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ASSESSMENT OF THE MATERIAL STRENGTH OF ANISOTROPIC MATERIALS WITH ASYMMETRY OF THE ELASTIC RANGE

SUMMARY

The aim of the paper is to apply the energy-based criterion of limit elastic states for the assessment of the material effort of anisotropic materials. The linear elastic anisotropic materials in the plane state of stress are considered. The theory of elastic eigen states determined by the symmetry of the Hooke elastic tensors (stiffness and compliance tensors) and the energy-based criterion of elastic limit states for anisotropic materials is used according to the theory proposed by Jan Rychlewski. Experimental data for paperboard and the results of atomic calculations were applied. The common feature of the aforementioned materials is the strength differential effect related to the asymmetry of the elastic range. Often, to determine the degree of this asymmetry one uses the ratio of the experimentally measured limit of elasticity (yield) in compression to the limit of elasticity in tension. Also, the failure criterion of P. S. Theocaris is mentioned within the discussion of the possibility of the extension of the criterion proposed by Rychlewski for anisotropic materials revealing asymmetry of elastic range and the related strength differential effect.

Keywords: *anisotropic materials, strength hypotheses, energy-based elastic limit criteria, elastic eigen states, the criteria of material effort, asymmetry of elastic range, strength differential effect*

OCENA WYTĘŻENIA MATERIAŁÓW ANIZOTROPOWYCH Z ASYMETRIĄ ZAKRESU SPRĘŻYSTEGO

Celem pracy jest zastosowanie energetycznego kryterium osiągnięcia sprężystych stanów granicznych do oceny wytężenia w anizotropowych materiałach liniowo-sprężystych w płaskim stanie naprężenia. Wykorzystano teorię sprężystych stanów własnych określonych przez symetrię tensorów sprężystości Hooke'a (tensorów podatności i sztywności) oraz energetyczne kryterium stanów granicznych sformułowane przez Jana Rychlewskiego w pracach. Wykorzystano wyniki badań doświadczalnych dla tektury oraz rezultaty atomowych obliczeń numerycznych symulujących deformację materiałów amorficznych. Wspólną cechą wymienionych materiałów jest asymetria własności wytrzymałościowych, a zatem i zakresu sprężystego. Często używa się ilorazu granicy sprężystości (plastyczności) przy ściskaniu do granicy sprężystości (plastyczności) przy rozciąganiu, aby określić stopień tej asymetrii, tzw. efekt różnicy wytrzymałości. Omówiono również kryterium P.S. Theocarisa pod kątem możliwości uogólnienia kwadratowego kryterium Rychlewskiego dla dowolnych materiałów anizotropowych wykazujących efekt różnicy wytrzymałości.

Słowa kluczowe: *anizotropowe materiały, kryterium wytężenia, sprężyste stany własne, efekt różnicy wytrzymałości, asymetria zakresu sprężystego*

1. INTRODUCTION

The proposed by Jan Rychlewski energy-based criterion of elastic limit states with the use of the theory of elastic eigen states and energy-orthogonal states gave rise to the creation of the theory of material effort of anisotropic materials (Rychlewski 1984, 1995; Kowalczyk *et al.* 2003). In the energetic criterion one should define limit energies for each elastic eigen state, which, in a particular case, is also energy-orthogonal state. These limit energies may be determined experimentally or calculated if an effective structural model of a material is given. A proposition how to calculate limit energies is given in (Nalepka and Pęcherski 2003) and discussed in more detail in (Janus-Michalska and Pęcherski 2003; Kordzikowski, Janus-Michalska and Pęcherski 2005). To some extent, the paper is based on the article published in Polish (Kordzikowski and Pęcherski 2010). In this paper, however, the presentation of the assessment of the material strength of anisotropic materials with asymmetry of the elastic range is given in a more concise and straightforward way. The discussion of the strength of foams has been omitted in order to focus on the study of limit states of metallic solids. Some new explanations and interpretations of the limit states are added and the relations to the results presented in the literature are more thoroughly discussed. Also the new references to pertinent papers are added.

2. THE QUESTION OF AN ENERGY-BASED MEASURE OF MATERIAL EFFORT

The precursor of the hypotheses of material effort, which are based on the concept of elastic energy is Eugenio Beltrami (1835–1899), who in 1885 suggested strain energy density as a measure of material effort (Beltrami 1885).

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Maksymilian Tytus Huber (1872–1950) in his seminal work of 1904 (Huber 1904) suggested, independently of Beltrami and using the original arguments having foundations in the contemporary knowledge about molecular structure of matter, that reaching the elasticity limit of a body is determined by the density of elastic strain energy. However, if the influence of pressure can be neglected, what is equivalent to the assumption that all hydrostatic states of stress are safe, the material effort is measured by the density of elastic energy of distortion. Recently, it is known that almost half a century earlier the same idea had been proposed by James Clerk Maxwell; however, his letter to William Thompson from 1856, in which the concept had been presented, was not published until 1936 (Maxwell 1936). The Huber criterion has come to be known and is widely used as an energy-based condition of material effort for isotropic bodies. Often, this criterion is combined with the names of Richard von Mises (1883–1953), who in 1913 received in an entirely different way a formula defining the criterion of plasticity (Mises 1913), and of Heinrich Hencky (1885–1951), who proved that both formulations are equivalent (Hencky 1924). The first proposition of a generalization of this hypothesis for isotropic as well as orthotropic materials with asymmetry of the elastic range was given by Włodzimierz Burzyński (1900–1970) in 1928. He proved that under certain restrictions posed on the elasticity coefficients corresponding to the assumption that the spherical part of the stiffness tensor is an eigen state (so called volumetrically isotropic materials (Rychlewski 1984), the total density of elastic energy can be additively decomposed to the part of volume change and the part of the change of shape (distortion) (Burzyński 1928). In the same year, a proposition of a limit condition for an anisotropic material was presented by Mises (Mises 1928) in the form:

$$\boldsymbol{\sigma}' \cdot \mathbf{H} \cdot \boldsymbol{\sigma}' \leq 1 \qquad (1)$$

where: $\mathbf{H}$ is a tensor of the fourth order called a material limit tensor and $\boldsymbol{\sigma}'$ – deviator of the stress tensor $\boldsymbol{\sigma}$. The German scientist, however, giving his condition categorically rejected its energy-based interpretation. The proof of the energetic nature of the Mises condition, was presented by Jan Rychlewski in the 80s of the XX century.

For the case of linear elastic materials J. Rychlewski (1984) proved the following theorem. For any solid body with the linear elastic properties defined by the compliance tensor $\mathbf{C}$ (stiffness tensor $\mathbf{S}$) and the limit properties described with the fourth order symmetric tensor $\mathbf{H}$, there exists one and only one energy-orthogonal decomposition of the space S (of symmetric tensors of the second order) $S = H_1 \oplus H_2 \oplus \ldots \oplus H_\kappa$, $\kappa \leq 6$, $H_L \perp H_K$, for $L \neq K$ and only one set of constans $h_1, h_2, \ldots, h_\kappa$, $h_\alpha \neq h_\beta$ for $\alpha \neq \beta$ such that for any stress tensor $\boldsymbol{\sigma} \in S$, $\boldsymbol{\sigma} = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 + \ldots + \boldsymbol{\sigma}_\kappa$, $\boldsymbol{\sigma}_\kappa \in H_\kappa$ and the limit condition $\boldsymbol{\sigma} \cdot \mathbf{H} \cdot \boldsymbol{\sigma} \leq 1$ takes the form:

$$\boldsymbol{\sigma} \cdot \mathbf{H} \cdot \boldsymbol{\sigma} = \frac{1}{h_1}\phi(\boldsymbol{\sigma}_1) + \ldots + \frac{1}{h_\kappa}\phi(\boldsymbol{\sigma}_\kappa) \leq 1 \qquad (2)$$

where $\phi(\boldsymbol{\sigma}_1) + \phi(\boldsymbol{\sigma}_2) + \ldots + \phi(\boldsymbol{\sigma}_\kappa) = \phi(\boldsymbol{\sigma})$, $\phi(\boldsymbol{\sigma}_K) = \frac{1}{2}\boldsymbol{\sigma}_K \cdot \mathbf{C} \cdot \boldsymbol{\sigma}_K = \frac{1}{2}\boldsymbol{\sigma}_K \cdot \boldsymbol{\sigma}_K$ (*no summation over* K) is the density of elastic energy accumulated in the K-th energy-orthogonal state and h_κ are the weight coefficients of elastic energy called in (Theocaris 1989a) the Rychlewski moduli of material effort. The aforementioned energetic limit criterion transforms into the classic criterion of Maxwell and Huber for isotropic solids. Theoretically, the condition (2) has a concise and an elegant form. However, it has rather apparent simplicity. The difficulty stems from an abstract formulation in the six-dimensional space of the energy-orthogonal eigen states of stress tensor. Although the physical interpretation of material effort moduli $h_1, h_2, \ldots, h_\kappa$, $\kappa \leq 6$ is clear as the limit energies of corresponding eigen states, the experimental determination of these limit constants is not so straightforward.

3. ANALYSIS OF FAILURE CRITERIA FOR ANISOTROPIC MATERIALS WITH ASYMMETRY OF THE ELASTIC RANGE

The procedure in determining the asymmetric energy-based condition of material effort involves the use of experimental data obtained from tests in uniaxial stress states. In order to determine the asymmetric limit surface, let us confine to the solid bodies revealing the same symmetry of a material in the elastic state and in the limit state (the tensor of compliance $\mathbf{C}$, or the tensor of stiffness $\mathbf{S}$, is parallel to the limit tensor $\mathbf{H}$). In such a case the elastic eigen states are at the same time the energy-orthogonal states. One of possible, rather straightforward, method is to proceed in the following way:

1. Determine the tensor of compliance $\mathbf{C}$ (or the tensor of stiffness $\mathbf{S}$).
2. Determine the eigen values (λ) and eigen axes ($\boldsymbol{\omega}$) of the tensor of compliance $\mathbf{C}$ (or the tensor of stiffness $\mathbf{S}$).
3. Determine the transformation matrix from the system of principal axes (the axes in which the experiment is usually performed) to the system of eigen axes of the tensor of compliance $\mathbf{C}$ (or the tensor of stiffness $\mathbf{S}$).
4. Transform the limit values of stress obtained from the experiment in the principal axes: σ_{T1}– tensile limit stress with respect the first axis, σ_{T2} – tensile limit stress with respect the second axis, σ_{T3} – tensile limit stress with respect the third axis, σ_{C1} – limit compression stress with respect the first axis, σ_{C2} – limit compression stress with respect the second axis, σ_{C3} – limit compression stress with respect the third axis) to the system of eigen axes (σ_T^I – tensile limit stress in the first eigen state, σ_T^{II} – tensile limit stress in the second eigen state, σ_T^{III} – tensile limit stress in the third eigen state, σ_C^I – limit compression stress in the first eigen state,

σ_C^{II} – limit compression stress in the second eigen state,
σ_C^{III} – limit compression stress in the third eigen state).

5. Substitute limit values σ_T^I, σ_T^{II}, σ_T^{III}, σ_C^I, σ_C^{II}, σ_C^{III}, to the criterion of Rychlewski in sequential quarters of the system of eigen axes, in which the limit surface takes the shape of deformed ellipsoid:

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_T^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{III}\right)}\le 1 \text{ in the first one,}$$

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_C^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{III}\right)}\le 1 \text{ in the second one,}$$

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_C^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{III}\right)}\le 1 \text{ in the third one,}$$

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_T^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{III}\right)}\le 1 \text{ in the fourth one,}$$

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_T^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{III}\right)}\le 1 \text{ in the fifth one,}$$

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_C^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_T^{III}\right)}\le 1 \text{ in the sixth one,}$$

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_C^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{III}\right)}\le 1 \text{ in the seventh one,}$$

$$\frac{\Phi\left(\boldsymbol{\sigma}^I\right)}{\Phi\left(\boldsymbol{\sigma}_T^I\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{II}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{II}\right)}+\frac{\Phi\left(\boldsymbol{\sigma}^{III}\right)}{\Phi\left(\boldsymbol{\sigma}_C^{III}\right)}\le 1 \text{ in the eighth one,}$$

where: $\boldsymbol{\sigma}=\boldsymbol{\sigma}_{T,C}^I+\boldsymbol{\sigma}_{T,C}^{II}+\boldsymbol{\sigma}_{T,C}^{III}$ – decomposition of the stress tensor into the eigen states for tension and compression, respectively, $\Phi(\boldsymbol{\sigma}_{T,C}^{I,II,III})$ – limit value of the density of elastic energy in the particular eigen state for tension or compression.

The verification of the proposed procedure was performed using experimental data for paperboard, which are given in (Suhling *et al*. 1985; Biegler and Mehrabadi 1995):

$$\begin{array}{lll} E_1=3510\ \text{MPa}, & E_2=3510\ \text{MPa}, & E_3=6930\ \text{MPa},\\ \nu_{13}=0.15, & \nu_{23}=0.15, & \nu_{12}=0.3,\\ G_{23}=1700\ \text{MPa} & G_{13}=1700\ \text{MPa}, & G_{12}=1500\ \text{MPa}. \end{array}$$

The stiffness tensor assumes the form:

$$\mathbf{S}=\begin{bmatrix} 4220 & 1520 & 1700 & 0 & 0 & 0\\ 1520 & 4220 & 1700 & 0 & 0 & 0\\ 1700 & 1700 & 7940 & 0 & 0 & 0\\ 0 & 0 & 0 & 1700 & 0 & 0\\ 0 & 0 & 0 & 0 & 1700 & 0\\ 0 & 0 & 0 & 0 & 0 & 1500 \end{bmatrix}\text{MPa.}$$

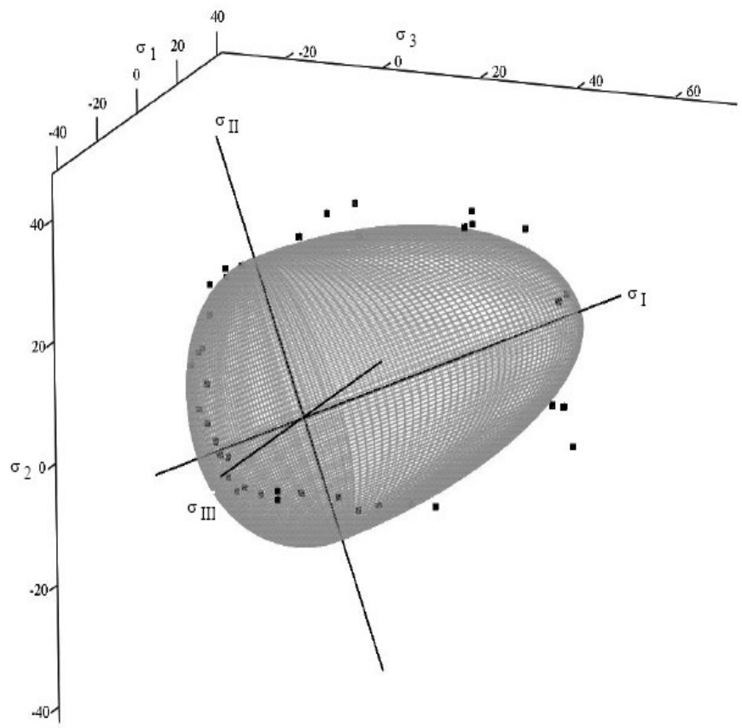


Fig. 1. Asymmetric limit surface determined for the calculated stress limits based on the energetic criterion of Rychlewski, applied to each part of the system of eigen axes

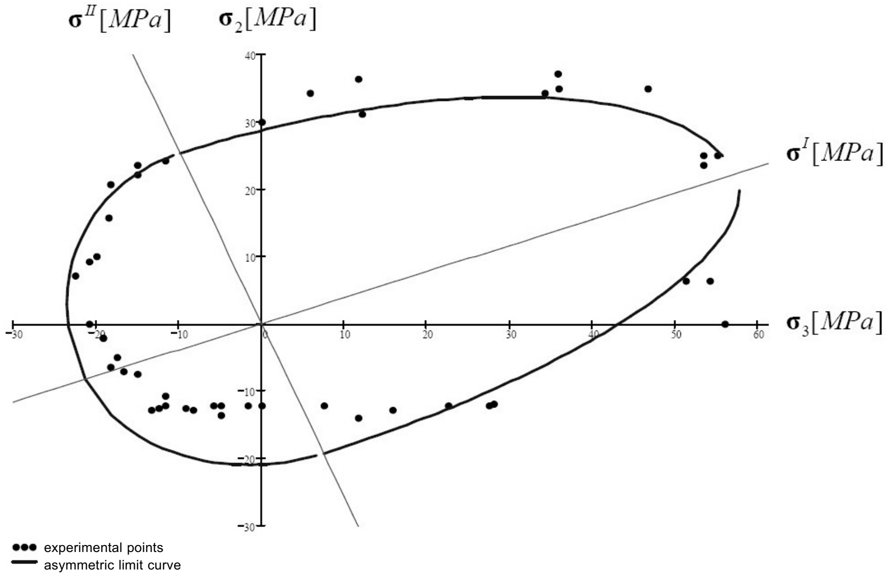


Fig. 2. Asymmetric limit curve for paperboard determined for the calculated limit stresses based on the energetic criterion of Rychlewski, applied to each quarter of the system of eigen axes

Following the steps given above, we obtain an asymmetric limit surface inscribed between points obtained from the experiment (Fig. 1).

Following the proposed scheme one can as well restrain to plane states only, which are discussed thoroughly and verified for thin films in (Ostrowska-Maciejewska and Pęcherski 2006) (Fig. 2).

Proceeding according to the scheme presented above and performing the calculations (Kordzikowski 2007), there was determined in this work an asymmetric limit curve inscribed between points obtained from the experiment for amorphous metal (Lund and Schuh 2005) (Fig. 3).

It is also worth to mention and compare limit curves by Rychlewski with the limit curves obtained from the Theocaris criterion (Theocaris 1989a), which could be used to generalize the Rychlewski criterion for arbitrary anisotropic material showing a strength differential effect (SDE):

$$\frac{\sigma_1^2+\sigma_2^2}{\sigma_{T1}\sigma_{C1}}+\frac{\sigma_{33}^2}{\sigma_{T3}\sigma_{C3}}-\left(\frac{2}{\sigma_{T1}\sigma_{C1}}-\frac{1}{\sigma_{T3}\sigma_{C3}}\right)\cdot$$

$$\cdot\sigma_1\sigma_2-\frac{\sigma_2\sigma_3+\sigma_3\sigma_1}{\sigma_{T3}\sigma_{C3}}+$$

$$+\left(\frac{1}{\sigma_{T1}}-\frac{1}{\sigma_{C1}}\right)(\sigma_1+\sigma_3)+\left(\frac{1}{\sigma_{T3}}-\frac{1}{\sigma_{C3}}\right)\sigma_3=1,$$

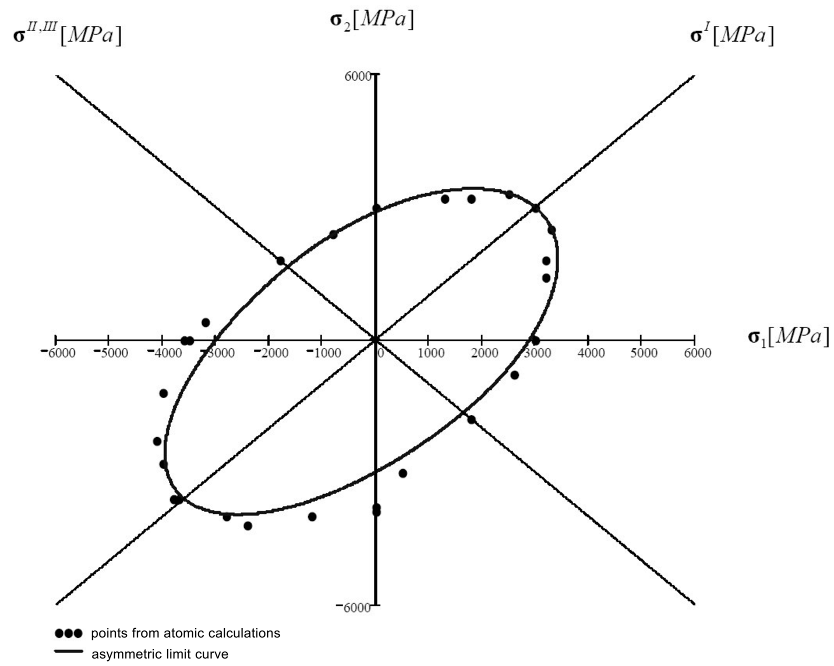


Fig. 3. Asymmetric limit curve for amorphous metal (Kordzikowski 2007). The points correspond to the results of atomic calculations (Lund and Schuh 2005)

Where: σ_{Ti}, σ_{Ci} denote respectively the limit values of stress in tension and compression in the principal direction i = 1, 2, 3.

Using the given above algorithm and the experimental data presented in (Theocaris 1989a, b, c) the following limit curves were determined (Fig. 4 and 5).

4. CONCLUSIONS

The presented approach makes it possible to apply the energy-based criterion of Rychlewski (1984, 1995) for evaluation of the material effort in the case of materials which are characterized by strength differential effect, and hence the

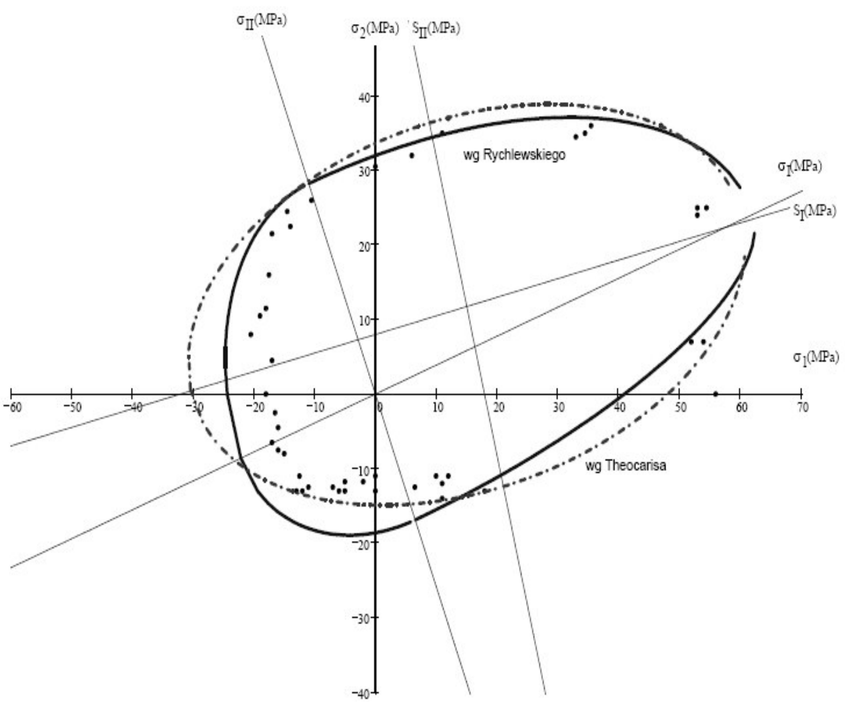
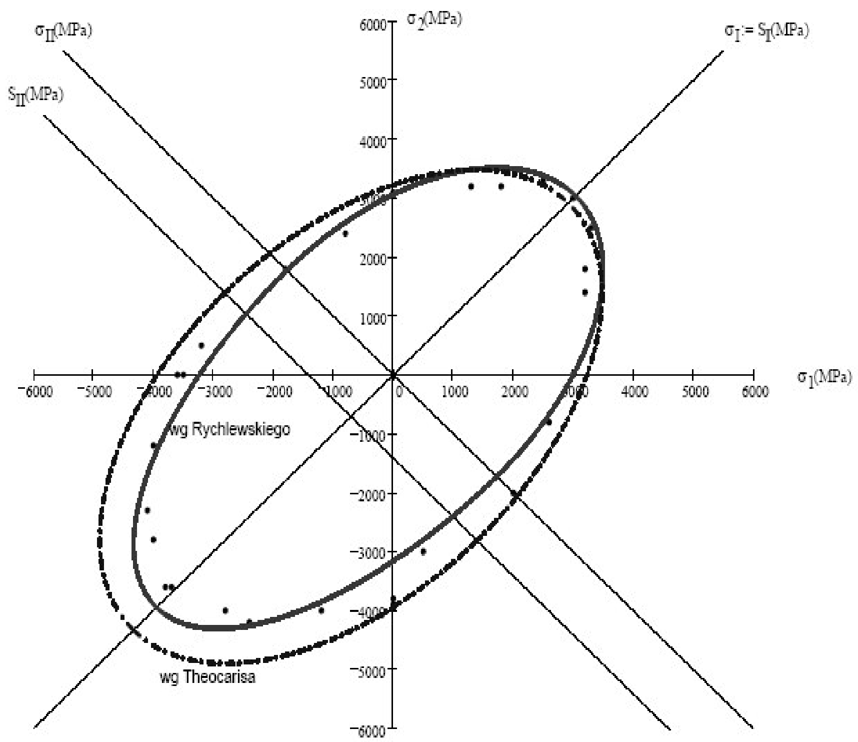


Fig. 4. Limit curves for paperboard determined for the calculated limit stresses based on the energetic criterion of Rychlewski, applied to each quarter of the system of eigen axes and according to Theocaris



Rys. 5. Limit curves for amorphous metal determined for the calculated limit stresses based on the energetic criterion of Rychlewski, applied to each quarter of the system of eigen axes and according to Theocaris

asymmetry in the elastic range. The given graphical interpretation of the asymmetric energetic condition in the system of eigen axes (in the space of eigen states) shows that in each part of this system there is a different limit surface defined, corresponding to material properties determined experimentally in the system of principal axes (in the space of principal stresses). Such an approach to the analysis of the energy-based material effort criterion by Rychlewski allows one to use uniaxial tests to determine material effort and also provides the basis to determine Rychlewski's modules h_κ, and hence the limit state tensor $\mathbf{H}$.

The discussed analysis creates also a possibility to relate the proposed method of a sectional description and identification of limit curves with the methodology developed by Oller *et al*. (2003), which is based on the paraboloid isotropic yield criterion as a starting point. The criterion is to be adjusted then by certain transformation to the behaviour of an orthotropic material with asymmetry of elastic range.

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