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FLOW–EXCITED ACOUSTIC PULSATIONS IN DUCTS WITH CLOSED SIDE BRANCHES

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In flow ducts with closed side branches strong acoustic pulsations are often induced. This was shown by test results performed for systems with a single side branch and co-axial branches with the same lengths. With growing the flow velocity an excitation of successive resonant modes was observed. Their frequencies were increased together with the flow velocity but at peaks of sound pressure there was an excellent agreement between measurements results and theoretical predictions. A conversion of fluctuating flow energy to energy of resonant acoustic field was included in theoretical consideration by means of negative resistance in impedance model of branches. Hence, it was possible to predict a stronger nonlinearity in the case of a duct with co-axial branches. It was found that a characteristic parameter of the analyzed phenomenon is Strouhal number. Its values for all modes are within the same range approximately and, in addition, it determines a change of acoustic inertance at the branch opening.

1. Introduction

In industrial air transport systems a closed side branch of main duct may be a potential source of strong acoustic pulsations [1, 2]. When dimensions of the branch cross-section are much smaller than a branch length, an excitation of resonant modes corresponding approximately to odd multiples of a quarter wavelength along the branch are observed [6, 18]. A high level of generated sound may induce vibration of the system construction which can cause serious damage.

The mechanism of sound excitation in the system with a single side branch is similar to that causing a generation of self-sustained oscillations in deep cavity [4, 10, 14] or Helmholtz resonator [7, 12, 13] exposed to the grazing flow. An increase of acoustic energy in the system is the result of interaction between unstable shear layer and resonant modes of the branch. First, at the point of flow separation the acoustic field in the branch opening causes a transfer of mean flow energy to shear layer which involves a transformation of continuous shear layer to large scale discrete vortices. These vortices are convected with the flow and interact with the downstream corner of the branch opening. At this point a conversion of fluctuating flow energy to energy of resonant acoustic field takes place [13].

On the other hand, in accord with classical acoustics a presence of the closed side branch in a duct causes substantial reflections of acoustic waves traveling along the duct at frequencies close to resonant modes [8]. This reduces a transmission of acoustic energy past a junction between branch and duct. The filter property of side branch resonators is often used in flow duct systems in order to suppress a narrowband noise produced by machines using atmospheric air as a working medium [15, 17].

From the above it follows that, depending on flow properties such as velocity or turbulence intensity, the side branch in the duct may cause an increase in sound level at resonant frequencies or may reduces transmission of acoustic energy during resonance. In this paper these opposite effects will be analyzed by means of simple models of acoustic waves transmission in two variants of flow systems: a duct with single side branch (Subsec. 2.1) and a duct involving co-axial branches (Subsec. 2.2). The next part of the work presents test results including measurements of frequency and pressure level of pulsations induced in the systems (Sec. 4).

2. Theoretical background

A long circular duct represents an acoustic system which possesses dimensions compared to a wavelength. Therefore, it is not possible to treat the system as one having lumped constants, and it must instead be considered as one having distributed constants. When a diameter of the duct is constant, the acoustic inertance and compliance are distributed uniformly along the duct, and the acoustic motion is wave-like as well as in unbounded space. If walls of the duct are sufficiently smooth to neglect viscous losses, then acoustic waves traveling along the duct may be considered as plane waves. In this case a wave impedance at any cross-section of the duct is

$$R_w = \frac{\varrho c}{\pi r_d^2}, \tag{1}$$

where ϱ denotes the air density, c is the sound speed and r_d is a radius of duct. When there is a motion of air in the duct with mean velocity U, an effect of sound waves convection causes a change of the wave impedance R_w. If the sound waves travel in the direction of the flow then the wave impedance is following

$$R_w^+ = R_w(1 + M), \tag{2}$$

where $M = U/c$ denotes Mach number. Otherwise

$$R_w^- = R_w(1 - M). \tag{3}$$

2.1. *Transmission of acoustic waves in duct with single side branch*

The presence of a single side branch in the duct causes the acoustic impedance at the junction differs from the wave impedance which is the characteristic value for a plane wave, and reflected and transmitted waves are produced consequently. Assume that an incident plane wave is propagated in the direction of the flow (Fig. 1). An acoustic

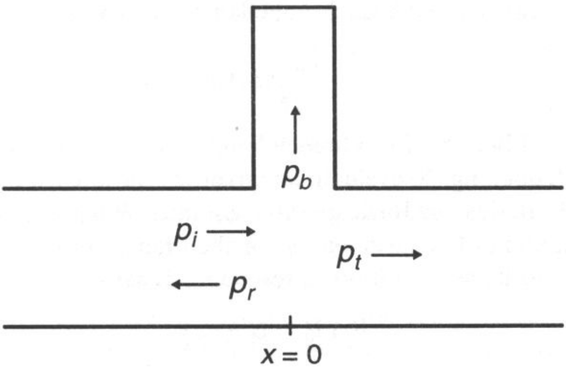

Fig. 1. Transmission of acoustic wave beyond junction of duct and single side branch.

pressure produced by this wave is

$$p_i = A_i e^{j(\omega t - k^+ x)}, \tag{4}$$

where $\omega = 2\pi f$ is an angular frequency, $k^+ = k/(1+M)$ and $k = \omega/c$ is a wave number. At the junction of the duct and the branch, the reflected wave

$$p_r = B_r e^{j(\omega t + k^- x)}, \tag{5}$$

where $k^- = k/(1-M)$ and transmitted wave

$$p_t = A_t e^{j(\omega t - k^+ x)}, \tag{6}$$

are created. If cross-sections of the duct and the branch are small in comparison to wavelength, then at the point of junction, chosen in Fig. 1 as the origin of the x coordinate, the following conditions of continuity of pressure and volume velocity are satisfied:

$$p_i + p_r = p_b = p_t, \tag{7}$$

$$U_i + U_r = U_b + U_t, \tag{8}$$

where

$$U_i = \frac{p_i}{R_w^+}, \qquad U_r = -\frac{p_r}{R_w^-}, \qquad U_b = \frac{p_b}{Z_b}, \qquad U_t = \frac{p_t}{R_w^+}, \tag{9}$$

p_b and U_b are a pressure and a volume velocity at a branch opening and Z_b is a branch impedance. An insertion of Eqs. (4) – (6) into Eqs. (7) – (9) leads to an expression for the sound power transmission coefficient α_t

$$\alpha_t = \left|\frac{A_t}{A_i}\right|^2 = \left|\frac{2Z_b}{2Z_b + R_w(1-M^2)}\right|^2, \tag{10}$$

where operator $|\cdot|$ denotes modulus of complex number. As follows from Eq. (10), an influence of sound wave convection on transmission of acoustic energy beyond the junction is negligible at low Mach number flows ($M^2 \ll 1$). In this case the transmission coefficient depends only on the branch impedance Z_b. The general expression for this impedance is

$$Z_b = R_s + j\frac{\varrho c}{\pi r_b^2}\left[k\Delta d - \cot(kd)\right], \tag{11}$$

where r_b is a radius of branch, d is a branch length and R_s is a resistance due to viscous action in the branch opening. To evaluate an unknown end correction Δd, the theoretical value $\Delta d_0 = 8r_b/3\pi$ derived by Rayleigh [8] is assumed. When viscous losses is negligible, there is not a dissipation of acoustic energy in the branch. In this case the transmission coefficient equals zero if the condition of resonance is satisfied

$$k\Delta d_0 = \cot(kd), \tag{12}$$

which can be approximated to the form

$$f_m \approx \frac{c(2m-1)}{4(d+\Delta d_0)}, \qquad m = 1, 2, 3\ldots . \tag{13}$$

This means that the incident sound wave is totally reflected from the junction and returned towards the source. As may be seen from Eqs. (10) and (11), in frequency ranges between resonant modes f_m the transmission coefficient α_t approaches unity. Therefore the single branch in the duct represents acoustic filter with the highest attenuation at resonant frequencies.

The analysis presented above is valid under the assumption that both the cross-sectional area of the duct and the flow velocity are small enough to maintain laminar motion of air in the duct. When a Reynolds number

$$\mathcal{R}e = \frac{r_d U}{\nu}, \tag{14}$$

where ν is the coefficient of kinematic viscosity, is much greater than 1160 the flow in the duct becomes turbulent [16]. To adopt the outline model of wave transmission to this situation, the incident sound wave will be now interpreted as acoustic perturbation generated by flow disturbances.

As mentioned in Introduction, an existence of unsteady motion in the area close to the branch opening may cause a conversion of fluctuating flow energy to energy of acoustic field inside the branch. Finally the high-level acoustic pulsations may be created in the system. An increase in the system acoustic energy can be incorporated in the theoretical analysis by putting the negative resistance R_n in the impedance model of the branch. Therefore, at low Mach number flows the equation for transmission coefficient may be rewritten in the form

$$\alpha_t = \left|\frac{\dfrac{R_n}{R_w} + jX}{\dfrac{R_n}{R_w} + \dfrac{1}{2} + jX}\right|^2, \tag{15}$$

where $X = (r_d/r_b)^2[k\Delta d_0 - \cot(kd)]$, It results from Eq. (15), that for values of R_n/R_w decreasing from 0 to -0.25 the system behaves like an acoustic filter with attenuation growing at resonant frequencies. When $R_n/R_w = -0.25$ the coefficient α_t is constant and equals unity for all frequencies. At values of R_n/R_w smaller than -0.25 the coefficient α_t is greater than unity at resonance. This means that initial acoustic perturbations travelling along the duct are amplified in the system.

As can be seen from Eq. (15), for values of R_n/R_w close to -0.5 the coefficient α_t may possess any high value at resonant frequencies. It corresponds to unbounded growth of pulsations amplitude. Theoretically this situation is possible in the undamped oscillator operating as linear system. In real conditions an unbounded increase in amplitude at resonance is limited by nonlinear effects. In the considered system the nonlinearity in the branch opening produces an additional loss resistance which increases with the growth of pulsations amplitude. When a balance between acoustic energy losses and the energy extracted from flow perturbations is reached, the stable acoustic pulsations with frequencies close to resonant modes as well as several harmonics will appear in the system.

2.2. Transmission of acoustic waves in duct with co-axial branches

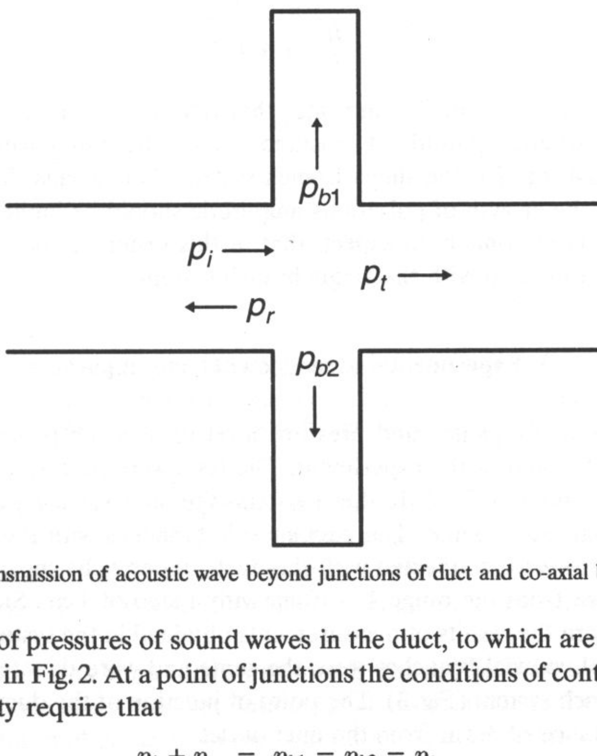

Fig. 2. Transmission of acoustic wave beyond junctions of duct and co-axial branches.

A distribution of pressures of sound waves in the duct, to which are attached co-axial branches, is shown in Fig. 2. At a point of junctions the conditions of continuity of pressure and volume velocity require that

$$p_i + p_r = p_{b1} = p_{b2} = p_t\,, \tag{16}$$

$$U_i + U_r = U_{b1} + U_{b2} + U_t\,, \tag{17}$$

where

$$U_i = \frac{p_i}{R_w^+}\,, \qquad U_r = -\frac{p_r}{R_w^-}\,, \qquad U_t = \frac{p_t}{R_w^+}\,, \qquad U_{b1} = \frac{p_{b1}}{Z_{b1}}\,, \qquad U_{b2} = \frac{p_{b2}}{Z_{b2}} \tag{18}$$

and Z_{b1}, Z_{b2} are impedances of branches. After inserting Eqs. (4) – (6) into Eqs. (16) – (18) the following expression for transsmision coefficient may be obtain

$$\alpha_t = \left| \frac{2Z_{b1}Z_{b2}}{2Z_{b1}Z_{b2} + R_w(1-M^2)(Z_{b1}+Z_{b2})} \right|^2, \tag{19}$$

Under the assumption that the branches have the same cross-sectional area and length, the expression (19) reduces to the form

$$\alpha_t = \left| \frac{Z_b}{Z_b + R_w\,(1-M^2)} \right|^2, \tag{20}$$

where Z_b is the impedance of single branch from Eq. (11). If the transfer of energy from flow disturbances to acoustic field will be included in a impedance model of the branches and, moreover, low Mach number flow will be assumed ($M^2 \ll 1$), then

$$\alpha_t = \left| \frac{\dfrac{R_n}{R_w} + jX}{\dfrac{R_n}{R_w} + 1 + jX} \right|^2. \tag{21}$$

A comparison of Eqs. (15) and (21) indicates that values of α_t calculated from the equations are identical when magnitude of resistance R_n for the system with co-axial branches is twice bigger than that for the single branch system. This means that in the duct with co-axial branches the growth of pulsations amplitude should be limited by stronger nonlinearity. Thus, it is reasonably to expect, that in this system a harmonic distortion will be much greater compared with the single branch system.

3. Experimental arrangements and apparatus

Measurements of frequency and pressure level of acoustic pulsations generated in the systems were the aim of the experiment. The tests were performed in the laboratory arrangements presented in Fig. 3. In these systems the duct was composed of the circular pipe with the radius $r_d = 8$ mm. The circular side branches with the radius $r_b = 5$ mm and variable lengths were connected with the ducts under right angle. The lengths used in experiment were from the range 1 – 10 cm with a step of 1 cm. Since the aim of the tests was to compare the results of measurements obtained in the systems, it was assumed that the lengths of co-axial branches were the same and were denoted as d, likewise as for the single branch system (Fig. 3). The point of junction of the duct and branches was chosen at the distance of 3.2 m from the duct outlet.

The systems were supplied with the compressed air at the maximum pressure 0.5 MPa. The maximum velocity $u_{\max}$ of the air stream was measured by using a Pitot tube with diameter of 1.6 mm and a liquid-column manometer in the shape of letter U. The probe was mounted in the center of the duct outlet. The mean flow velocity U of the air in the duct was calculated from formula [11]

$$U = \frac{49}{60} u_{\max}. \tag{22}$$

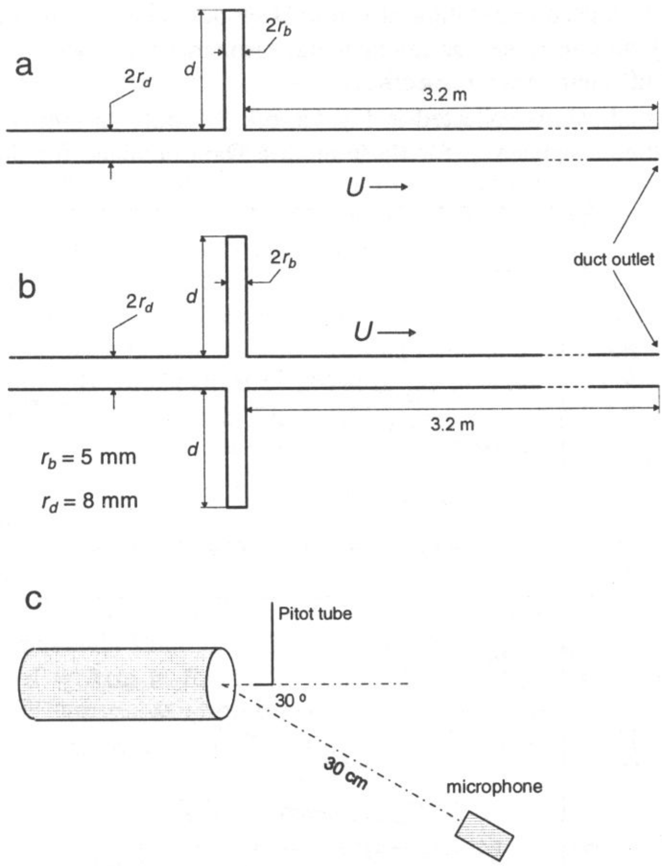

Fig. 3. Experimental setups: a) duct with single side branch, b) duct with co-axial branches, and c) position of Pitot tube and microphone at duct outlet.

The tests were carried out in the range of U from 40 m/s to 128 m/s in which a generation of acoustic pulsations was observed. The maximum value of U corresponds to a maximum efficiency of flow installation. The Reynolds number calculated from Eq. (14) possesses values from the range of $2 \cdot 10^5$ to $7 \cdot 10^5$. Thus, it should be expected that an air flow in the duct was turbulent.

Acoustic measurements were made with the Brüel & Kjæar instrument setup consisting a 1" microphone and a high resolution signal analyzer 2033. The microphone was mounted at the distance of 30 cm from the duct outlet. The line joining the centre of the microphone and the centre of the duct outlet made 30° angle with air stream axis.

4. Analysis of test results

At mean flow velocity U from the range 40 – 128 m/s and lengths of branches d = 1 – 10 cm the pulsations corresponding to the resonant modes f_1, f_3 and f_5 were observed in experiment. A used denotation of resonant modes is given from Eq. (13). Therefore, f_1 mode may be interpreted as fundamental resonant mode, while f_3 and f_5 modes as its third and fifth harmonic, respectively.

Experimental results collected in Fig. 4 a, b, c illustrate an influence of flow velocity U on pulsations frequency f for these modes. Data obtained for the duct with single

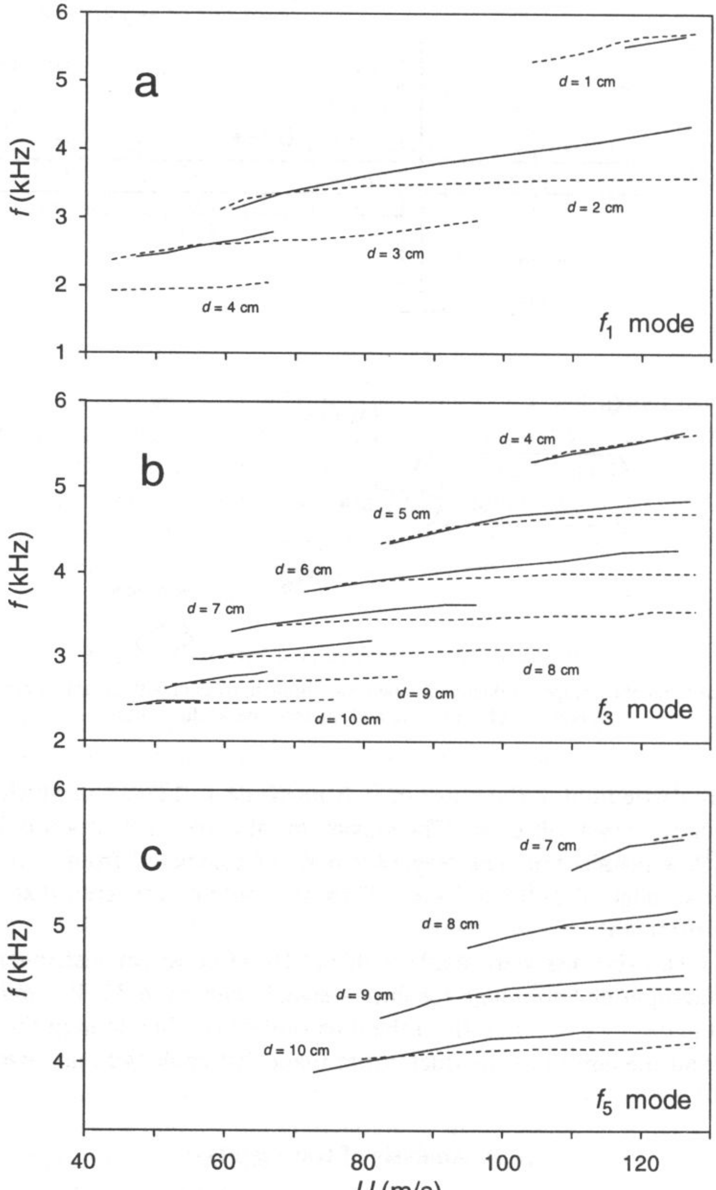

Fig. 4. Dependence of frequency of resonant modes on flow velocity U for single side branch system (———) and sytem with co-axial branches (- - - -).

branch and co-axial branches are denoted by solid and dotted lines, respectively. As may be seen, in the both systems the frequency f of acoustic pulsations grows with velocity U. However, the observed increase in f for the duct with co-axial branches is for nearly all lengths of branches smaller distinctly than for the second system. Moreover, pulsations corresponding to particular mode appear for certain values of d at different ranges of velocity U. Additionally, at length $d = 4\,\text{cm}$ a pulsations of fundamental f_1 mode are generated only in the duct with co-axial branches (Fig. 4 a). Note that plots obtained for each mode at the lowest values of d finish at velocity $U = 128\,\text{m/s}$ which corresponds to maximum efficiency of flow installation.

For each resonant mode the different value of branch length d results in a various frequency range of acoustic pulsations. With growing value of d a decrease in pulsations frequency occurs which is accompanied by a shift of the flow velocity U to lower values. This correlation explains the plots in Fig. 5, which present a dependence of Strouhal number S

$$S = \frac{2fr_b}{U} \tag{23}$$

on the flow velocity U. As may be seen, for all modes generated in the both systems, the maximum values of Strouhal number are close together ($S = 0.45 - 0.54$). The biggest differences are noted in minimum values of S. When U is smaller than the maximum value 128 m/s the minimum value of S is approximately twice smaller in the duct with co-axial branches ($d = 3\,\text{cm}$ in Fig. 5 a, $d = 7 - 10\,\text{cm}$ in Fig. 5 b). For all remaining values of d the flow velocities, at which generation of particular mode would be possible, exceed the velocity $U = 128\,\text{m/s}$ making a precise determination of minimum value of S impossible.

The fact that for certain branch lengths d the values of Strouhal number are very similar has a simple physical meaning. From Eq. (23) and the relation

$$U_c = \mu U\,, \tag{24}$$

where $\mu = 0.62$ [5] and U_c is the mean convection velocity of flow disturbances, one can obtain

$$\lambda = \frac{2\mu r_b}{S}\,. \tag{25}$$

In the above equation $\lambda = U_c/f$ is a distance between two succeeding flow disturbances which cyclically shed from an upstream edge of branch opening. More precisely, λ represents the distance between the two neighbouring points within turbulent shear layer with the highest concentration of fluid vorticity or the distance between centers of two succeeding vortices when the vorticity is accumulated into vortices. In the works [3, 9], where an assumption of wave-like flow disturbances was made, λ is called the hydrodynamic wavelength. The presented experimental results show that λ assumes values from the range $1.1 - 2\,\text{cm}$ and increases with a growth of flow velocity U. The increase in λ with U is evidently larger in the case of the duct with co-axial branches.

The data presented in Fig. 6 a show a change of frequencies f, at which the pressure level L_p reaches the maximum value, with the branches length d. By solid lines are indicated the results of frequency calculations based on Eq. (12). As may be seen, there is an

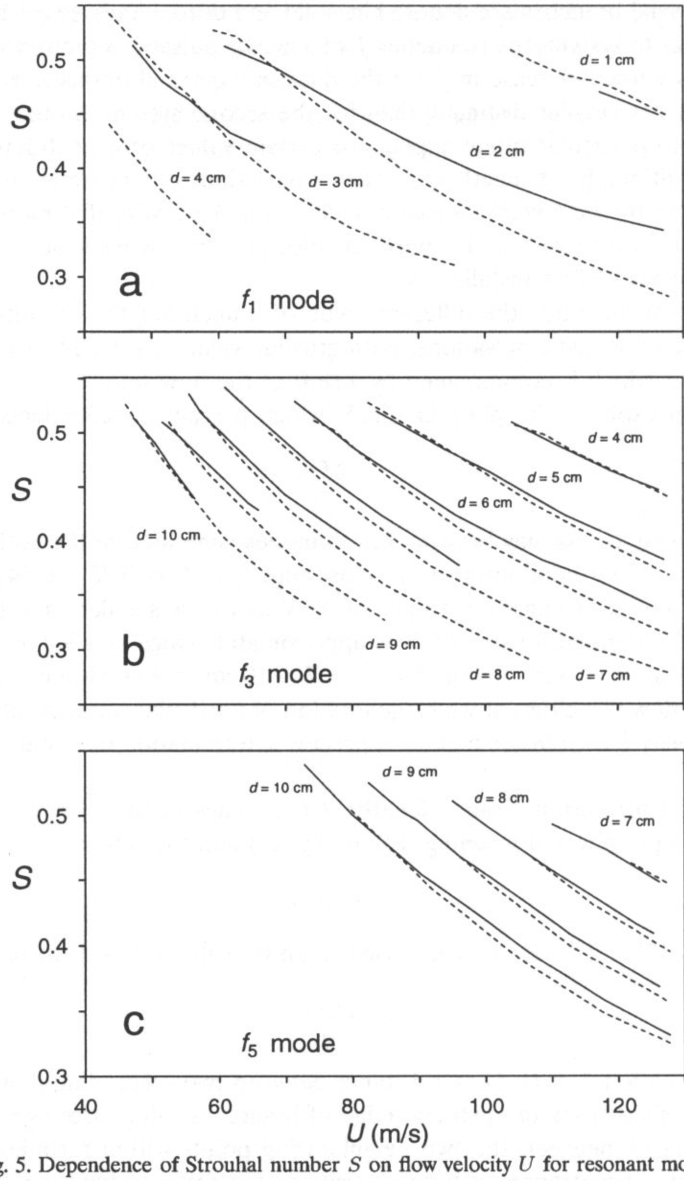

Fig. 5. Dependence of Strouhal number S on flow velocity U for resonant modes; (——) single side branch system, (- - - -) system with co-axial branches.

excellent agreement between experimental results and theoretical predictions. Therefore, an important conclusion may be drawn that the acoustic condition of resonance (12) is not influenced by air flow in the duct.

In Fig. 6 b maximum values of sound pressure level L_p of generated pulsations as a function of d are presented. As it results from experimental data, maximum values of L_p are from the range 70 – 100 dB and, as a rule, they decrease with increasing d.

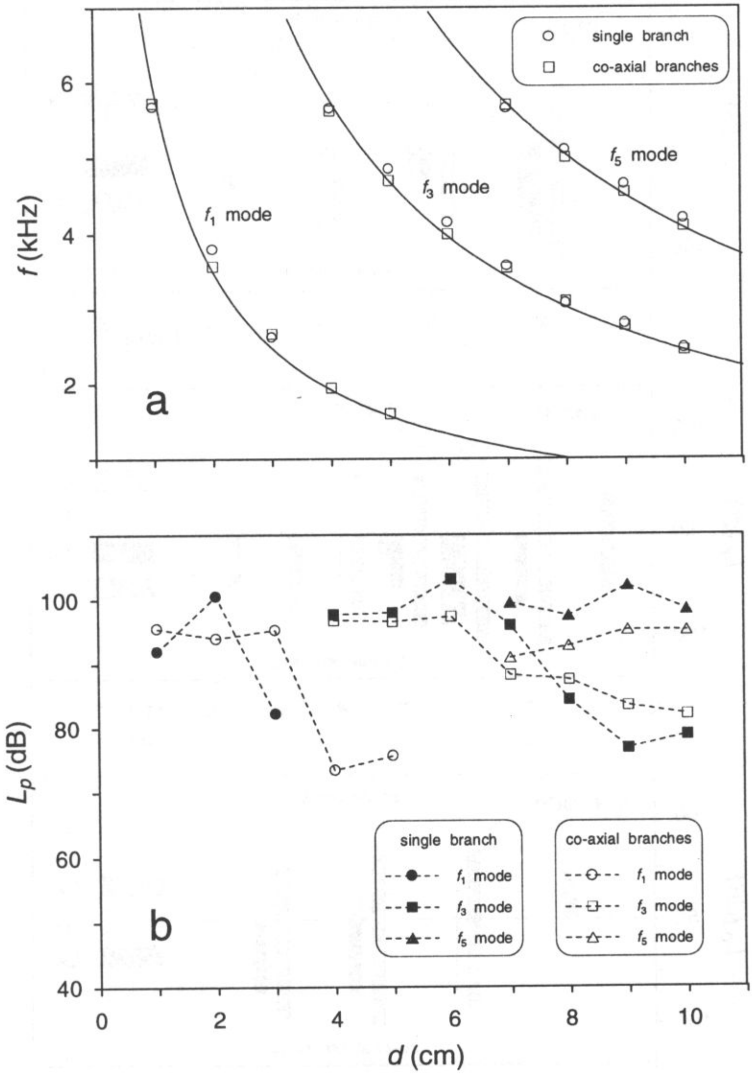

Fig. 6. a) Frequencies of resonant modes at different lengths of branches at maximum sound pressure level; (——) theoretical results, b) maximum pressure level of resonant modes.

As was well predicted by theoretical analysis (Sec. 2), a process of sound generation is associated with harmonic distortion. To illustrate this effect in Fig. 7 maximum values of L_p corresponding to harmonics of resonant modes f_1, f_3 and f_5 are displayed. For the harmonics a different denotation is used. For example, f_{3h} denotes the third harmonic of f_1 mode, whereas f_{15h} is the fifth harmonic of f_3 mode or the third harmonic of f_5 mode. From a comparison of data in Fig. 7 it follows, that in the single branch system the harmonic distortion is much weaker than in the system with co-axial branches. It can be clearly observed in the experimental data obtained for f_1 and f_3 modes. At some values

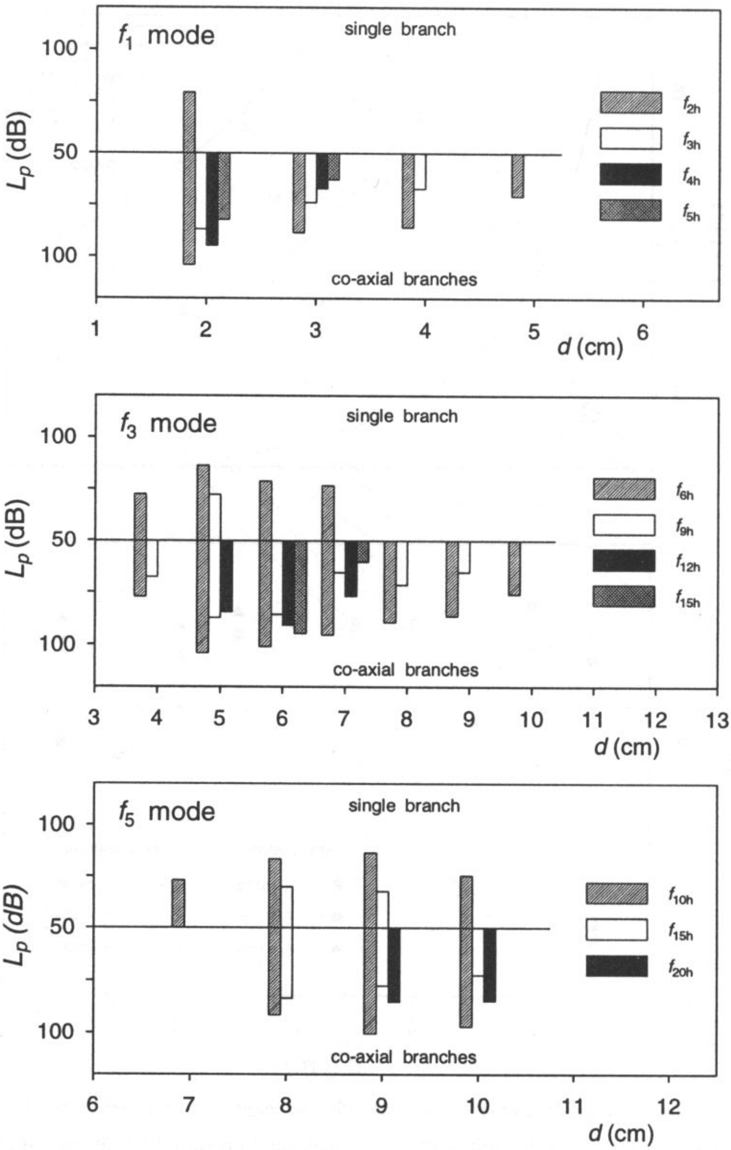

Fig. 7. Maximum pressure level of harmonics of resonant modes.

of d as much as four successive harmonics occur ($d = 2, 3$ cm for mode f_1, $d = 6, 7$ cm for mode f_3). Moreover, in some cases the level of harmonic component is higher than the level of resonant mode ($d = 2$ cm for f_1 mode, $d = 5 - 7$ cm for f_3 mode, $d = 9$ cm for f_5 mode).

As follows from previously presented data, the pulsations frequency f is generally a function of the flow velocity U (Fig. 4) and values of f calculated from the resonance condition (12) agree with measurements only in the case of maximum level of pulsations

(Fig. 6 a). These facts indicate, that in Eq. (11) describing the branch impedance the end correction Δd is the parameter which must vary with flow velocity U. Because Strouhal number seems to be a characteristic quantity of analyzed phenomenon, then it will be reasonable to seek rather a relation between Δd and S.

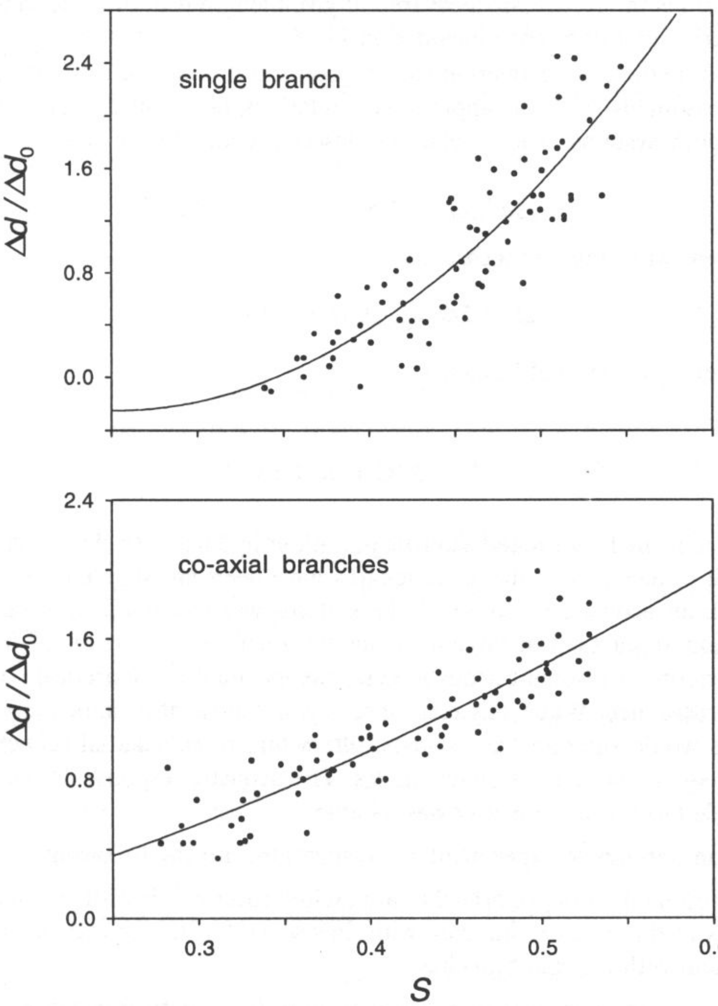

Fig. 8. Dependence of end correction Δd on Strouhal number S for systems with a) single side branch, b) co-axial branches; (——) best fit lines.

A dependence of nondimensional end correction $\Delta d/\Delta d_0$ on the Strouhal number for both systems is shown in Fig. 8. The values of Δd were calculated from Eq. (12), in which the theoretical value Δd_0 was replaced by unknown Δd

$$\Delta d = \frac{\cot(kd)}{k}. \tag{26}$$

The wave number k was evaluated from experimental results

$$k = \frac{2\pi f(d,U)}{c}, \tag{27}$$

where $f(d,U)$ is the frequency measured at given length d of branches and flow velocity U. All experimental data were included in Fig. 8.

The used method of correlation gave a good result. Since sets of data points in Fig. 8 correlate reasonably well, an approximate relations between $\Delta d/\Delta d_0$ and S may be found for both systems. These relations describe equations of best fit lines drawn in Fig. 8:

$$\Delta d/\Delta d_0 = 1.53 - 14.2S + 28.2S^2 \tag{28}$$

for the system with single branch and

$$\Delta d/\Delta d_0 = -0.32 + 1.9S + 3.2S^2 \tag{29}$$

for the system with co-axial branches.

5. Conclusions

In the paper the flow-excited acoustic pulsations in ducts with closed single side branch and co-axial branches with the same lengths have been investigated. In theoretical part of the work an acoustic response of the systems was examined by means of a simple model of sound waves transmission along the duct. A conversion of fluctuating flow energy to energy of resonant acoustic field was included in theoretical consideration by a negative resistance. When this resistance was not present in impedances of branches, the systems would operate like acoustic filters due to substantial reflections of sound waves at frequencies near resonant modes. The acoustic response of the systems could be unbounded unless nonlinearity was included.

The main findings of experimental investigations are the following:

1. Odd resonant modes of branches are excited successively with growing flow velocity. Frequencies of these modes increase with flow speed but this growth is distinctly smaller for the system with co-axial branches.

2. Pulsations in both systems reach maximum level at frequencies corresponding to classical resonance condition for quarter-wave resonator.

3. The flow-resonant response of branches occurs within the range 0.25 – 0.55 of Strouhal number.

4. The excitation of resonant modes is accompanied by high nonlinearity. It was especially observed in the case of duct with co-axial branches as was well predicted by theory.

5. Changes of the end correction of side branch resonators depend on Strouhal number.

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