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# MICROMECHANICAL MODELLING OF PACKING AND SIZE EFFECTS IN PARTICULATE ELASTIC-PLASTIC COMPOSITES

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## 1. Introduction

The micromechanical approaches employed in the present study originate from the concept of composite sphere Hashin model and its modification to the generalized self-consistent (GSC) model by Christensen, cf. [1]. The concept was later employed in [2], within the morphologically representative pattern-based approach (MRP), to describe the packing and size effects in an elastic composite made of a continuous matrix and dispersed spherical particles. In the simplest example of 2-pattern approach the Representative Volume Element (RVE) is idealized by two representative patterns (see Fig. 1): the first one governed by the N-phase GSC scheme (Fig. 1b), and the second one described by the classical self-consistent scheme (SC) (Fig. 1c). To account for the size effect in the first pattern an interphase is introduced, having a thickness  $t_{int}$  independent of the particle radius and different properties than the basic two phases. To describe the packing effect, the coating thickness in this pattern is specified by half the mean distance  $\bar{\lambda}$  between nearest-neighbor particles in the RVE. The volume fraction of the GSC pattern can be calculated as

$$(1) \quad c = f_{inc} (1 + \bar{\lambda})^3$$

where  $f_{inc}$  is the volume fraction of the inclusions. For a specified volume fraction  $f_{inc}$ , the parameter  $\bar{\lambda}$  may vary from 0 (nearest particles are in contact) to the value determined by the matrix phase fraction. Using the corresponding localization relations valid for the GSC and SC schemes [1] the average strain in the phases can be related to the overall strain in the composite. The method is extended to the case of non-linear constitutive laws by performing incremental tangent or secant linearization of the local material behaviour.

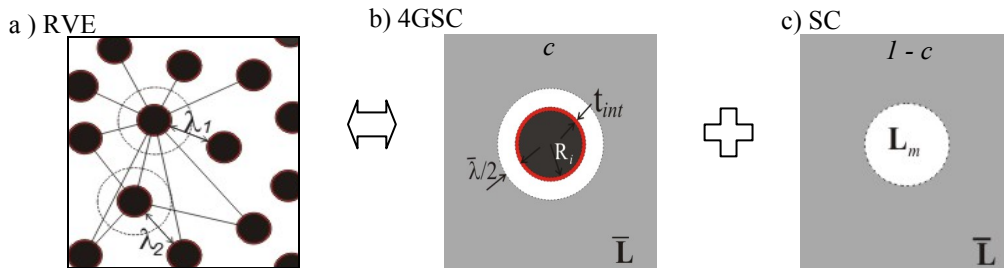


Figure 1. Scheme of the morphologically representative pattern-based approach

In the paper the validity of this analytical scheme as concerns the predictions of packing and size effects on overall elastic-plastic composite properties are compared with results of computational homogenization by the finite element method (FEM) performed on the generated RVEs of prescribed statistical characteristics.

## 2. Results

Assume a composite consisting of a linear elastic ceramic phase C and an elasto-plastic metal phase M with the Huber-Mises yield surface and power law hardening. Let us demonstrate the predicted influence of the packing parameter  $\bar{\lambda}$  on effective properties for the elasto-plastic composite.

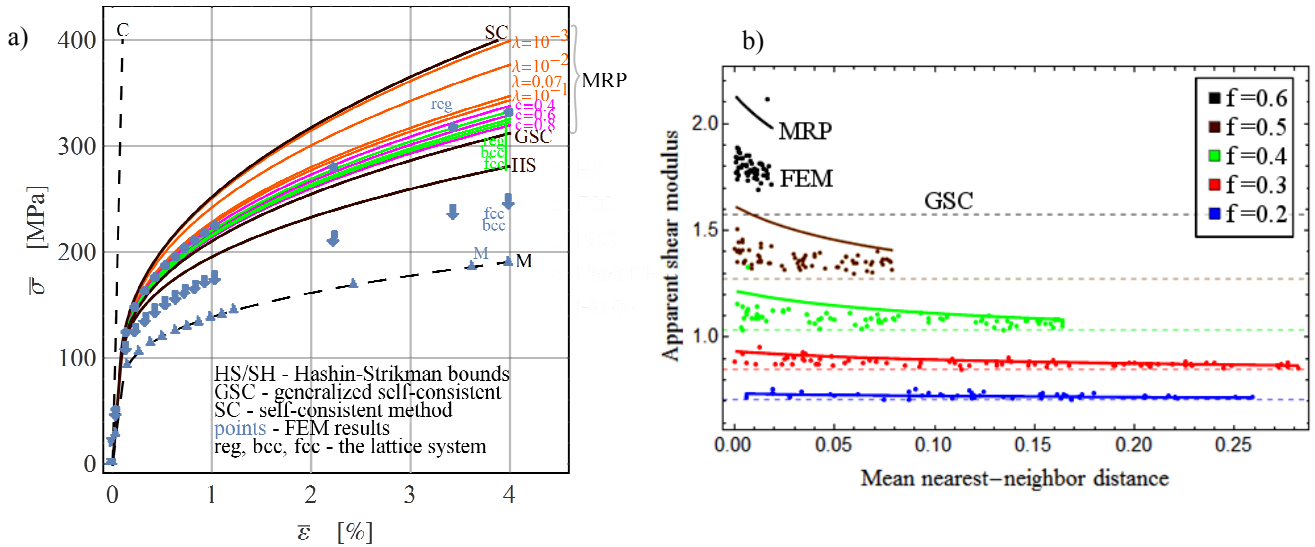


Figure 2. Results derived by different micromechanical estimates and FEM:  
a) stress-strain response in uniaxial tension of an elastic-plastic composite with  $f_{inc}=30\%$ ,  
b) relation between shear modulus and  $\bar{\lambda}$  of a two phase elastic composite for various  $f_{inc}$ .

In Fig. 2a the estimates of the macroscopic stress-strain curve obtained for uniaxial tension of the composite is shown. The results were derived by incremental tangent linearization and isotropization of the elasto-plastic tangent stiffness of the metal matrix. The smaller  $\bar{\lambda}$  the closer MRP-based estimate is to the SC scheme prediction. Obviously, when  $\bar{\lambda} \rightarrow 0$  the results tend to the SC scheme result. Analogically, increasing the volume fraction of the GSC-type pattern ( $c \rightarrow 1$ ) brings results closer to the generalized self-consistent scheme. For completeness, other classical bounds and estimates for a two-phase isotropic composite are also included in the figure. Additionally, in Fig. 2a the MRP-based estimates of  $\bar{\sigma}$  are shown for the regular cubic, BCC and FCC distributions of particles, or more precisely, for  $\bar{\lambda}$  varying in the same way with inclusion volume fraction as for these ordered ways of particle distribution. It results with a constant  $c$  value, independent of  $f_{inc}$ . For comparison, the numerical simulations of three lattice structures were performed with periodic boundary conditions of the Dirichlet type. For the regular cubic cell very good correlations between the MRP and FEM predictions were observed, while for the face centered cubic and body centered cubic FEM  $\bar{\sigma}$  is lower than the micromechanical results, probably due to the cubic symmetry of unit cells.

In order to verify the micromechanical estimates presented in the previous section, computational homogenization is also performed on representative volume elements with randomly distributed non-overlapping spherical inclusions and controlled value of  $\bar{\lambda}$ . The bulk and shear moduli were assessed for the RVE. In Fig. 2b, FE results show less dependence on  $\bar{\lambda}$  than MRP and they are closer to the GSC predictions, but the MRP results have the same qualitative character as FEM results.

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### 3. References

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