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Prestressing for local isolation of forced vibrations

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ABSTRACT

The problem of dynamic response stabilization is a crucial issue in many engineering applications or structures subjected to an external source of excitation or dynamic load. At present, owing predominantly to advances in measurement technology, microprocessor control and development of smart materials it is possible to solve many of these problems. Semi-active or active damping systems, which are used to improving structure response, requires additional dampers or absorbers.

Contrary, in the article we present approach of suppressing local vibration via introducing initial prestressing into the chosen element or elements of the structure. In that way it is possible to change properties of the structure and its modes of vibrations.

We present the results of numerical simulations of the mechanical structure subjected to external excitations. Our results show that by introducing prestresing it is possible to significantly influence on eigenfrequances and eigenmodes. Also effectiveness of vibration amplitudes reduction can be significantly larger, at least one order of magnitude larger.

Keywords: *local suppresion of forced vibrations, prestressing, sensitivity analysis and prestress optimization*

1 INTRODUCTION - DYNAMIC RESPONSE STABILIZATION AND CHALLENGES IN AEROSPACE ENGINEERING

The problem of dynamic response stabilization is a crucial issue in many engineering applications. The problem can be addressed to: a) impact load absorption at the source of this excitation, b) damping of the impact born, free vibrations or c) damping of continuously, externally forced vibrations. The first problem in all activities devoted to stabilization of dynamic response of the structure is identification (usually in real time) of dynamic excitation. It is especially a challenging issue for the impact loads, when in few milliseconds not only the kinetic energy the impact, but also the impact velocity has to be determined (cf. Ref. 1). Our strategy for impact load absorption will be different for a "slow" impact (high kinetic energy but low velocity) and for a "fast" impact (low kinetic energy but high velocity). Of course, dynamic structural response depends very much from the natural, material damping, which can be significantly improved applying additional passive dampers (eg. using well selected elastomeric components). In the case of predictable impact loads, various types of passive shock-absorbers can be sufficiently effective (eg. oil damper + gas spring in a regular landing gear). For predictable externally forced vibrations, so-called tuned mass-dampers TMD, with properly designed location for extra mass + spring can be also effective. However, when the excitation is variable and random, the characteristic of our damping device should be adaptable to the identified on-line load case. We can call this class of shock absorbers AIA (Adaptive Impact Absorption). The group of pneumatic/hydraulic flow control shock-absorbers with actively controlled piezo-valves determines the first class of AIA systems (cf. Ref. 2), requiring real time fead-back control of piezo-actuators. The second class is

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determined by pneumatic systems (eg. airbags) with controlled special release valves (cf. Ref. 3). In this case, less demanding open-loop control can be also very effective. The next (semi-active) class requires precise in time switching between two active interfaces of adaptive inerter (so-called SPINMAN), causing switching of the spin of rotating inertial cylinders for conversion of linear impact energy into rotation of the inerter (cf. Ref. 4). Still, semi-active AIA systems can be some time replaced by a passive (simpler, cheaper and lighter), smart solutions and not far in effectiveness from the first one. Finally, so-called PAR (Prestress Accumulation-Release) adaptive system for damping of impact born vibrations has been demonstrated as extremely effective algorithm. (cf. Refs. 5, 6).

Concentrating on aerospace engineering, the following requirements are crucial: a) necessity of artificial vibration damping system due to flexibility of the structure and its low natural damping, b) necessity of low weight of the proposed damping system, c) necessity of high reliability (simplicity) of the proposed system. Therefore, semi-active (or even smart-passive) AIA damping systems will be preferable for aero-applications. The concept of pre-designed pre-stressing seems to be one of possible options for aero-applications leading to effective reduction of externally forced (impact born or continuously excited) vibrations.

Feasibility study for control of eigen vibrations in flexible truss structures of parabolic mirrors on the orbit, by means of inducing self-stresses has been already performed (Ref. 7). It has been demonstrated that ca 15% reduction of local vibration amplitude is available via proper pre-stressing of the structure. The challenging issue is to find out, how far we can reduce local vibrations (caused by continuous external excitation) with introduced self-stresses (eventually adapting to variable excitation). The second question is, how far we can increase the effectiveness of the PAR technique described above with initial prestressing.

The main objective of this paper is to analyze the case study of a flexible structure (Fig. 1) with excited vibration. The effectiveness of eventual reduction of these vibrations in selected location by prestressing will give us a good motivation for further research on development of the mentioned above techniques.

2 FUNDAMENTAL THEORY OF NATURAL FREQUENCY ANALYSIS FOR PRESTRESSED STRUCTURES

General equations of motion can be expressed as follows:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}. \qquad (1)$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}$ denote the mass, damping and stiffness matrix, respectively. The symbols: $\ddot{\mathbf{x}}, \dot{\mathbf{x}}, \mathbf{x}$ denote vectors of space variables (accelerations, velocities and displacements, respectively), while the vector $\mathbf{F}$ denotes external forces. Neglecting damping effect and external forces, the free vibration problem takes the following form:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \qquad (2)$$

and the corresponding solution leads to the formula:

$$\mathbf{x} = \mathbf{\Phi}\, e^{j\omega t} \qquad (3)$$

where $\mathbf{\Phi}$ denotes the amplitude vector, ω denotes the eigen frequency vector and t denotes time. Substitution of the above relation to the equation (2) leads to:

$$\left[\mathbf{K} - \omega_i^2 \mathbf{M}\right]\Phi_i = 0 \qquad (4)$$

We are interested in non-trivial solutions satisfying the following formula:

$$\det\left[\mathbf{K} - \omega^2 \mathbf{M}\right] = 0 \qquad (5)$$

Equation (5) does not depend on eventual initial stresses (so-called prestressing: internal, self-equilibrated stresses, caused by geometrical incompatibilities, eg. thermal distortions). Influence of initial stresses on modal analysis can be taken into account via modification of the stiffness matrix $\mathbf{K}$ and including additional geometrical stiffness matrix $\mathbf{K}_G$. Then, the equation (5) takes its final form:

$$\det\left[\mathbf{K} + \mathbf{K}_G - \omega^2 \mathbf{M}\right] = 0 \tag{6}$$

In commercial FEM systems, like Abaqus, the modal analysis of prestressed structures is performed in two steps. In the first step, the static analysis $\mathbf{Kx} = \mathbf{F}$ defined for nonlinear geometry has to be solved. Then, the determined solution allows calculation of the geometric stiffness matrix $\mathbf{K}_G$ and modification of the stiffness matrix $\mathbf{K}$ (cf. Eq. 6) in the second step, leading to the final modal analysis.

3 SENSITIVITY ANALYSIS FOR PRESTRESSED STRUCTURES

Eigenvalue problem for prestresed structure can be expressed as follows:

$$\left[\mathbf{K} + \mathbf{K}_G - \omega_i^2 \mathbf{M}\right]\mathbf{\Phi}_i = 0 \tag{7}$$

Direct differentiating of Eq. (7) leads to formula:

$$\frac{\partial \mathbf{K}}{\partial F_k}\mathbf{\Phi}_i + \mathbf{K}\frac{\partial \mathbf{\Phi}_i}{\partial F_k} + \frac{\partial \mathbf{K}_G}{\partial F_k}\mathbf{\Phi}_i + \mathbf{K}_G\frac{\partial \mathbf{\Phi}_i}{\partial F_k} - \frac{\partial \omega_i^2}{\partial F_k}\mathbf{M}\mathbf{\Phi}_i - \omega_i^2\frac{\partial \mathbf{M}}{\partial F_k}\mathbf{\Phi}_i - \omega_i^2\mathbf{M}\frac{\partial \mathbf{\Phi}_i}{\partial F_k} = 0 \tag{8}$$

where F_k is initial stress introduced in k element. Both mass $\mathbf{M}$ and stiffness $\mathbf{K}$ matrix do not depend on the initial stresses F_k, so:

$$\frac{\partial \mathbf{K}}{\partial F_k} = 0; \frac{\partial \mathbf{M}}{\partial F_k} = 0\,. \tag{9}$$

Substitution of the above relations to the Eq. (8) leads to following form:

$$\left[\mathbf{K} + \mathbf{K}_G - \omega_i^2 \mathbf{M}\right]\frac{\partial \mathbf{\Phi}_i}{\partial F_k} + \frac{\partial \mathbf{K}_G}{\partial F_k}\mathbf{\Phi}_i - \frac{\partial \omega_i^2}{\partial F_k}\mathbf{M}\mathbf{\Phi}_i = 0\,. \tag{10}$$

Premultiply both side of Eq. (10) by $\mathbf{\Phi}_i^T$ and assuming that geometrical stiffness matrix $\mathbf{K}_G$ is symetric, leads to formula:

$$\mathbf{\Phi}_i^T\frac{\partial \mathbf{K}_G}{\partial F_k}\mathbf{\Phi}_i - \frac{\partial \omega_i^2}{\partial F_k}\mathbf{\Phi}_i^T\mathbf{M}\mathbf{\Phi}_i = 0 \tag{11}$$

In vibration problems the mode shapes $\mathbf{\Phi}$ are often normalized with respect to the mass matrix:

$$\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi} = \mathbf{I} \tag{12}$$

Then, the equation for eigenvalue sensitivity takes its final form:

$$\frac{\partial \omega_i^2}{\partial F_k} = \mathbf{\Phi}_i^T\frac{\partial \mathbf{K}_G}{\partial F_k}\mathbf{\Phi}_i\,. \tag{13}$$

Numerical evaluation of the *i* eigenvalue sensitivity require modeshape associated with *i* eigenvalue and derivative of geometrical stiffness matrix.

4 NUMERICAL EXPERIMENTS ON THE MODEL OF FLEXIBLE STRUCTURE

The flexible frame (Fig. 1a) was built using rectangular steel tubes and two types of structural joints, called passive and semi-active joints (for a detailed description see [6,8]). During experiments semi-active joints worked in a passive mode. Geometrical dimension of the experimental setup is shown in Fig. 1b.

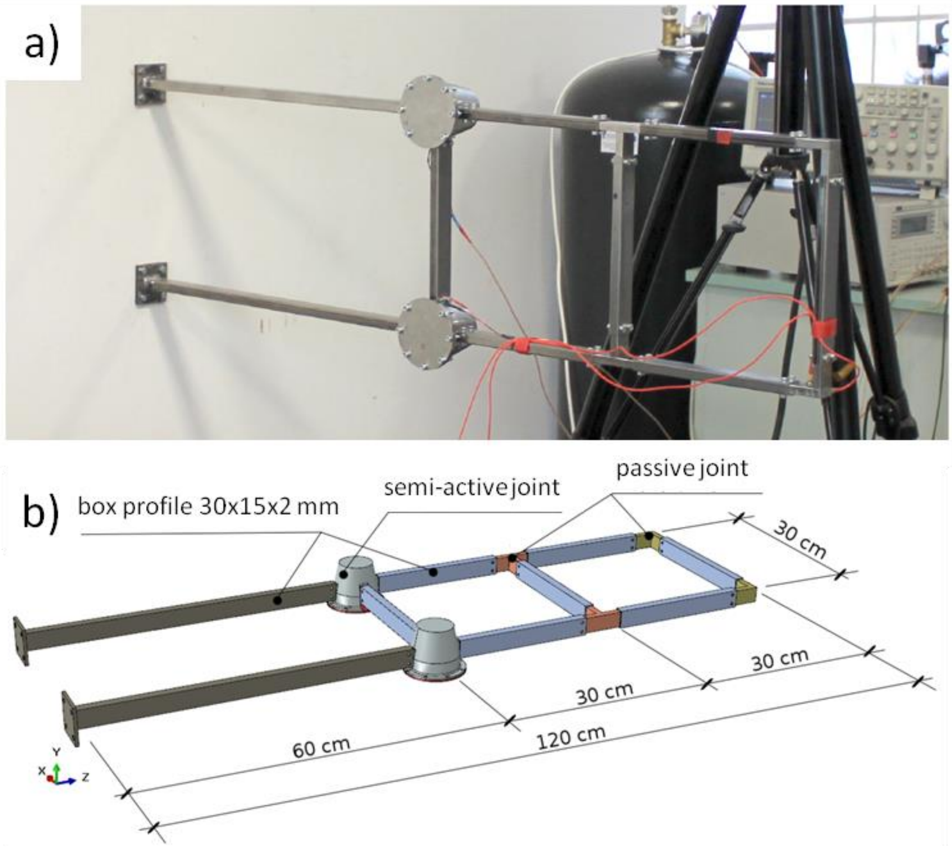

Figure 1- Flexible frame used in experiments a) general view, b) geometrical dimension.

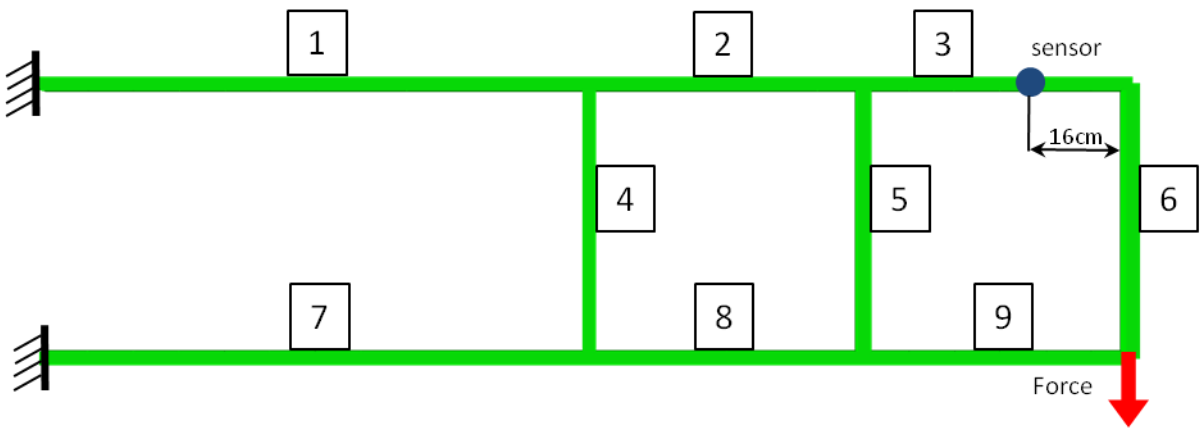

Figure 2 – Numerical model of the experimental setup.

A numerical model of flexible structure mockup has been elaborated taking advantage of Abaqus program. The finite element model was assembled from 70 beam elements, 18 linear springs, 12 dashpot elements and six point masses, added in nodes corresponding to the structural joint locations. The structural joints were modeled using linear and rotational springs and the dashpot elements. The boundary conditions have been defined by fixing two supporting points on the left hand side.

In order to identify the properties of the structural joints, the model updating procedure has been conducted in two steps: in the first step the stiffness of linear and rotational springs has been tuned in such a way, that the first eigen-frequency corresponds to the measured data. It was assumed that the numerical model has only one type of a structural joint. The first fourth natural frequencies and mode shapes determined from the model are shown in Fig. 3. In the second step damping coefficient of the dashpot elements has been identified based on the time response.

The lower joint on the right hand side of the structure has been pulled down to an initial displacment of about 1 mm and then released. The Polytec OFV-505 laser vibrometer has been used to measure the velocity of the flexible frame in point shown in Fig 2. The response has been recorded using a National Instruments Data Acqusition device (NI USB-621) at a sampling frequency of 500Hz. As shown in Fig. 4 very good agreement has been obtained between simulation results and measured response.

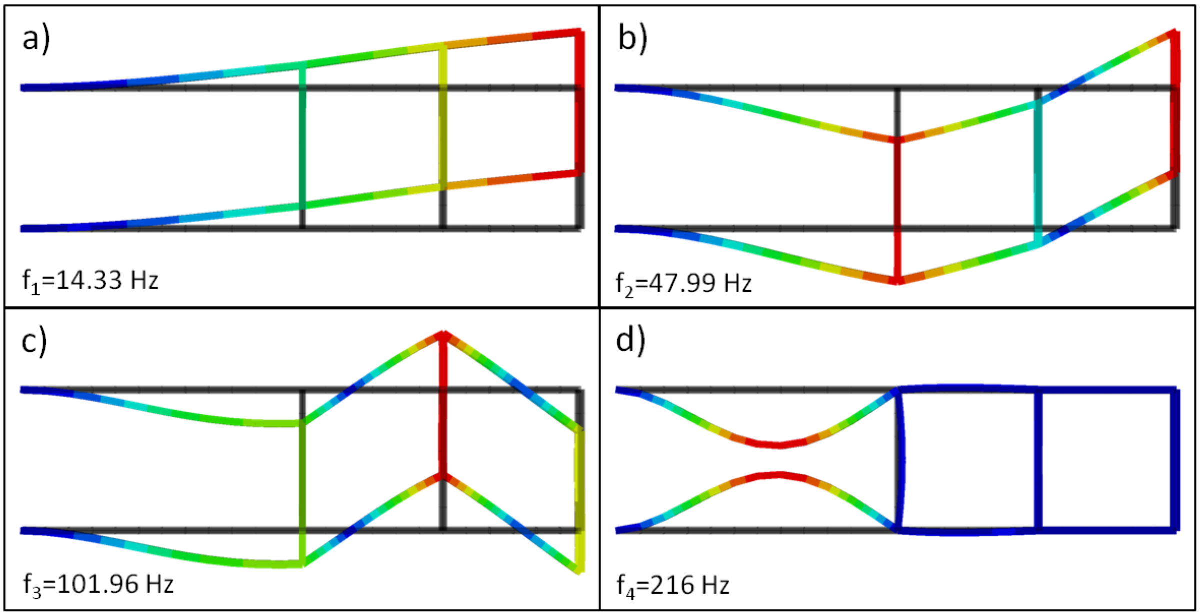

Figure 3 - First four eigenvalues and mode shapes of the flexible frame.

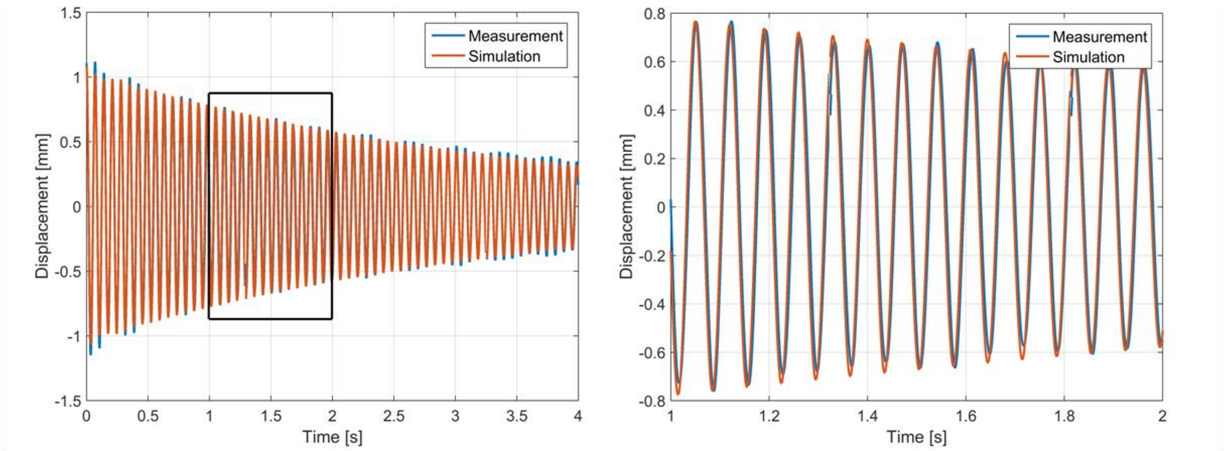

Figure 4 - Comparison of measured time response and simulated displacment of numerical model.

5 MODIFICATION OF MODAL RESPONSE DUE TO PRESTRESSING

Assuming danger of externally forced critical harmonic excitation (close to the first eigen-frequency 14,33Hz), reduction of deflection for the monitored node (sensor, see Fig. 2) by prestressing has to be pre-designed. The sensitivity analysis has been performed, determining the influence of linear prestress – distortions (geometric incompatibilities) imposed into the elements of numerical model for four first eigen-frequencies. During the sensitivity analysis only cases when a couple of axial forces have compresed elements have been considered. The results are shwon in Fig. 5, the colors of the bars indicate the number of the eigen-frequencies (blue – first, red – second, green – third and violet – fourth). Numbers on the x-axis indicate the number of an element in numerical model where prestress has been imposed. Positive value of the bar corresponds to a decreass in eigenfrequency value. One can see, that the most effective way to shift the first eigenfrequency is to introduce prestressing in horizontal elements, especially in elements 2 and 8.

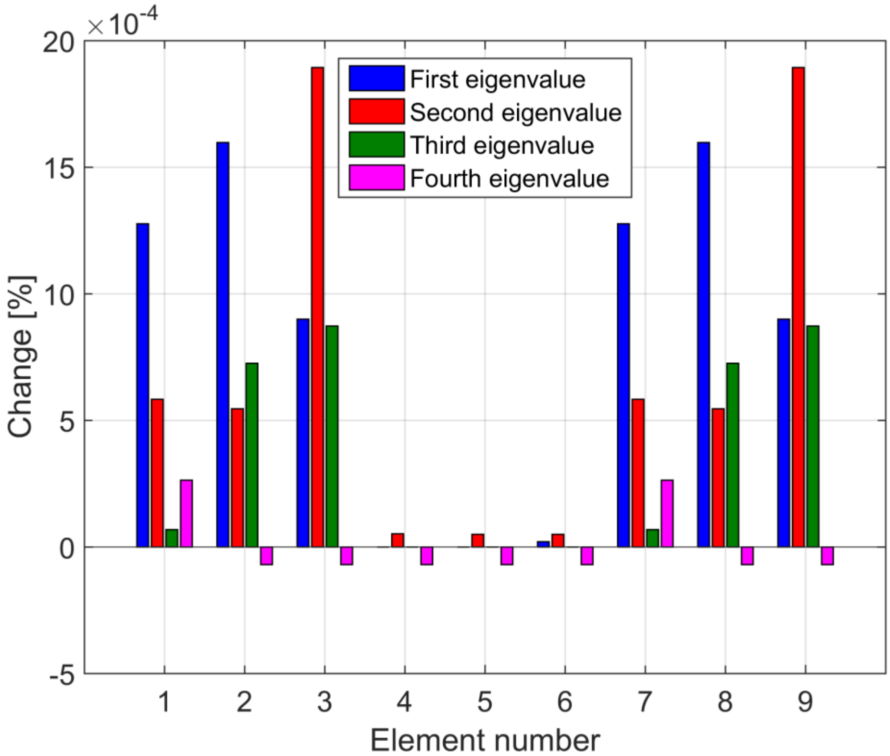

Figure 5 - Sensitivity responses to linear prestress-distortion.

Therefore, let's decide to impose linear prestress-distortion into the element No.2 with such a magnitude, that the observed amplitude of vibration is minimized.

The obtained result, corresponding to prestressing in the element No.2 with the prestress-force 1000N is demonstrated in Fig.7. One can observe shift of the eigen-frequency of the first eigen-mode and significant reduction of the corresponding vibration amplitude in the observed point.

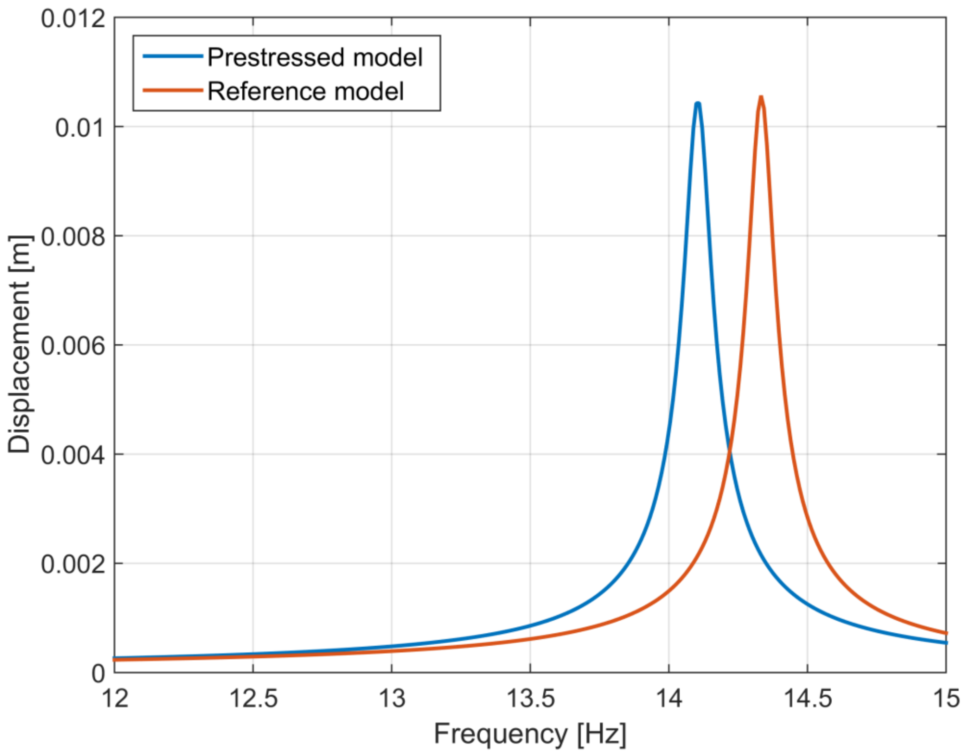

Figure 6 – Comparison of the FRF of the reference and prestressed model.

6 CONCLUSIONS

The analyzed case study shows that small shifting of the structural eigen-frequency is achievable via introducing of prestress. It has been also demonstrated, that properly tuned initial stresses induced in the structure via initial distortions (incompatibilities) forced in one location can effectively reduce vibration amplitudes in other, selected location (in our case, ca 80% local reduction can be observed). However, the side effect of such operation can be increase of vibration amplitudes in other location on the structure. One can say, that prestressing technique can be effective in local isolation of structural vibration amplitudes, caused by some critical, external excitation. Important advantage of the proposed technique, especially for aeronautic and aerospace applications, is new option for reducing unwanted vibrations, without increasing the structural mass.

The magnitude of modification of the dynamic, structural response depends on overall structural stiffness and sensitivity of defined objective function for admissible structural modifications in selected localizations. The above results motivate authors for further, more methodological studies on control of structural dynamic response, based on modifiable prestressing technique, what will be published in a separate paper. The challenges to overcome require including material damping effect, what will make the computational task more heavy, but also will lead to more realistic results. Also, the problem of prestress optimization (location for generation of prestress distortions and their magnitude) and technologies for prestress inducing (preferable in a modifiable manner) are open questions.

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