Examination of the Effect of a Sound Source Location on the Steady-State Response of a Two-Room Coupled System

Mirosław MEISSNER

Institute of Fundamental Technological Research
Polish Academy of Sciences
Pawińskiego 5B, 02-106 Warszawa, Poland
e-mail: mmeissn@ippt.gov.pl

In this paper, the computer modelling application based on the modal expansion method is developed to study the influence of a sound source location on a steady-state response of coupled rooms. In the research, an eigenvalue problem is solved numerically for a room system consisting of two rectangular spaces connected to one another. A numerical procedure enables the computation of shape and frequency of eigenmodes, and allows one to predict the potential and kinetic energy densities in a steady-state. In the first stage, a frequency room response for several source positions is investigated, demonstrating large deformations of this response for strong and weak modal excitations. Next, a particular attention is given to studying how the changes in a source position influence the room response when a source frequency is tuned to a resonant frequency of a strongly localized mode.

Keywords: coupled room system, steady-state room response, potential and kinetic energy densities, localization of modes.

1. Introduction

Theoretical and numerical predictions of the sound field in concert halls, theatres and sacral objects have recently attracted considerable attention within the field of architectural acoustics (GOŁAŚ, SUDER–DĘBSKA, 2009; KAMISIŃSKI *et al.*, 2009; KOSAŁA, 2009), since this interest is connected with a growing demand for the acoustical comfort in public spaces. The acoustic behaviour of coupled rooms has also been studied in the context of the architectural acoustics (ERMANN, 2005; BRADLEY, WANG, 2010) because coupled-volume systems, composed of two or more spaces that are connected through acoustically transparent openings, can be found in various buildings or constructions. The orchestra pit

and balconies in opera houses or theatres coupled to the main floor as well as churches with several naves and chapels are typical examples of architectural objects with a structure of coupled rooms (MARTELLOTTA, 2009). In order to obtain a better understanding and control of the acoustics in such room systems, it is vital to have an efficient theoretical or computational method for predicting a structure of sound field in coupled rooms. Nowadays, many numerical methods can be used to estimate the sound field in coupled rooms, like diffusion-equation models (BILLON *et al.*, 2006; XIANG *et al.*, 2009), statistical-acoustic models (SUMMERS *et al.*, 2004), the geometrical acoustics (SUMMERS *et al.*, 2005; PU *et al.*, 2011) and the modal expansion method (MEISSNER, 2007, 2008b, 2010), also known as the modal analysis. The geometrical acoustics applies at best to rooms with dimensions large compared to the wavelength. Moreover, this method neglects diffraction phenomena since a propagation in straight lines is its main postulate. A theory that fully represents the phenomenology of the sound field (but more difficult) is the modal analysis because it bases upon the wave acoustics. The wave approach can be used in a low-frequency range, thus this theory has a practical application for rooms with dimensions comparable with the sound wavelength.

In the low-frequency limit, the coupled-room systems exhibit some interesting effects like: the mode degeneration due to modification of the coupling area, confinement of an acoustic energy in a part of room system, called the localization of modes, and a considerable difference between a rate of sound decay in early and late stages of the reverberant process, known as a double sloped decay. These phenomena have been investigated by the author in recent papers (MEISSNER, 2008c, 2009a,b,c, 2011). The current work focuses on the examination of an effect of a sound source location on a steady-state response of coupled rooms. The research explores the geometry that often occurs in the reality when two rectangular subrooms with the same heights are connected to one another. A room response is described theoretically by means of a modal expansion of the sound field for a weakly damped room system. Eigenfunctions, resonant frequencies and modal damping coefficients were calculated numerically by the use of a computer implementation that exploits the forced oscillator method.

2. Room acoustics in low-frequency range

In the low-frequency range, the sound pressure field inside an air-filled enclosure is determined via a solution of a three-dimensional wave equation with specified initial and boundary conditions. In this approach, the acoustic room response is found as a superposition of individual responses of normal acoustic modes generated inside the room by a harmonic sound source (KUTTRUFF, 1973). Acoustic modes are inherent properties of the enclosure, and are determined by a room geometry and impedance conditions at the room walls. Each mode is defined by

the natural (resonant or modal) frequency ω_n, the modal damping coefficient r_n and the mode shape specified by the eigenfunction $\Phi_n(\mathbf{r})$, where $\mathbf{r} = (x, y, z)$ determines the position of the observation point and $n = 0, 1, 2, \ldots, N$. The natural number N determines the amount of modes, that should be included in the room response. It depends on the total room absorption $\mathcal{A}$ because the range of modal frequencies is bounded from above by the frequency (SCHROEDER, 1996)

$$f_s = c\sqrt{6/\mathcal{A}}, \tag{1}$$

where c is the sound speed. Below f_s the modal density is low and particular modes can be decomposed from the room response. Thus, in multi-mode resonance systems the frequency f_s marks the transition from individual, well-separated resonances to many overlapping modes. The functions Φ_n are mutually coupled through the impedance condition on absorptive walls

$$\frac{\partial p}{\partial n} = -\frac{\rho}{Z}\frac{\partial p}{\partial t}, \tag{2}$$

where $p(\mathbf{r}, t)$ is the sound pressure, ρ is the air density, Z is the wall impedance and $\partial/\partial n$ is the derivative taken in a direction normal to the surface of the room walls. However, in the low-frequency limit typical materials covering room walls are characterized by a low absorption: $\Re(Z)/\rho c \gg 1$, thus it is possible to assume that the shapes of modes are well approximated by the uncoupled eigenfunctions Φ_n determined for hard room walls (DOWELL *et al.*, 1977).

Taking all this into account and assuming that a source term in the wave equation has the form $-q(\mathbf{r})\cos(\omega t)$, where $q(\mathbf{r})$ and ω are the volume source distribution and the source frequency, the sound pressure in a steady-state can be determined by (MEISSNER, 2008a)

$$p(\mathbf{r}, t) = \sum_{n=0}^{N} [A_n \cos(\omega t) + B_n \sin(\omega t)]\Phi_n(\mathbf{r}), \tag{3}$$

$$A_n = \frac{c^2(\omega_n^2 - \omega^2)Q_n}{(\omega_n^2 - \omega^2)^2 + 4r_n^2\omega^2}, \qquad B_n = \frac{2c^2\omega r_n Q_n}{(\omega_n^2 - \omega^2)^2 + 4r_n^2\omega^2}, \tag{4}$$

where Q_n are factors describing the sound source strength

$$Q_n = \int_V q(\mathbf{r})\Phi_n(\mathbf{r})\,\mathrm{d}v, \tag{5}$$

where V is the room volume, and the modal damping coefficients r_n are determined by

$$r_n = \frac{\rho c^2}{2}\int_S \frac{\Phi_n^2}{Z}\,\mathrm{d}s, \tag{6}$$

where S is the surface of all the room walls. The functions Φ_n are mutually orthogonal and are normalized in the volume V by the relation

$$\int_V \Phi_m \Phi_n \, \mathrm{d}v = \delta_{mn}, \tag{7}$$

where $\delta_{mn} = 1$ for $m = n$ and $\delta_{mn} = 0$ for $m \neq n$. A zero-order mode is the so-called Helmholtz mode having the resonant frequency ω_0 equal to zero and the following normalized eigenfunction: $\Phi_0 = 1/\sqrt{V}$, which corresponds to a trivial solution of the eigenvalue equation

$$\nabla^2 \Phi_n + \left(\frac{\omega_n}{c}\right)^2 \Phi_n = 0. \tag{8}$$

In a steady-state, a sound source located in a room produces the acoustic energy field with a spatial density that depends on the room shape, the surface impedance on the room walls and the sound source parameters. The potential acoustic energy is stored in the form of pressure, while the kinetic one is manifested as a particle velocity. In terms of these quantities, the potential and kinetic energy densities are written as

$$w_p(\mathbf{r}) = \frac{1}{2\rho c^2} \left\langle p^2 \right\rangle, \qquad w_k(\mathbf{r}) = \frac{1}{2}\rho\langle \mathbf{u} \cdot \mathbf{u}\rangle, \tag{9}$$

where $\langle \cdot \rangle$ is the time-averaging, $\mathbf{u}$ is the particle velocity vector, and a dot denotes the scalar product. After using Eqs. (3), (4) and the momentum equation

$$\rho\frac{\partial \mathbf{u}}{\partial t} = -\nabla p, \tag{10}$$

the potential energy density $w_p(\mathbf{r})$ can be expressed as

$$w_p(\mathbf{r}) = \frac{1}{4\rho c^2}\left\{ \left[\sum_{n=0}^{N} A_n \Phi_n(\mathbf{r})\right]^2 + \left[\sum_{n=0}^{N} B_n \Phi_n(\mathbf{r})\right]^2 \right\} \tag{11}$$

and the kinetic energy density $w_k(\mathbf{r})$ as

$$w_k(\mathbf{r}) = \frac{1}{4\rho\omega^2}\left[\left(\sum_{n=0}^{N} A_n \frac{\partial \Phi_n}{\partial x}\right)^2 + \left(\sum_{n=0}^{N} A_n \frac{\partial \Phi_n}{\partial y}\right)^2 + \left(\sum_{n=0}^{N} A_n \frac{\partial \Phi_n}{\partial z}\right)^2 \right.$$
$$\left. + \left(\sum_{n=0}^{N} B_n \frac{\partial \Phi_n}{\partial x}\right)^2 + \left(\sum_{n=0}^{N} B_n \frac{\partial \Phi_n}{\partial y}\right)^2 + \left(\sum_{n=0}^{N} B_n \frac{\partial \Phi_n}{\partial z}\right)^2 \right]. \tag{12}$$

These equations indicate that time-averaged energetic quantities of the acoustic field depend directly on the modal parameters: ω_n, r_n, Φ_n and the source frequency ω, and indirectly on the position and spatial distribution of the sound source through the parameter Q_n.

When a room is excited by a point sound source with the power P, the source function $q(\mathbf{r})$ has the form of delta function i.e. $q(\mathbf{r}) = Q\delta(\mathbf{r} - \mathbf{r}_0)$, where $\mathbf{r}_0 = (x_0, y_0, z_0)$ determines the position of source point and the parameter Q is given by (KINSLER, FREY, 1962)

$$Q = \sqrt{8\pi\rho cP}. \tag{13}$$

In this case, the potential and kinetic energy densities are as follows:

$$w_p(\mathbf{r}) = \frac{Q^2}{4\rho c^2}\left\{\left[\sum_{n=0}^{N} a_n\Phi_n(\mathbf{r}_0)\Phi_n(\mathbf{r})\right]^2 + \left[\sum_{n=0}^{N} b_n\Phi_n(\mathbf{r}_0)\Phi_n(\mathbf{r})\right]^2\right\}, \tag{14}$$

$$w_k(\mathbf{r}) = \frac{Q^2}{4\rho\omega^2}\left\{\left[\sum_{n=0}^{N} a_n\Phi_n(\mathbf{r}_0)\frac{\partial\Phi_n}{\partial x}\right]^2 + \left[\sum_{n=0}^{N} a_n\Phi_n(\mathbf{r}_0)\frac{\partial\Phi_n}{\partial y}\right]^2\right.$$
$$+ \left[\sum_{n=0}^{N} a_n\Phi_n(\mathbf{r}_0)\frac{\partial\Phi_n}{\partial z}\right]^2 + \left[\sum_{n=0}^{N} b_n\Phi_n(\mathbf{r}_0)\frac{\partial\Phi_n}{\partial x}\right]^2$$
$$\left.+ \left[\sum_{n=0}^{N} b_n\Phi_n(\mathbf{r}_0)\frac{\partial\Phi_n}{\partial y}\right]^2 + \left[\sum_{n=0}^{N} b_n\Phi_n(\mathbf{r}_0)\frac{\partial\Phi_n}{\partial z}\right]^2\right\}, \tag{15}$$

where $a_n = A_n/Q_n$ and $b_n = B_n/Q_n$. Since the right-hand side of Eq. (14) is a symmetric function of the sound source and the observation points coordinates, in the case of the potential energy density the reciprocity principle is valid. This means that when we put the source at $\mathbf{r}$, we observe at point $\mathbf{r}_0$ the same potential energy density as we did before at $\mathbf{r}$, when the source was located at $\mathbf{r}_0$. As it is evident from Eq. (15), the reciprocity principle is not satisfied for the kinetic energy density.

3. Computer simulation of steady-state room response

A computer program written in Pascal language was developed for this research to investigate the influence of the sound source position on the potential and kinetic energy densities in a steady-state. The numerical simulation was performed for the enclosure consisting of two connected subrooms. A shape of this room system is shown schematically in Fig. 1. This geometry is an example of the situation often occurring in practice when two rectangular subrooms with the same heights are joined together in such a way that an energy can be transmitted between them, thus they constitute a two-room coupled system. In the numerical study, subrooms had the height h of 3 m and their lengths and widths were the following: $l_1 = 5.7$ m, $l_2 = 4$ m, $w_1 = 8$ m and $w_2 = 5$ m. The opening realizing an acoustic coupling between subrooms had the height h, the width w of 2 m and the thickness d of 0.3 m. The room system was excited by a point sound source having the power P of 0.1 W.
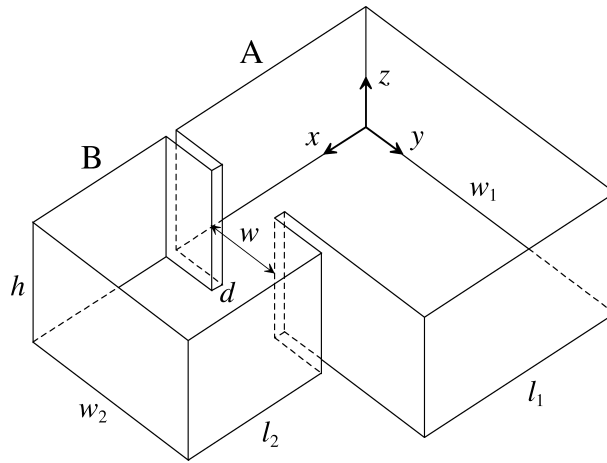
Fig. 1. Irregularly shaped enclosure consisting of two connected rectangular subrooms denoted by A and B.

In the simulation it was assumed that the walls of the subrooms are covered by an absorbing material providing a low sound damping, and the random-absorption coefficient α characterizes the damping properties of this material. For the sake of the model simplicity, the wall impedance corresponding to this absorption coefficient was purely real, i.e. the mass and stiffness of the absorbing material were neglected. This is equivalent to the damping of a sound wave with no phase change upon reflection. In this case, for a given value of the coefficient α, the wall impedance on the subrooms' walls was found from the equation (KINSLER, FREY, 1962)

$$\alpha = \frac{8}{\xi} \left[1 + \frac{1}{1+\xi} - \frac{2}{\xi} \ln(1+\xi) \right], \tag{16}$$

where $\xi = R/\rho c$ is the impedance ratio and R is the wall resistance.

In the case of an irregular room geometry, the first step towards determining the room response in a steady-state is a computation of the eigenfunctions Φ_n, the resonant frequencies ω_n and the modal damping coefficients r_n. Since a lightly damped room is considered, the functions Φ_n may be approximated by eigenfunctions computed for perfectly rigid room walls. Thus, the expression for Φ_n can be written as

$$\Phi_n(\mathbf{r}) = \frac{\Theta_k \cos\left(\dfrac{\pi k z}{h}\right) \Psi_m(x, y)}{\sqrt{h}}, \tag{17}$$

where $k = 0, 1, 2, \ldots, K$ and $\Theta_k = 1$ for $k = 0$ and $\Theta_k = \sqrt{2}$ for $k > 0$. The eigenfunctions Ψ_m, $m = 0, 1, 2, \ldots, M$, are normalized over the room horizontal cross-section and $\Psi_0 = 1/\sqrt{S_c}$, where $S_c = V/h$ is the surface of this cross-section. The distributions of Ψ_m were found by means of a direct solution of the

two-dimensional wave equation by a numerical procedure employing the forced oscillator method. This method is based on the principle that the response of a linear system to a harmonic excitation is large when the driving frequency is close to the resonant frequency (NAKAYAMA, YAKUBO, 2001). Using Eq. (17) in Eq. (8) it is easy to find that a formula for the resonant frequencies ω_n is as follows

$$\omega_n = \sqrt{\left(\frac{\pi k c}{h}\right)^2 + \omega_m^2}, \tag{18}$$

where ω_m are the resonant frequencies corresponding to Ψ_m. When the eigenfunctions Ψ_m were known, ω_m were calculated from the eigenvalue equation, $\nabla^2 \Psi_m + (\omega_m/c)^2 \Psi_m = 0$, multiplying it by Ψ_m, integrating and applying the orthogonal property. Next, using Eq. (6) the modal damping coefficients r_n were computed.

The computer simulation of a steady-state room response was performed for the absorption coefficient α of 0.1. Since the total room absorption $\mathcal{A}$ is simply equal to αS, where, as earlier, S denotes the surface of all the room walls, then, using Eq. (1), it is easy to calculate that the frequency f_s, corresponding to this value of α, is 165 Hz, approximately. Below this frequency 130 acoustic modes were found. The distribution of the modulus of the eigenfunction Ψ_m for some modes is plotted in Fig. 2. These graphs illustrate four fundamental shapes of resonant modes. In the first case (Fig. 2a), the acoustic mode may be identified as a delocalized mode because its energy is reasonably uniformly distributed
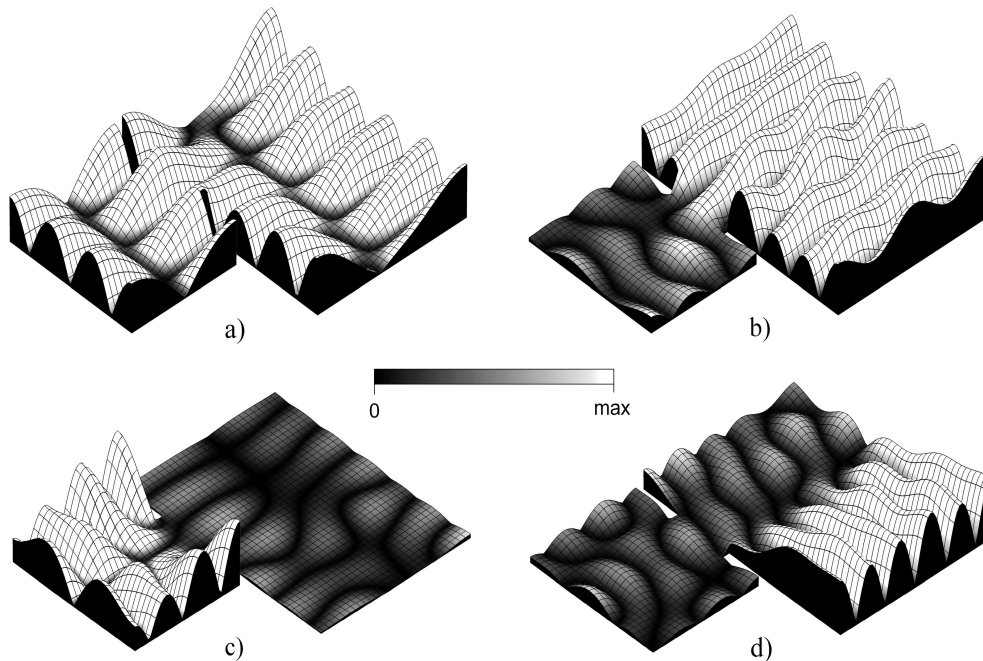

Fig. 2. Modulus of the eigenfunction Ψ_m for the mode number m:
a) 27, b) 34, c) 36, d) 45.

inside the room. On the other hand, the next three modes are recognized as localized modes since their energy is concentrated in different parts of the room system: the subroom A (Fig. 2b), the subroom B (Fig. 2c) and a portion of the subroom A (Fig. 2d). Confinement of the acoustic energy in a restricted part of rooms, known as the mode localization, is characteristic for systems of coupled rooms and enclosures with an irregular geometry (MEISSNER, 2005), because in rectangular rooms all eigenmodes are delocalized.

When a lightly damped room is excited by the harmonic point source, a sound signal received at the observation point is dominated by an individual response of the normal acoustic mode whose modal frequency is very close to the source frequency ω. Thus, if the source frequency varies, the acoustic signal perceived at this point may change considerably because the response of a single mode depends on the value and slope of its eigenfunction at the source and observation points, as it results from Eqs. (14) and (15). This is clearly confirmed by the
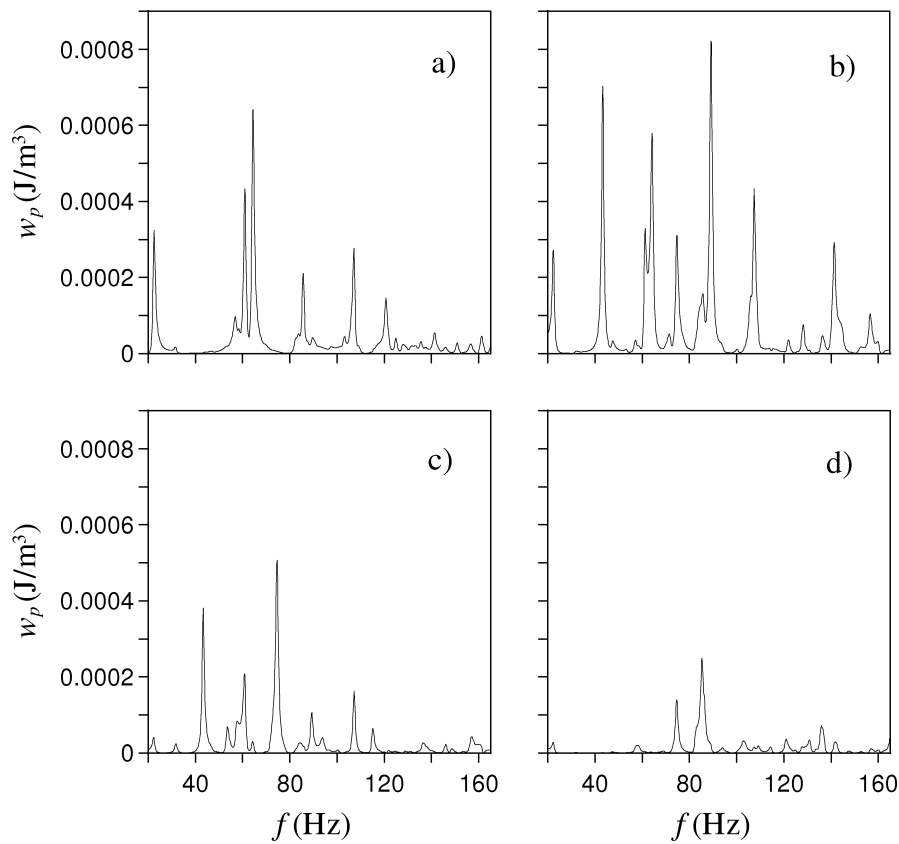

Fig. 3. Frequency dependence of the potential energy density w_p at the observation point $x = 3$ m, $y = 5$ m, $z = 1.8$ m for the source positions (in meters): a) $x_0 = 2$, $y_0 = 6$, $z_0 = 1$, b) $x_0 = 2$, $y_0 = 3$, $z_0 = 1$, c) $x_0 = 5$, $y_0 = 3$, $z_0 = 1$, d) $x_0 = 8$, $y_0 = 3$, $z_0 = 1$.

simulation data depicted in Fig. 3 showing, for four different source positions, an evolution of the potential energy density w_p with the source frequency $f = \omega/2\pi$. As it may be seen, there is a substantial influence of the source position on the value of w_p. However, more interesting from the practical point of view is the finding that the frequency response of the room is highly deformed by modes which strongly respond to a sound excitation because it manifests itself as a high, narrow peak in the frequency dependence of w_p.

Now, if simulation results from Fig. 3b are shown in a logarithmic scale, one can easily discover that there are some modes which, for a given position of the source and observation points, respond very weakly to a sound excitation. In Fig. 4 such examples are modes with the mode number n of 3, 78, 107 and 124. Equation (14) shows that, for the potential energy density, the weak modal response occurs when the absolute value of the product $\Psi_m(x_0, y_0)\Psi_m(x, y)$ has a minimum. In the extreme case this value may be equal to zero, since the eigenfunction Ψ_m possesses both positive and negative values due to the orthogonality property. Obviously, the number of sign changes of Ψ_m increases for the growing mode number m. This regularity is illustrated by graphs in Fig. 5, where solid lines indicate zeros of the eigenfunction Ψ_m for different acoustic modes.

As mentioned previously, localization of eigenmodes is a phenomenon intimately associated with systems of coupled rooms. This effect is of special interest in this study because the location of a sound source in a part of the room where
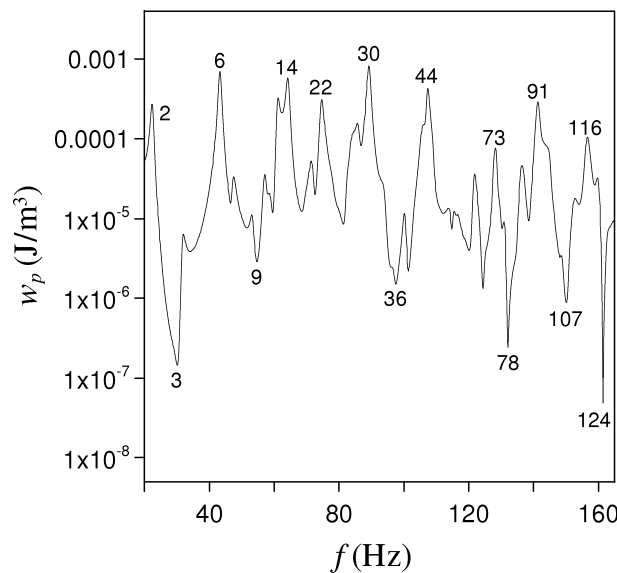

Fig. 4. Frequency dependence of the potential energy density w_p at the observation point $x = 3$ m, $y = 5$ m, $z = 1.8$ m for the source position $x_0 = 2$ m, $y_0 = 3$ m, $z_0 = 1$ m. Numbers above maxima and below minima represent the mode number n.
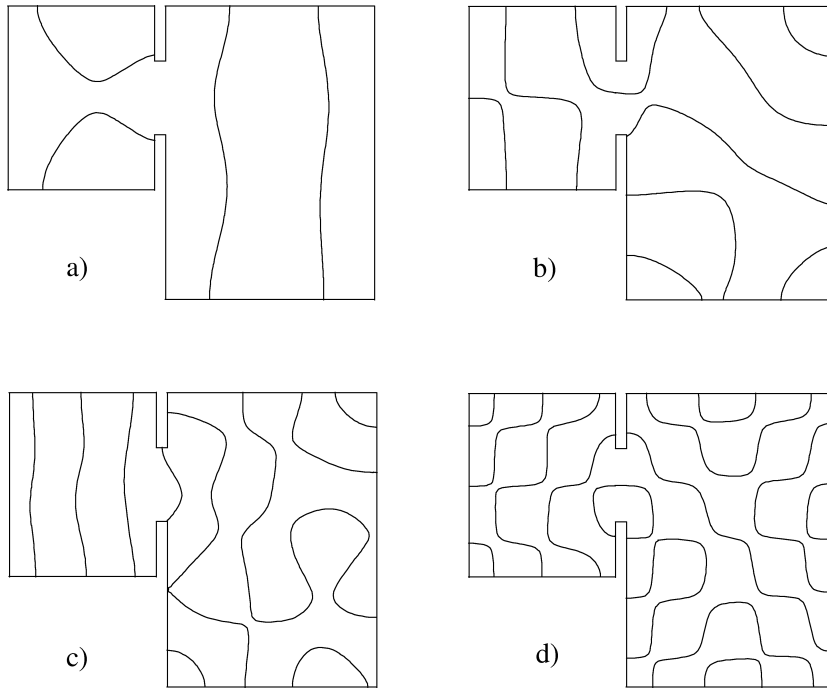
Fig. 5. Lines indicating zeros of the eigenfunction Ψ_m for the mode number m:
a) 10, b) 20, c) 35, d) 50.

the modal energy is very small may result in large decreases in the potential and kinetic energy densities. This problem will be analysed in detail for a mode with the mode number n of 73, which was found to be localized in the subroom A. The modulus of the eigenfunction Ψ_m for this mode is depicted in Fig. 2b. The results of computer simulations of w_p and w_k in the observation plane, situated at a constant height from the floor, are shown in Figs. 6 and 7. In the simulation it was assumed that a source frequency is tuned to the resonant frequency of the mode and there are two positions of the source: one in the subroom A and another one in the subroom B. Figures 2b and 6a show that placing of the sound source at an appropriate point of the subroom A results in a strong modal response because the distribution of the potential energy density w_p in (x, y) plane reproduces the square of the eigenfunction Ψ_m corresponding to the mode 73. On the other hand, the room response is very weak when the source is situated in the subroom B where the energy of this mode is small (Fig. 6b). Moreover, the distribution of w_p looks completely different from the previous case because the room response is now created by the surrounding modes. The kinetic energy density w_k, as seen in Fig. 7, is several dozen times smaller than the potential one, and according to Eq. (15), in the case of strong room response, its distribution in (x, y) plane is proportional to $(\partial \Psi_m / \partial x)^2 + (\partial \Psi_m / \partial y)^2$, where Ψ_m is the eigenfunction for the mode 73. This means that maxima of w_p correspond to
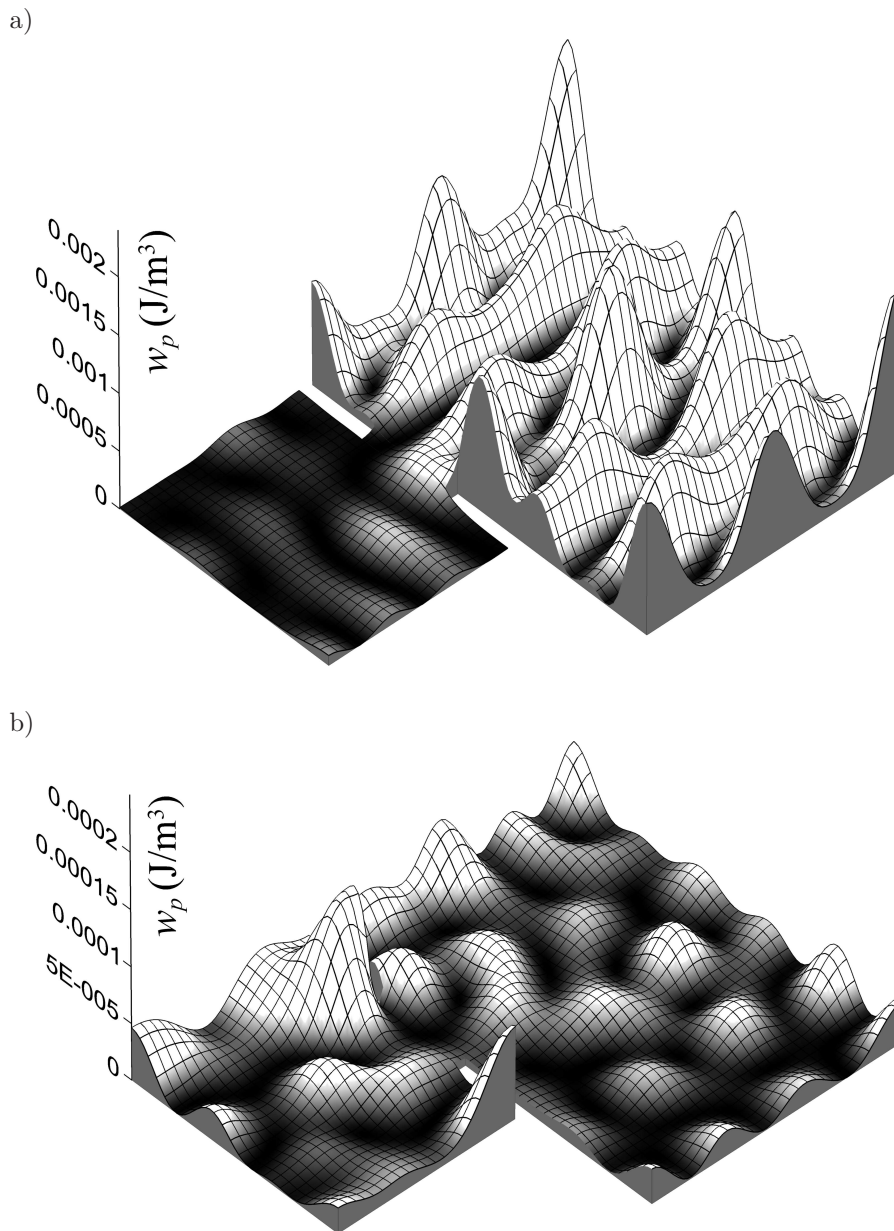
a)

b)

Fig. 6. Potential energy density w_p in the observation plane $z = 1.8$ m for the source positions (in meters): a) $x_0 = 2.8$, $y_0 = 4.2$, $z_0 = 1$, b) $x_0 = 7.5$, $y_0 = 2$, $z_0 = 1$. The source frequency is 128.6 Hz.

minima of w_k and *vice versa*. Of course, as earlier, in the case of the weak room response the distribution of w_k looks quite different from the case of the strong response (Fig. 7b). In order to illustrate more clearly the differences between
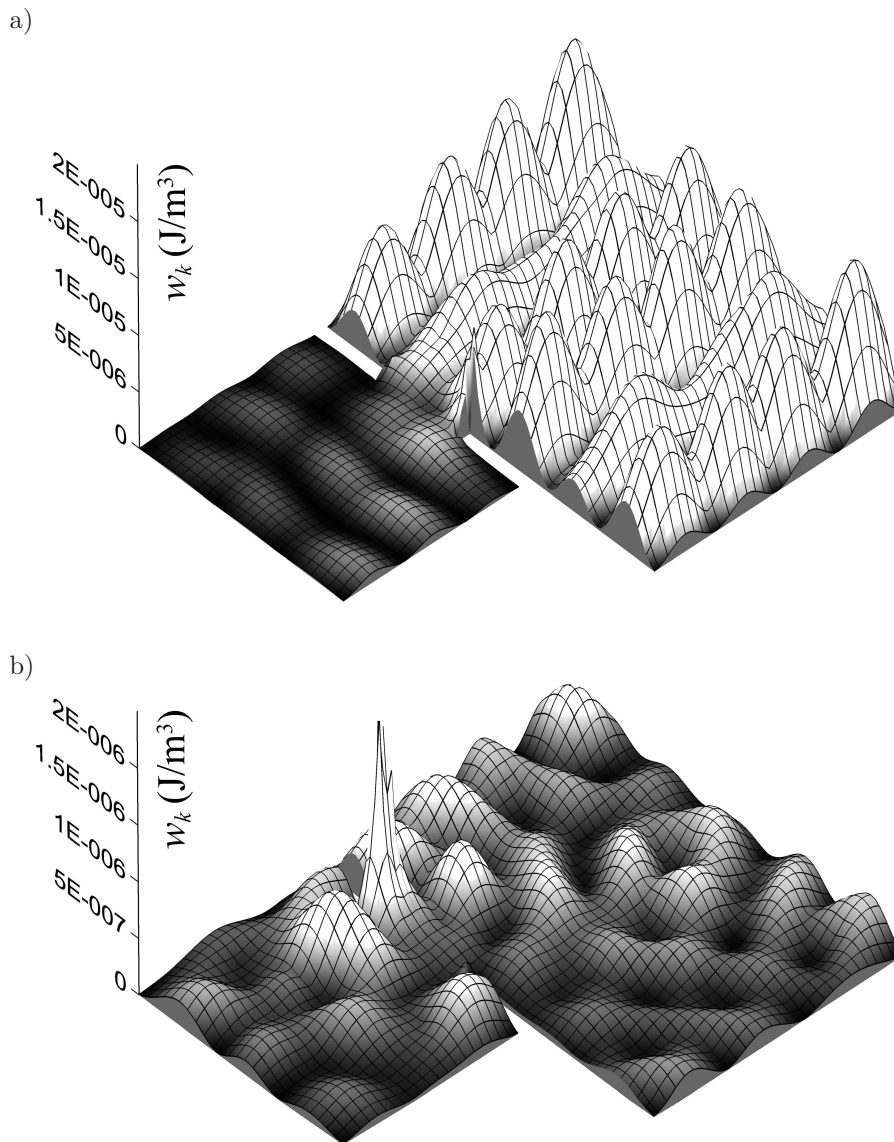
a)

b)

Fig. 7. Kinetic energy density w_k in the observation plane $z = 1.8$ m for the source positions (in meters): a) $x_0 = 2.8$, $y_0 = 4.2$, $z_0 = 1$, b) $x_0 = 7.5$, $y_0 = 2$, $z_0 = 1$. The source frequency is 128.6 Hz.

the strong and weak room responses, in Fig. 8 the dependencies of w_p and w_k on the coordinate y for a constant value of the coordinate x are shown. For the strong response, the intense wavy changes in w_p and w_k are noted, because for the mode considered acoustic resonance is excited between the walls separated by the distance w_1 (Fig. 1). The weak response also has a wave-like nature but its
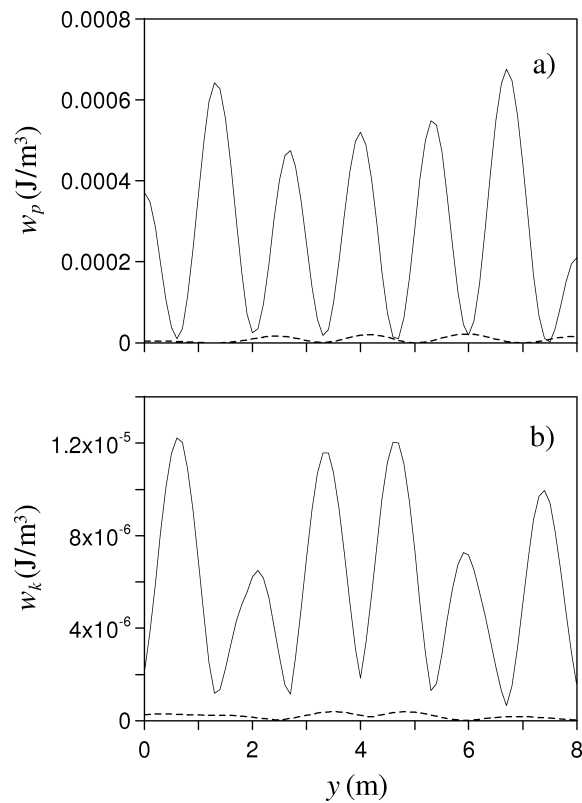
Fig. 8. Changes in the potential and kinetic energy densities w_p and w_k with the coordinate y for $x = 2$ m, $z = 1.8$ m and the source positions (in meters): $x_0 = 2.8$, $y_0 = 4.2$, $z_0 = 1$ (solid lines), $x_0 = 7.5$, $y_0 = 2$, $z_0 = 1$ (dashed lines). The source frequency is 128.6 Hz.

maximal amplitude is several dozen times smaller as compared to this amplitude for the strong response.

4. Summary and conclusions

In this paper, the modal expansion method is used to examine the effect of the sound source location on the steady-state response of coupled rooms in the low-frequency range. In the theoretical part, underlying assumptions of the modal expansion method are briefly presented and influence of modal and sound source parameters on the potential and kinetic energy densities is discussed. Details of the numerical method, together with the results of computer simulations, are presented in the next part of the work. The room system under consideration consisted of two connected rectangular subrooms denoted by A and B, and in numerical simulations, a configuration of the sound-absorbing material on the

room walls was assigned to the system. The simulation results have shown that in the case of a harmonic excitation the sound signal received at the observation point is dominated by an individual response of the normal acoustic mode whose frequency is close to the source frequency. Therefore, if the source frequency varies, the acoustic signal perceived at this point changes considerably because the response of a single mode depends on the value and slope of its eigenfunction at the source and observation points. It was found that for different source positions this results in large deformations of the frequency room response for strong and weak modal excitations because they manifest themselves either as high peaks or as large drops in the response. The influence of the source position on the room response is especially evident for localized modes. This problem was examined in detail for the mode strongly localized in the subroom A. Simulations revealed that when a source frequency is tuned to a resonant frequency of this mode placing of the point source inside the subroom A may result in the strong room response, while location of the source inside the subroom B, where there is a small concentration of the modal energy, always leads to a very weak response.

References

1. BILLON A., VALEAU V., SAKOUT A., PICAUT J. (2006), *On the use of a diffusion model for acoustically coupled rooms*, Journal of the Acoustical Society of America, **120**, 2043–2054.

2. BRADLEY D.T., WANG L.M. (2010), *Optimum absorption and aperture parameters for realistic coupled volume spaces determined from computational analysis and subjective testing results*, Journal of the Acoustical Society of America, **127**, 223–232.

3. DOWELL E.H., GORMAN G.F., SMITH D.A. (1977), *Acoustoelasticity: general theory, acoustic natural modes and forced response to sinusoidal excitation*, Journal of Sound and Vibration, **52**, 519–542.

4. GOŁAŚ A., SUDER–DĘBSKA K. (2009), *Analysis of Dome Home Hall theatre acoustic field*, Archives of Acoustics, **34**, 3, 273–293.

5. ERMANN M. (2005), *Coupled volumes: aperture size and the double-sloped decay of concert halls*, Building Acoustics, **12**, 1–14.

6. KAMISIŃSKI T., BURKOT M., RUBACHA J., BRAWATA K. (2009), *Study of the effect of the orchestra pit on the acoustics of the Kraków Opera Hall*, Archives of Acoustics, **34**, 4, 481–490.

7. KINSLER L.E., FREY A.R. (1962), *Fundamentals of acoustics*, John Wiley & Sons, New York.

8. KOSAŁA K. (2009), *Calculation models for acoustic analysis of sacral objects*, Archives of Acoustics, **34**, 1, 3–11.

9. KUTTRUFF H. (1973), *Room acoustics*, Applied Science Publishers, London.

10. MARTELLOTTA F. (2009), *Identifying acoustical coupling by measurements and prediction-models for St. Peter's Basilica in Rome*, Journal of the Acoustical Society of America, **126**, 1175–1186.

11. MEISSNER M. (2005), *Influence of room geometry on low-frequency modal density, spatial distribution of modes and their damping*, Archives of Acoustics, **30**, 4 (Supplement), 205–210.

12. MEISSNER M. (2007), *Computational studies of steady-state sound field and reverberant sound decay in a system of two coupled rooms*, Central European Journal of Physics, **5**, 293–312.

13. MEISSNER M. (2008a), *Influence of wall absorption on low-frequency dependence of reverberation time in room of irregular shape*, Applied Acoustics, **69**, 583–590.

14. MEISSNER M. (2008b), *Acoustics of irregular rooms in buildings: low-frequency mathematical modelling and numerical simulation*, [in:] *Renewable Energy, Innovative Technologies and New Ideas*, Chwieduk D., Domański R., Jaworski M. [Eds.], pp. 265–272, Publishing House of Warsaw University of Technology, Warsaw.

15. MEISSNER M. (2008c), *Influence of absorbing material distribution on double slope sound decay in L-shaped room*, Archives of Acoustics, **33**, 4 (Supplement), 159–164.

16. MEISSNER M. (2009a), *Computer modelling of coupled spaces: variations of eigenmodes frequency due to a change in coupling area*, Archives of Acoustics, **34**, 2, 157–168.

17. MEISSNER M. (2009b), *Spectral characteristics and localization of modes in acoustically coupled enclosures*, Acta Acustica united with Acustica, **95**, 300–305.

18. MEISSNER M. (2009c), *Application of Hilbert transform-based methodology to computer modelling of reverberant sound decay in irregularly shaped rooms*, Archives of Acoustics, **34**, 4, 491–505.

19. MEISSNER M. (2010), *Simulation of acoustical properties of coupled rooms using numerical technique based on modal expansion*, Acta Physica Polonica A, **118**, 123–127.

20. MEISSNER M. (2011), *Numerical modelling of coupled rooms: evaluation of decay times via method employing Hilbert transform*, Acta Physica Polonica A, **119**, 1031–1034.

21. NAKAYAMA T., YAKUBO K. (2001), *The forced oscillator method: eigenvalue analysis and computing linear response functions*, Physics Reports, **349**, 239–299.

22. PU H., QIU X., WANG J. (2011), *Different sound decay patterns and energy feedback in coupled volumes*, Journal of the Acoustical Society of America, **129**, 1972–1980.

23. SCHROEDER M. (1996), *The "Schroeder frequency" revisited*, Journal of the Acoustical Society of America, **99**, 3240–3241.

24. SUMMERS J.E., TORRES R.R., SHIMIZUD Y. (2004), *Statistical-acoustics models of energy decay in systems of coupled rooms and their relation to geometrical acoustics*, Journal of the Acoustical Society of America, **116**, 958–969.

25. SUMMERS J.E., TORRES R.R., SHIMIZUD Y., DALENBÄCK B.L. (2005), *Adapting a randomized beam-axis-tracing algorithm to modeling of coupled rooms via late-part ray tracing*, Journal of the Acoustical Society of America, **118**, 1491–1502.

26. XIANG N., JING Y., BOCKMAN A.C. (2009), *Investigation of acoustically coupled enclosures using a diffusion-equation model*, Journal of the Acoustical Society of America, **126**, 1187–1198.