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Full-wave analysis of finite baffle system for linear phased array applications

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Abstract

Mixed boundary-value problem for a finite system of rigid baffles in acoustic medium is solved for the case of sound radiation with the help of the method developed earlier in electrostatics. The solution is sought in spectral domain. The method described here enables direct evaluation of the spatial spectrum of pressure distribution on the baffle plane, which is used for the far-field radiation pattern evaluation. The approach can be used for the phased array modelling with the finite element size and the inter-element interactions taken into account. Some illustrative numerical examples present the far-field radiation pattern and the wave-beam steering in a baffle system that may be considered as a model of one dimensional ultrasonic transducer array.

1. INTRODUCTION

A typical one-dimensional ultrasonic array transducer consists of alternate sets of acoustically different materials: piezoelectric, which responds to the incident waves by electric signal, and acoustically isolating material (like epoxy) between them. By controlling the phase and the amplitude of the excitation signal of different elements the beam generated by the array can be dynamically steered in the region of interest and focused at a given point. The phased arrays are widely and successfully used in medical ultrasonic diagnostics for a long time. Their properties give rise to their advantages over conventional ultrasonic transducers, including flexible control and signal processing, high inspection speed and fast imaging capabilities. The nondestructive evaluation and testing is another area where the ultrasonic phased arrays has been receiving great attention recently. There are different approaches to the linear phased array modeling described in the literature. The beam profile modeling is based on the intuitive representation of an array as a set of simple point sources [1,2]. In the point spread function modeling [3,4] the ability of a phased array to visualize a point reflector (by means of certain imaging algorithm) is modeled. For this purpose the ultrasonic data from the array due to a point reflector at a particular spatial position are simulated first. Then the image of the reflector is plotted using the appropriate imaging algorithm applied to the simulated data.

Both these methods must apply certain model of the individual element of the array (they are typically piezoelectric beams separated by epoxy layers). There are different methods of modeling the array elements, including finite element analysis [5] or Huygens principle [6]. In the later case, usually the integration of a series of point or line sources is performed to obtain the element directivity function due of the finite size of the array element. The above approaches to the modeling of array transducer assume that the individual elements respond to the incident wave pressure independently on each other yielding the electric signal proportional to the incident wave amplitude.

However, since piezoelectric materials are closer to hard, and epoxy is closer to soft acoustic materials, the Bragg scattering occurs when the incident wave scatters from the array. This phenomenon must somehow distort the local acoustic pressure on piezoelectric elements of the array affecting its electric response. In this paper the model of ultrasonic linear array transducer based on the full-wave analysis of the corresponding boundary-value problem is proposed. The wave excitation problem is considered particularly, however the scattering or sound detection problem can also be addressed by this method. The considered system consists of a finite number of acoustically hard strips (baffles) where the normal acoustic vibration vanish, and between them are acoustically soft domains where the acoustic pressure is given constant values in the considered excitation problem. This is a mixed (Dirichlet-Neumann) boundary-value problem that we deal with: the given pressure between baffles models the wave-beam generation. A similar system of baffles modeling the phased array transducer was considered for example in [7]. An efficient method developed in electrostatics [8] is found suitable for rigorous solution of the above-mentioned problem for finite planar system of baffles. The paper is organized as follows. In the next section the boundary value problem for strips is formulated. In sec. 3 the method of solution is discussed and in sec. 4 some numerical results are presented.

2. BOUNDARY-VALUE PROBLEMS FOR STRIPS

Two-dimensional harmonic wave-field $\exp\{j(\omega t-\xi x-\eta z)\}$ is considered independent of y in the acoustic media governed by equations for acoustic potential φ, pressure p and particle velocity $\boldsymbol{v}$, (t - time, x, y, z - spatial coordinates, ω, ξ, η - angular temporal and spatial frequencies)

$$\nabla^2\varphi+k^2\varphi=0,\ \boldsymbol{v}=-\nabla\varphi,\ p=j\omega\rho_a\varphi \qquad (1)$$

where $k=\omega/c$. Standard notations are applied: c is sound velocity and ρ_a is mass density of the media. We pay special attention to the wave-field at the plane z=0. Assuming known pressure there of amplitude p: $p\cdot\exp\{-j\xi x\}$, the resulting z-component of the particle velocity $v_z(z=0+)$ on the upper side of this plane (denoted by v to shorten notations), can be easily evaluated:

$$v=G\,p,\ G=\eta/(\omega\rho_a),\ \eta=\sqrt{k^2-\xi^2}=-j\sqrt{\xi^2-k^2} \qquad (2)$$

neglecting the exponential term $\exp\{j(\omega t-\xi x)\}$; G is the surface harmonic admittance of acoustic half-space. Above, η is chosen in order to satisfy the radiation condition of the acoustic field at $z\to\infty$. Inside the media, the acoustic potential generated by surface p (given at z=0) is

$$\varphi(x,z)=-j\,p/(\omega\rho_a)\mathrm{e}^{-j(\xi x+\eta z)} \qquad (3)$$

In the presented method of analysis, we will need the x-derivative of pressure $p(x)$ at z=0, denoted here by $q(x)$

$$q\equiv p_x=-j\,\xi\,p \qquad (4)$$

which in spectral domain yields

$$v=(jG/\xi)\,p_{,x}=g(\xi)q\,;\ g(\xi)=\frac{j}{\omega\rho_a}\frac{\eta}{\xi}\,;$$
$$g(\xi\to\pm\infty)=g_\infty S_\xi,\ g_\infty=\frac{1}{\omega\rho_a} \qquad (5)$$

where S_ξ=1 for $\xi\geq 0$ and -1 otherwise, for arbitrary real ξ.

Let us consider a finite system of N acoustically hard baffles distributed along the x-axis on the boundary z=0 of the acoustic medium spanning for z>0, as shown in Fig. 1.

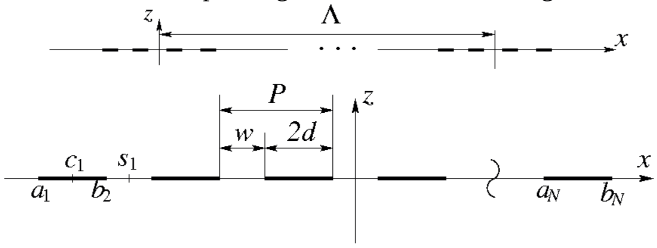

Fig. 1. A system of N rigid baffles (strips) on the boundary of acoustic media spanning z>0.

Their edges are defined by x-coordinates (a_i,b_i), i=1..N. The baffles assumed to be infinitely long along the y-axis.

For the case of the acoustic field generation considered here the normal component of the particle velocity $v\equiv v_z$ vanishes on baffles. A harmonic pressure of amplitude p_l (constant over entire slot) excites the wave-field in the medium z>0; $l\Lambda$ describes the position of the given l-th slot center along the x-axis. Thus, the boundary conditions are

$$\begin{aligned}&q=0,\ x\in(b_l,a_{l+1}),l=0..N,\ \text{between baffles}\\&v=0,\ x\in(a_l,b_l),\ l=1..N,\ \text{on baffles}\\&p(s_l)=p_l,\ l=1..N-1,\ \text{at the slot center}\end{aligned} \qquad (6)$$

where $s_l=(a_{l+1}+b_l)/2$ - coordinate of the center of slot between l and l+1 baffles; b_0 and a_{N+1} corresponds to (–/+) ∞ respectively; p_l are given values. They are constant in given slots between baffles due to the condition q=0 there. The solutions to the boundary-value problem are the functions $p(x)$ and $v(x)$ at z=0 plane. The field in the medium, z>0, can be evaluated from (3).

3. METHOD OF SOLUTION

To find the solution of (6) for the system of N hard acoustic baffles the method developed earlier in electrostatics for the finite system of conducting strips [8,9] can be successfully adopted. The set of template functions introduced in [8] as the partial solutions of the corresponding electrostatic problem is referred below

$$\Phi^{(N)}(x)=j^{N-1}\prod_{m=1}^{N}\frac{1}{\sqrt{d_m^2-(x-c_m)^2}}$$
$$\Phi^{(N,i)}(x)\sim x^i\Phi^{(N)},\ i=0...N-1 \qquad (13)$$

where d_m and c_m are the half-width and center coordinate of i^{th} baffle. The function $\Phi^{(N)}$ is the basis template function and the rest of $\Phi^{(N,i)}$ can be derived from $\Phi^{(N)}$ as in (13). The above functions have known spectral representation being multiple convolutions of Bessel functions of the first kind $J_0(\xi d_m)$ and $J_1(\xi d_m)$. For the basis template function $\Phi^{(N)}$ the spectral representation is given below

$$\Phi^{(N)}(\xi)=\Phi_1(\xi)*\Phi_2(\xi)*\cdots*\Phi_N(\xi), \qquad (14)$$

where

$$\Phi_i(\xi)=\mathrm{F}\left\{\frac{1}{\sqrt{d_i^2-(x-c_i)^2}}\right\}=J_0(d\,\xi)\,\mathrm{e}^{j\xi c_i},\xi\geq 0,$$

and F denote the Fourier transform. Note the semi-finite support of the above functions, which feature is of great importance for further numerical analysis. The function $\Phi^{(N)}(x)$ has the property that its real and imaginary parts vanish in subsequent domains of the *x*-axis, as required by the boundary conditions (6). We introduce the template functions for the acoustic field generation problem considered here as follows

$$Q^{(N)}(\xi)=\begin{Bmatrix}\Phi^{(N)}(\xi),\xi\geqslant 0\\ \Phi^{*(N)}(-\xi),\xi<0\end{Bmatrix},\quad V^{(N)}(\xi)=S_{\xi}Q^{(N)}(\xi)=\begin{Bmatrix}\Phi^{(N)}(\xi),\xi\geqslant 0\\ -\Phi^{*(N)}(-\xi),\xi<0\end{Bmatrix}. \tag{15}$$

As shown in [8], the functions defined in (15) have their spatial counterparts vanishing on the *x*-axis in accordance with (6), namely $Q^{(N)}(x)$ (as $q(x)$) vanishes between baffles and $V^{(N)}(x)$ vanishes on baffles (as $v(x)$). These functions, evaluated at discrete values of the spectral variable $\xi_n=n\Delta\xi$, are the discrete series in the numerical analysis applied here and actually they represent, on the basis of the theory of FFT [10] periodic functions in spatial domain with a certain large period $\Lambda=2\pi/K$, $K=\Delta\xi$ (see Fig.1)

$$(Q,V)^{(N)}(x)=\sum_n (Q,V)_n^{(N)}e^{-j\xi_n x} \tag{16}$$

The functions (16) will help us to satisfy the boundary conditions (6). Namely, the wave-filed $(q,v)(x)$ can be represented by the inverse Fourier transform written in discrete form for the assumed large period Λ (formally, $\Lambda\rightarrow\infty$, but in the applied approximation Λ is large but finite)

$$(q,v)(x)=\sum_n (q,v)_n e^{-j\xi_n x}, \tag{17}$$

where the corresponding spectral representations (spectral samples $(q,v)_n$) are given by the following expansion [11]

$$q_n=\sum_m \alpha_m Q_{n-m}, v_n=\sum_m \beta_m S_{n-m}Q_{n-m}. \tag{18}$$

The expansions (18) represent the convolutions in spectral domain written in discrete form, which in spatial domain corresponds to the product of the template functions (15) with certain unknown functions represented by their Fourier transforms (in discretized form)

$$(\alpha,\beta)(x)=\sum_n (\alpha,\beta)_n e^{-i\xi_n x}, \tag{19}$$

given by corresponding spectral samples $(\alpha,\beta)_n$ which occur in (18) as unknown expansion coefficients that have to be determined. Apparently, the functions (17,18), being the solutions of the considered boundary-value problem for finite system of *N* baffles satisfy the boundary conditions (6) due to the properties of the template functions (15,16). Now we only need to check if the applied solutions (17,18) satisfy the wave equation inside the media, which equation is represented at the baffle plane $z=0$ by the harmonic admittance $g(\xi)$ (5). Only this part of the wave-field represented by $(q,v)(x)$ which satisfies the radiation condition at $z\rightarrow\infty$ are involved in the solution. This yields the relation for the n^{th} spectral sample having wave-number ξ_n:

$$v_n=g(\xi_n)q_n, \tag{20}$$

which must be satisfied for all *n*, particularly for large $|n|$ where $g(\xi_{N_1})=\pm g_\infty$ (formally, $N_1=\infty$, but in the applied approximation N_1 is assumed large but finite - this is the only approximation applied in the method). This is only possible if [13]:

$$\beta_m=g_\infty\alpha_m \tag{21}$$

which substituted into the earlier equation and accounting for (15) and (5) yields:

$$g_\infty\sum_m \alpha_m[S_{n-m}-j(\eta_n/\xi_n)]Q_{n-m}^{(N)}=0. \tag{22}$$

The equations for α_m outside the limits $n\in[-N_1,N_1]$ and $m\in[-N_1,N_1]$ are satisfied automatically, what can be checked by inspection [13]. To obey the condition that the pressure takes given constant values in the slots between baffles we use the similar technique as described in [14]. Having *N* baffles there are $N_s=N$-1 slots and constraints which have to be satisfied. For this purpose the number of coefficients α_m in (22) is chosen to be $2N_1+1+N_s$ and the above N_s constrains are added to the system of equations (22):

$$p_i\equiv p(x=s_i)=\left(\int q(x)\,dx\right)_{x=s_i},\ i=..N_s, \tag{23}$$

where s_i is the i^{th} slot center (see Fig.1). Here we benefit from the spectral representation of template solution $Q^{(N)}(\xi)$ (15) and following the same considerations as described in [8] we can numerically evaluate (13) as follows

$$p_i=\sum_m \alpha_m \mathrm{F}^{-1}\left\{jQ^{(N)}(\xi_{n-m})/\xi_n\right\}_{x=s_i}, i=1..N_s \tag{24}$$

Summarizing, the system of linear equations for α_m, $m\in[-N_1-M_l, N_1+M_u]$, where $M_u=M_l=N_s/2$ for even N_s and $M_l=(N_s-1)/2$ and $M_u=(N_s+1)/2$ for odd N_s, is

$$[A_{mn}][\alpha_m]=b_n,\ n\in[-N_1,N_1+N_s]. \tag{25}$$

The elements of matrix A_{mn} are given by (22) and $b_n=0$ for $n\in[-N_1, N_1]$, and

$$A_{mn}=\mathrm{F}^{-1}\left\{jQ^{(N)}(\xi_{n-m})/\xi_n\right\}_{x=s_i},\quad b_n=p_i,\ n\in[N_1+1,N_1+N_s], i\in[1,N_s]. \tag{26}$$

Thus, solving the system of equations (25) for unknown α_m, $m\in[-N_1-M_l,N_1+M_u]$ the solution of the considered boundary-value problem can be obtained from (17,18).

4. NUMERICAL RESULTS AND DISCUSSION

In this section some numerical examples of the sound beam-forming by given pressure between baffles are given. It should be noted, that the method of analysis discussed here yields the spatial spectrum of the acoustic pressure on the baffle plane directly. Taking the advantage of this, the radiation pattern can be simply evaluated from the inverse Fourier transform of $p(\xi)$ which can be easily evaluated from $q(\xi)$ defined in (4) (note, $p(\xi\rightarrow 0)=0$). At the level *z* above the baffle plane $z=0$, the acoustic pressure behaves according to (3), thus introducing spatial angular variables $x=R\sin\theta$, $z=R\cos\theta$, the pressure in polar coordinates is

$$p(R,\theta)=\int_{-\infty}^{\infty} p(\xi)\,\mathrm{e}^{-jR(\xi\sin\theta+\eta\cos\theta)}d\xi, \tag{27}$$

where η is given by (2). At large distance $R\rightarrow\infty$, we may drop this part of the integral representing the localized field at the baffle plane, which depends on imaginary valued η (2). This is made by constraining γ to the domain $(-\pi/2,\pi/2)$ in the transformed integration, where $\xi=k\sin\gamma$, $\eta=k\cos\gamma$. Substituting these into (27) one obtains the transformed integration for evaluation the pressure at large distance from the baffle plane, denoted here as $p_R(\theta)$

$$p_R(\theta)=\int_{-\pi/2}^{\pi/2} p(k\cos\gamma)\cos\gamma\, e^{-jkR\cos(\gamma-\theta)}d\gamma, \quad (28)$$

and (28) can be easily calculated by the stationary phase method [14] (the stationary point of interest here is $\theta=\gamma$)

$$p_R(\theta)=p(k\sin\theta)\cos\theta\, k\sqrt{\frac{2\pi}{jkR}}e^{-jkR}, \quad (29)$$

or, substituting $p(\xi)$ from (4), the angular dependence in the far-field region can be deduced

$$p_R(\theta)\sim q(k\sin\theta)\cot\theta. \quad (30)$$

In the numerical examples presented in Fig. 2. the far-field radiation pattern is shown for N=8 baffles when the given pressure is $p_l=\exp\{jl\Lambda k\sin\beta\}$, l = 1..7, where β is a steering angle. The cases of β=0° and β=15° are considered. In Fig. 3 the corresponding distribution of the pressure field on the baffle plane is shown.

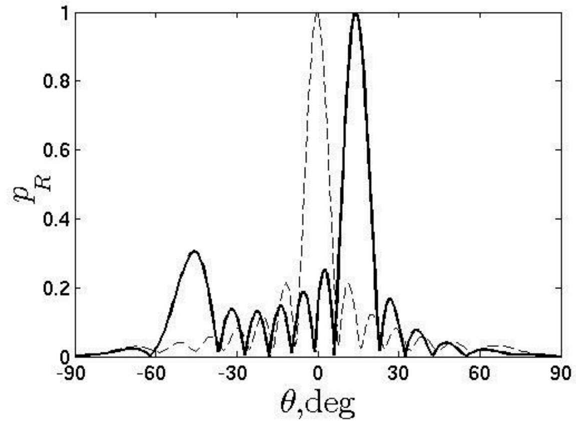

Fig. 2. Far-field radiation pattern for 8 baffle array; steering angle β=15° (solid line) and β=0° (dashed line); $P=\lambda$, w=0.75λ.

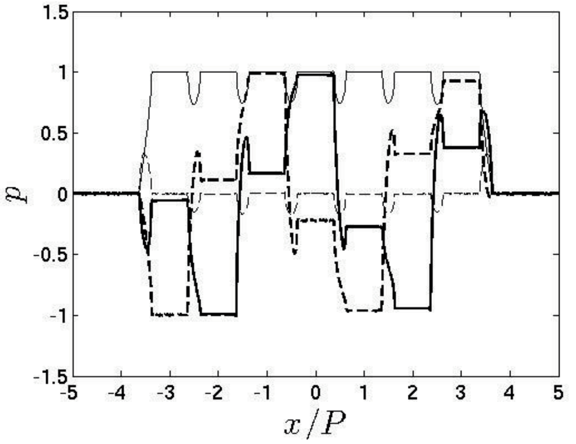

Fig. 3. Spatial distribution of $p(x)$ on the baffle plane for 8 baffle array; steering angle β=15° (thick lines) and β=0° (thin line); solid lines – Re(p), dashed lines – Im(p); P=λ, w=0.75λ.

5. CONCLUSION

In this work the mixed boundary-value problem for a finite system of rigid baffles in acoustic medium is solved for the case of sound radiation. The method developed earlier in electrostatics is adopted here. The presented examples confirm that the applied method is worth consideration for numerical experiments concerning the beam-forming systems. It yields all interesting characteristics of the system within the same simple analysis. The method described here enables direct evaluation of the spatial spectrum of pressure distribution on the baffle plane, which is used for the far-field radiation pattern evaluation. Thus, a phased array can be modeled by this approach with the finite element size and the inter-element interactions taken into account. The method can be directly applied for analyzing finite transducer array having different transducer elements with different width and spacing. Such modification may help reducing spurious effects connected with abrupt ends of the transducer system, which is considered as quite difficult for analysis. Besides, the problem of wave detection can be also address with this approach, which requires solving the corresponding boundary-value problem formulated for the case of plane acoustic wave scattering by finite baffle system.

REFERENCES

[1] S. C. Wooh, Y. Shi. *Influence of phased array element size on beam steering behavior*, Ultrasonics (36), 1998, 737-749.

[2] R. Ahmad, T. Kundu, *Modeling of phased array transducers*, J. Acoust. Soc. Am. 117-4, 2005, 1762-1776.

[3] R. Y. Chiao, L. J. Thomas, *Analytic evaluation of sampled aperture ultrasonic imaging techniques for NDE*, IEEE Trans. Ultrason., Ferroelect., Freq. Cont. (41), 1994, 484-493.

[4] F. Lingvall, T. Olofsson, T. Stepinski, *Synthetic aperture imaging using sources with finite aperture: deconvolution of the impulse response*, J. Acoust. Soc. Am. 114-1 2003, 225-234.

[5] R. Lerch, *Simulation of piezoelectric devices by two- and three-dimensional finite elements*, IEEE Trans. Ultrason., Ferroelect., Freq. Cont. (32), 1990, 233-247.

[6] A. McNab, A. Cochran, M. A. Campbell, *The calculation of acoustic fields in solids for transient normal surface sources of arbitrary geometry and apodisation*, J. Acoust. Soc. Am. 87-4, 1990, 1455-1465.

[7] E. Kühnicke, *Phase arrays - Fundamental investigations for correct steering by means of sound field calculations*, Wave Motion (44), 2007, 248-261.

[8] Y. Tasinkevych, E. J. Danicki, *On electrostatics of finite planar system of strips applied in surface acoustic wave interdigital transducers*, J. Tech. Phys. (46), 2005, 175-193.

[9] Y. Tasinkevych, *Methods of IDT charge spatial spectrum evaluation*, J. Tech. Phys. (45), 2004, 155-172.

[10] W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, Numreical recipes in C. The art of scientific computing, Cambridge University Press, 2nd ed, 1992.

[11] E. J. Danicki, *Electrostatics of interdigital transducers*, IEEE Trans. Ultrason., Ferroelect., Freq. Cont. (51),4, 2004, 444-452.

[12] K. Blotekja., K. A. Ingebrig., H. Skeie, *Methods for analyzing waves in structures consisting of metal strips on dispersive media*, IEEE Trans. Electron. Devices, ED20, 12, 1973, 1133-1138.

[13] E. Danicki, Spectral theory of interdigital transducers of surface acoustic waves, ch.III.3, http://www.ippt.gov.pl/edanicki/danickibook.pdf

[14] M. W. Fedoryuk, The Steepest Descent Method, Nauka, Moscow 1977, Chapt. 3.