_n \leftarrow \min(v_n, d_n - 1)$,
3. Randomization with the probability P: $v_n \leftarrow \max(v_n - 1,\ 0)$,
4. Vehicle movement: $x_n \leftarrow x_n + v_n$.

The initial density $\rho = m/D$ is the number of vehicles m divided by the total number of cells in the link, D. For given $v_{\max}$ there exists a maximum density for which all the vehicles can move freely with $v_{\max}$. In the deterministic limit $P = 0$, $\rho_{\max} = (v_{\max} + 1)^{-1}$ since for $\rho > \rho_{\max}$ there exists at least one vehicle which has less than $v_{\max}$ occupied cells in front of it, and therefore it is forced to slow down ($v_{\max} = 5$ give $\rho_{\max} = 0.16(6)$ for $P = 0$, and $\rho_{\max} \approx 0.15$ for $P = 0.1$). With each link there is associated the mean flux J' (number of vehicles leaving the link per unit time), for the entire network $\bar{J} = N^{-2} \sum_1^{N^2} J'_i$ is just the average of mean flux J'_i for each link i. Note that the assumed $v_{\max} = 5$ should be equivalent to about 50 km/h in a real city traffic flow, assuming that a single cell corresponds to a real size of 7.5 m (a vehicle length with safety distance in front and behind it), each step is about 2 s in real time.

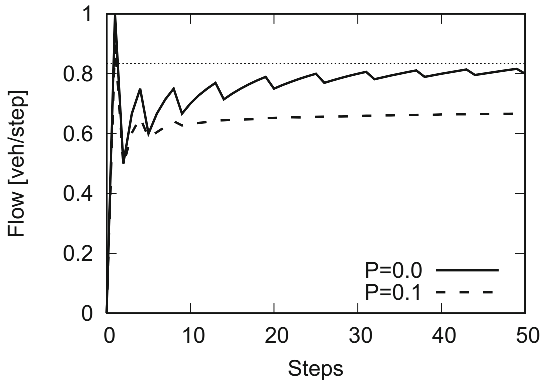

Fig. 2. Flux as a function of steps after opening an intersection for a CA fully filled with vehicles ($P = 0$ is exact, $P = 0.1$ is the average for 10^6 simulations). The horizontal line represents the exact limiting flux for $P = 0$, $J_{\max} = v_{\max}/(v_{\max} + 1) = 5/6$.

Boundary conditions can be either periodic or stochastic. If periodic boundary conditions are assumed, each vehicle leaving the network at east/north side will be placed at the beginning of corresponding links at west/south side. As the stochastic BCs, the so called expanded stochastic boundaries are applied [19]. These are formed by placing an additional CA with length equal to $v_{\max}$ as a source of vehicles. Vehicles appear at the beginning of such small CA with given probability P_{ins} and accelerate according to the CA rules. Such treatment is a proper insertion strategy which makes sure that all possible system states can be obtained. Here, in networks with stochastic sources, the right-most and the top-most nodes act as simple sinks, i.e. nothing prevents a vehicle from leaving the system.

2.1 Periodic Switching

The simplest possible strategy for control is to use cycle-based switching. For each node the cycle is: *(a)* "red light" for $(T - 2\tau)/2$ steps; *(b)* setup time for τ steps; *(c)* "green light" for $(T - 2\tau)/2$ steps; and *(d)* setup time for τ, giving T steps in total. Additionally, there can be phase shifts $T^{\phi}_{(i,j)}$ for different nodes in network. This means that the first step of the cycle for $N_{(i,j)}$ is realized at the time step $t + T^{\phi}_{(i,j)}$. It is easy to show that for unidirectional networks one can form "green waves" along a single direction by selecting the phase shifts as $T^{\phi}_{(i,j)} = (i + j - 2)T_{\text{delay}} \bmod (2T + 2\tau)$, $T_{\text{delay}} = D/v_{\max}$.

2.2 Self-controlling Strategy

As the responsive self-organizing controller a CA version of the LH strategy [10] is implemented. Below only brief summary of the most important principles is presented, see the original paper for detailed formulation and related proofs (the symbols used here are the same as in the cited work).

Let σ denote the stream which get "green light",

$$\sigma = \begin{cases} \text{head } \Omega & \text{if } \Omega \neq \emptyset \\ \arg\ \max_i \pi_i & \text{otherwise,} \end{cases} \tag{1}$$

where Ω is an ordered set containing stream indices π_i is a priority index for the corresponding stream i (the regular lattice networks have $i = 0$ or $i = 1$). The stabilization strategy assures that each stream i will be placed into the queue Ω at least once in $T_{\max}$ and, on average, once in T_{avg}. The priority index for stream i, provided that currently served stream is σ, is defined as

$$\pi_i = \frac{\hat{n}_i}{\tau_{i,\sigma}^{\text{pen}} + \tau + \hat{g}_i}, \tag{2}$$

where $\hat{n}_i$ is the number of vehicles expected to be served in time $\tau + \hat{g}_i$ for the stream i, τ is the remaining setup time, $\hat{g}_i$ is time required to clear existing queue at the intersection and all vehicles arriving just after clearing, provided that they arrive with the maximum flow rate (i.e., as a platoon traveling with $v_{\max}$), $\tau_{i,\sigma}^{\text{pen}}$ is the additional penalty term for switching from stream σ to i.

Originally, the authors have formulated the strategy using continuous equations based on kinematic waves approach [13]. Implementing it in a CA is not a straightforward task, especially if a nondeterministic NS model is considered, $P > 0$. It has been done in previous work [12], however, here calculating predictive variables is improved and the more realistic $P > 0$ is implemented. Note that non-zero randomization, $P > 0$, is of fundamental importance for the NS model. It makes it possible to reproduce such phenomena as spontaneous jam formation and destroys any artificial metastable states.

The difficulty for implementing $P > 0$ comes from the fact, that in order to calculate the priority index (2), one has to find variables characterizing the state of a crossing node at the current time step and also in the future. For each node, it is necessary to calculate the anticipated amount of the green time $\hat{g}_i$ which is the largest possible solution of

$$N_i^{\text{dep}}(t) + \hat{g}_i(t) Q_i^{\max} = N_i^{\exp}(t + \tau_i(t) + \hat{g}_i(t)), \tag{3}$$

where $N_i^{\text{dep}}(t)$ denotes the number of vehicles which have departed from the crossing, $N_i^{\exp}(t)$ is the number of vehicles which are expected to arrive at the node by the time t, $\tau_i(t)$ i the remaining setup time, $Q_i^{\max}$ is the saturation flow rate. The number of vehicles expected to leave the intersection is $\hat{n}_i(t) = \hat{g}_i(t) Q_i^{\max}$.

In the discussed CA model, it is trivial to keep track of N_i^{dep}: for each intersection one has to count the number of vehicles which have left the node. In order to find the number of vehicles which will approach the node in the following steps $t + \Delta t$ (t being the current step), $N_i^{\exp}(t + \Delta t)$, a temporary CA is created, which consists of a link connecting to the node and a link which leaves this node. Then this temporary automata is advanced for Δt steps according to the NS rules. Joining the two links is necessary in order to take into account any spill-back effects arising when there is some congestion immediately after the intersection. This procedure may be a bit time consuming but it can be efficiently implemented using appropriate caching mechanisms.

Note that this method of calculating N_i^{exp} will inevitably lead to inefficiency of the controlling method if $P > 0$. The reason for this is that advancing the temporary CA may give different value of N_i^{exp} then the "real" value obtained when advancing the entire CA system. This is desirable since in any realistic traffic model, there will be some velocity fluctuations making it impossible to exactly predict the value of $N_i^{\text{exp}}(t + \Delta t)$.

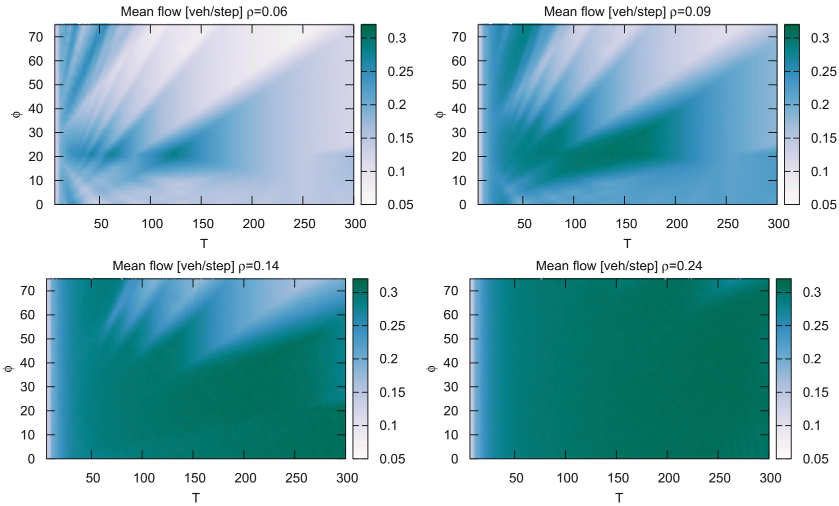

Fig. 3. A map of the mean flow $\bar{J}$ in the regular periodic network as a function of periods T and phase-shifts ϕ for four different densities for the fixed cycle controlling. $N = 6, D = 100, P = 0.1$.

Additionally, there is an important difference in defining the maximum flow rate $Q_i^{\max}$ in the continuous approach and the one using a cellular automata. In the former it can be assumed as a constant value, whereas in the latter it depends on time. Consider an infinitely long CA fully filled with vehicles and connected to an intersection with "red light". Assuming that at the moment $t = 0$, the light will turn green, vehicles will leave the intersection at the flow rate J which is presented in Fig. 2. It can be shown that in the deterministic limit $P = 0$, the limiting maximum flux is $J_{\max} = v_{\max}/(v_{\max}+1)$ (p. 240 in [3]). For non-deterministic models, $0 < P < 1$, there is no analytic solution for the limiting $J_{\max}$. However, in order to properly implement the LH mechanism, one has to use $Q_i^{\max}(t_g)$ which depends on the time t_g which denotes for how many steps the considering link has been granted "green light". In any case considered here, $Q_i^{\max}(t_g)$ has been precalculated: averaged over 10^6 stop-and-go CA simulations and tabularized in order to be useful for finding $\hat{g}_i$.

Finally, if there is more than one CA which belong to the same stream i (multiple lanes, bidirectional networks), the corresponding values of $N_i^{\exp}$, $Q_i^{\max}$, etc., are simply summed up for all the CA and a single value of π_i is calculated.

3 The Results

The correctness of implementation end efficiency of the LH strategy has been validated using three various scenarios: periodic network; periodic network with the possibility of vehicle turning; a bidirectional network with stochastic BCs and random intersection blocking.

3.1 Periodic Network $N = 6$

The dynamics of fixed cycle based switching for periodic networks with $P = 0$, has been discussed in detail in [18]. Here it is shown how the mean flow $J(\rho, T)$ depends on ρ, T and T^{ϕ} for wide range of relevant parameters for $N = 6$ and the non-deterministic $P = 0.1$. All the results are averaged by performing 10^5 steps for 10 different initial conditions.

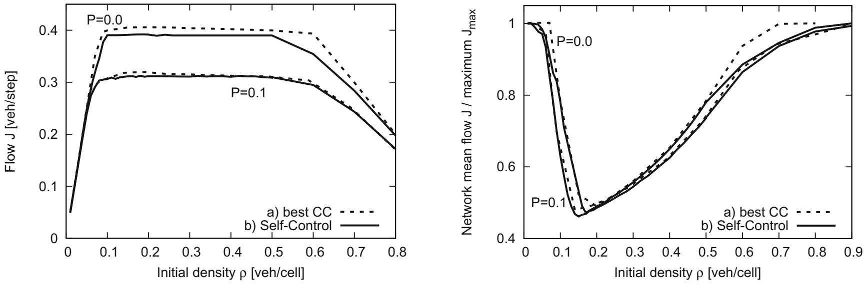

Fig. 4. Left: mean flow $J_{\text{best}}^{\text{CC}}$ in the $N = 6$ network for CC and J^{SC} for SC. Right: the same $\bar{J}$ but normalized with the maximum flux $J_{\max}$ taken from the fundamental diagram for $P = 0$ and $P = 0.1$ (precalculated and interpolated).

Figure 3 displays how the mean flow $\bar{J}(T, \phi)$ depends on the fixed cycle length T and phase-shifts ϕ for four different densities. Naturally, this CC strategy imposes a certain dynamical situation rather than being responsive to the current traffic state. If T and ϕ are properly adjusted, vehicle platoons which are formed get "green wave" giving maximum possible flow rate $\bar{J}$. If density is small enough, i.e., platoon length $\rho v_{\max} D$ per link is shorter than $D/2 - \tau v_{\max}$, that is $\rho < (2v_{\max})^{-1} - \tau/D$, then there exists cycles and for which vehicles can move without stopping and the resulting mean flow $\bar{J} = J_{\max}$. On the other hand, for some values of T platoons are always stopped when arriving to the intersection. Consequently one can observe significant variations (by $\approx 100\%$) in $\bar{J}$ especially for smaller densities, $\rho < \rho_{\max}$, where clearly there is the largest potential for optimization. If density is too large, nothing can be done in terms of adjusting T and ϕ and there is no optimization which can significantly improve situation.

Comparison between CC and SC strategies for various densities is shown in Fig. 4. In these plots $J^{\text{CC}}_{\text{best}}$ for CC represent the maximum possible value, i.e., is calculated for given ρ by simulating flows for all $1 \le T \le 300$ and $0 \le \phi \le 300$ and choosing the largest $\bar{J}$ (the same procedure is done in the next section). The decentralized SC converges to the optimum in the region where optimization is possible (the stabilization parameters are $T_{\text{avg}} = 150$ and $T_{\max} = 300$).

3.2 Periodic Network with Non-deterministic Turning

Introducing the possibility of vehicle turning (with the probability P_{turn}) makes an important difference when comparing to the previous case. Regular vehicle platoons can not be formed anymore, since they are separated with empty spaces resulting from changing a vehicle's direction (which in turn can form other platoons). A constant cycle controller can not adjust to such situation.

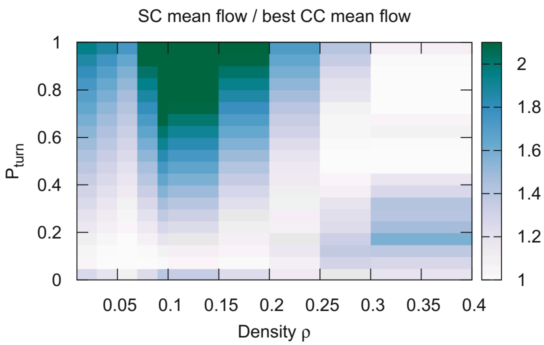

Fig. 5. The ratio $J^{\text{SC}}/J^{\text{CC}}_{\text{best}}$ as a function of mean density $\bar{\rho}$ and vehicle turning probability P_{turn}.

Figure 5 depicts the ratio of mean flows for the self-controlled and the best CC outcome. It is clear that in the region where optimization is possible (sufficiently small density), the SC outperforms the best possible CC by a factor of 2.

3.3 Network with Stochastic Input

As a final example we use a network with more realistic, stochastic boundaries (as described earlier) at the east and the south side, and open BCs at the west and the north side. For a single lane, $P_{\text{ins}} = 1.0$ will produce a flow with the maximum $J_{\max}$. Obviously for concurring streams, $J_{\max}$ can not be reached for $\rho > \rho_{\max}$, hence there must be a maximal P_{ins} above which one the mean flow will not increase. Figure 6(a) shows how $\bar{J}$ depends on the vehicle insertion probability. Also in this case, the decentralized strategy is able to form vehicle

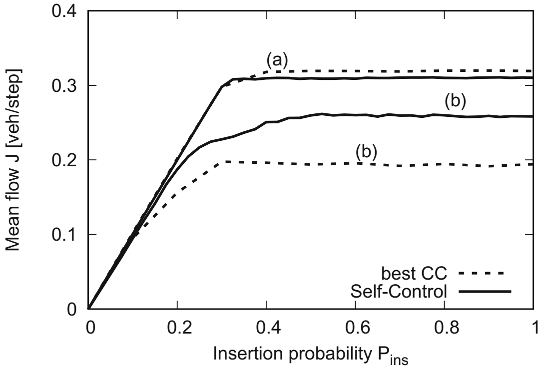

Fig. 6. Mean network flow as a function of vehicle insertion probability. (a) no turning, (b) turning into south-north direction with the probability $P_{\rm turn} = 1/4$ completely breaks down the CC control.

platoons and green waves so the optimal J is reached. Moreover, if heterogeneous turning is introduced – vehicle can turn from east-west towards south-north lanes with $P_{\rm turn} = 0.25$ – all the coordination in CC controlled network is lost. On the other hand, SC is able to recover quite well.

4 Conclusions

It has been shown how the self-controlled strategy proposed in [10] can be implemented in the classical cellular automata model of traffic [14] in the context of urban road networks [18]. Since the original formulation of the SC control is with continuous model based on kinematic waves, it is not straightforward to apply it in a CA model. In particular, the problems arise if the nondeterministic breaking in the CA is applied, $P > 0$. This is solved by using appropriate precalculated time-depended maximum fluxes $Q_i^{\rm max}$.

The presented simulations demonstrate that SC, by means of self organization, converges to the best possible fixed cycles in the case of regular networks with periodic and stochastic BCs. Additionally, if randomized scenarios are considered (e.g., vehicle turning), CC can not control flow in an optimal way since some responsiveness is required. In these cases SC significantly outperforms the best fixed cycle networks. In the future work, the presented model will serve as a test bed for other optimization methods for more complex network topologies and vehicle routing algorithms.

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