Determining the optimal locations of piezoelectric transducers for vibroacoustic control of structures with general boundary conditions

Ł. Nowak, T. G. Zieliński
Institute of Fundamental Technological Research, Polish Academy of Sciences
ul. Pawińskiego 5B, 02-106 Warsaw, Poland
e-mail: **lnowak@ippt.gov.pl**

Abstract
Vibroacoustic control of thin beam, plate and panelled structures with arbitrary boundary conditions is investigated. The study focuses on determining optimal locations of piezoelectric sensors and actuators on the surfaces of structures under vibroacoustic control. The work consists of three parts. In the first part, the undertaken assumptions and some governing equations are briefly introduced. Then, in the second part of the study, the piezo-transducers' locations which ensure optimal sensing/actuating capabilities for specific vibration modes are determined, basing on the derived analytical formulas and on some results of numerical simulations, as well as on the actuator/sensor equations given in the first part of the study. The relevant modes are selected by taking into account that the main purpose is to minimise the acoustic field generated by the vibrating structure. The third part of the work discusses some experimental investigations aimed for the verification of the results obtained theoretically. Some technical aspects of creating the composite structures for active control systems are briefly described in appendix.

1 Introduction

Active control of noise generated by vibrating structural elements has been the topic of numerous scientific investigations over the past several decades (see, for example [1], or some more recent [2, 3] and references within). The necessary elements of control systems developed for such applications are sensors and actuators: the sensors allow to determine the current state of vibrating structures (or some parameters of the generated acoustic field), whereas the actuators are used to apply the control loads. Among a variety of available techniques of implementation, one of the most commonly used are piezoelectric transducers attached to the surface of structures under control. Such solution preserves compactness of the controlled system while providing good electro-mechanical properties. However, the control performance depends on many factors of which the proper positioning of the transducers is certainly one of the most important.

Many of the studies devoted to the problem focused on specific types of structures, which may be accurately described using analytic formulas – like, for example, beams or simply supported plates. Solutions obtained for these cases allowed to design piezoelectric sensors and actuators sensitive only to specific sets of structural modes [1] or even to a single structural mode [4, 5]. However, the results of those investigations cannot be easily generalized onto a more general case of plates with arbitrary boundary conditions.

Taking into account the parameters of the closed-loop feedback control system it is desirable to use collocated piezoelectric sensor-actuator pairs. Two different solutions which ensure this feature can be found in literature. The first one – which is simpler and more practical, yet not always feasible due to the possible lack of access to both sides of a structure – is to attach the transducers symmetrically to the both surfaces of a thin beam or plate [1]. The second solution involves the use of a single piezoelectric element as a sensor and an actuator simultaneously [6, 7, 8, 9]. The advantages of such a solution with respect to the functionality of the control system are significant, but the necessary complications of the corresponding electronic circuits together with a requirement to meet very stringent parameters make it impractical.

Optimization algorithms for the placement of sensors and actuators may be based on different cost functions, depending on the types of structures, their purposes, and also some restrictions related with using various types of transducers [10, 11, 2]. This problem is usually solved numerically with different iterative algorithms.

This paper aims to describe a general procedure of designating the optimal parameters of the piezoelectric sensors/actuators for active vibroacoustic control of thin beams, plates and panelled structures with arbitrary boundary conditions. The basis for the following considerations is a formula derived for a multi-modal feedback control system, whose coefficients are computed using the piezoelectric actuator and sensor equations that describe the dynamic response of structure.

2 Theoretical considerations and analysis

2.1 Problem statement

The present study concerns the problem of reduction of low-frequency noise generated by vibrating structural elements. In this context, optimal locations of piezoelectric sensors and actuators for active vibroacoustic control system are sought. Beam, plate and panelled structures with arbitrary boundary conditions are considered. It is assumed that the structures and the piezoelectric transducers attached to their surfaces are rectangle in shape and that their edges are parallel to the axes of the global coordinate system. The typical geometry of the problem is depicted in figure 1. Vibrational motion of the structures is assumed to occur only in the z direction, so only one, corresponding component of the displacement field is considered, namely, the deflection $w = w(x, y, t)$.

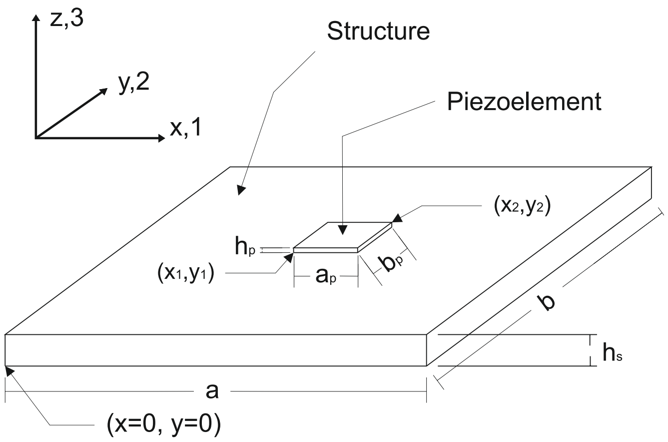

Figure 1: Geometry of the considered problem.

In case of the so-called beam structures it is assumed that the length a of a structure is much greater than its width b and its thickness h_s. The flexural waves propagate along the x direction only and the deflection w is constant along the y direction, that is: $\frac{\partial w}{\partial y} = 0$. The vibrations are described using the classical Euler-Bernoulli beam theory by the following equation (see, for example, [12]):

$$EI \frac{\partial^4 w}{\partial x^4} + \rho S \frac{\partial^2 w}{\partial t^2} = F_L, \tag{1}$$

where E is the Young's modulus of the isotropic material of the beam, I is the cross-sectional moment of inertia, ρ is the density of the material, S is the constant cross-section area of the beam, and finally, $F_L = F_L(x,t)\ \left[\frac{\mathrm{N}}{\mathrm{m}}\right]$ is the external load applied per length of the beam.

Similarly, plate and panelled structures considered in this study are thin in the sense of the classical Kirchhoff's plate theory. They are considered to be made of homogeneous, isotropic material (thus, in case of composites, such approach can be applied provided that the relevant effective material constants are known). Their vibrations are then described by the following equation of motion (see, for example, [12]):

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho h_s \frac{\partial^2 w}{\partial t^2} = F_S, \tag{2}$$

where $D = \frac{E h_s^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate, which depends on its thickness h_s, as well as on the Young's modulus E and Poisson's ratio ν of the material, while $F_S = F_S(x,y,t)\ \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ is the function of external load applied per surface area of the plate.

Due to the fact that the main interest of the present study is the acoustic emission of vibrating structural elements, the considered types of external excitation loads are limited to steady-state harmonic forces. The system is linear and the structural damping is neglected, therefore the response of the structure is also harmonic, with the same frequency and phase as the excitation. This study focuses only on a low-frequency range (up to about 400 Hz), since higher frequency vibrations can be rather easily suppressed using the well-known passive techniques. Taking all these assumptions into account, the response of a structure can be approximated by a finite sum of N structural modes as follows:

$$w(\boldsymbol{x},t) \cong \sum_{n=1}^{N} \Phi_n(\boldsymbol{x}) w_n(t) = e^{\mathrm{i}\omega t} \sum_{n=1}^{N} \Phi_n(\boldsymbol{x}) W_n \cong e^{\mathrm{i}\omega t} \tilde{w}(\boldsymbol{x}), \tag{3}$$

where $(\boldsymbol{x}) \equiv (x)$ in case of beam structures and $(\boldsymbol{x}) \equiv (x,y)$ in case of plate structures, while Φ_n is the normalized shape function of mode n with and w_n as the time-varying coefficients. When a harmonic motion is considered – with $\omega = 2\pi f$ as the angular frequency of the external excitation force (f being the frequency) – these coefficients are also time-harmonic and can be expressed as $w_n(t) = e^{\mathrm{i}\omega t} W_n$, where W_n are the (frequency-dependent) modal amplitudes; $\tilde{w}(\boldsymbol{x})$ in the (frequency-dependent) amplitude function of harmonic vibrations. Here and below, it is understood that when the time-harmonic term $e^{\mathrm{i}\omega t}$ is involved, eventually only real (or imaginary) part of the whole expression has physical meaning (and should be eventually taken as the final result).

Modal shape functions Φ_n are found by solving the corresponding eigen-problems to the motion equations (1) or (2), that is, by setting their right-hand-side terms to zero and seeking non-trivial (i.e., non-zero) solutions in the form $e^{\mathrm{i}\omega t}\tilde{w}(\boldsymbol{x})$. In the case of beams, regardless of their boundary conditions (and, as a matter of fact, because of their 'unidimensional' simplicity), it is always possible to find analytical solution consisting of a sum of trygonometric and hyperbolic functions [12]. In the case of plate structures, however, even when they are rectangular in shape, the analytical solutions can be found only for some specific ('geometrically-homogeneous') boundary conditions and – in general – it is required to use numerical methods, such as the Finite Element Method, in order to solve such problems.

The acoustic pressure field generated (in air) by a vibrating structure is, in general, complex and varies strongly with the frequency, the distance from the source, and the boundary conditions. According to that fact, it is necessary to deliver a specific cost function that will be minimized by the developed active control system to make the present investigation unambiguous. Thus, the far-field acoustic pressure in the half-space over one side of the vibrating structure placed in an infinite, rigid baffle will be considered here [13]. Under such conditions and taking into account the expression (3), the acoustic pressure in any point $\overrightarrow{R}$ of the half-space with given radial coordinates (r,ϕ,Θ) is described using Rayleigh's integral with the following equation:

$$p(\overrightarrow{R}) = \frac{\mathrm{i}\omega\rho_0 e^{jk|\overrightarrow{R}|}}{2\pi|\overrightarrow{R}|}\sum_{n=1}^{N}\left(W_n\int\limits_0^a\int\limits_0^b \Phi_n\left(x,y\right)e^{jk\Delta r}dxdy\right), \tag{4}$$

where k is the wavenumber of the acoustic wave, ρ_0 is the density of air and Δr is approximated by the following relation:

$$\Delta r \approx -x\sin\phi\cos\Theta - y\sin\phi\sin\Theta \tag{5}$$

It is assumed, that dimensions of the considered structures are smaller than the acoustic wavelength in air in the considered frequency range. This assumption justifies the fact of neglecting the influence of inertial loading of the surrounding medium on the vibration characteristics of the structure and also allows to make some important remarks, that concern influence of different interacting structural modes on the resultant acoustic pressure field. Every mode will reveal either acoustic monopole or dipole-like radiation characteristic at a frequency close to its corresponding eigenfrequency; the "dipole" modes will be very weak radiators. Due to these facts, the mean, resultant multi-modal acoustic pressure in the far field can be approximated by a linear combination of the modal amplitudes with coefficients computed using equation (4). This observations agree with the results of numerical simulations which will be described later on.

2.2 Piezoelectric sensors and actuators

The behavior of piezoelectric transducers is governed by the constitutive equations which include coupling between mechanical and electrical phenomena. Assuming that the summation convention is used (i.e., the summation is carried out over the repeating indices $i, j, k, l = 1, 2, 3$) these equations can be presented as follows, for example, in the so-called stress-charge form:

$$T_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \tag{6}$$

$$D_k = e_{kij}S_{ij} + \epsilon_{ki}E_i, \tag{7}$$

where $T_{ij}\ \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ is the second-order stress tensor, $S_{ij}\ \left[\frac{\mathrm{m}}{\mathrm{m}}\right]$ is the second-order strain tensor, $c_{ijkl}\ \left[\frac{\mathrm{N}}{\mathrm{m}^2}\right]$ is the fourth-order elasticity tensor, $e_{kij}\ \left[\frac{\mathrm{C}}{\mathrm{m}^2}\right]$ is the third-order tensor of piezoelectric coefficients (for the so-called stress-charge form), $D_k\ \left[\frac{\mathrm{C}}{\mathrm{m}^2}\right]$ is the electric displacement vector, $E_k\ \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$ is the electric field vector, and $\epsilon_{ki}\ \left[\frac{\mathrm{F}}{\mathrm{m}}\right]$ is the second-order tensor of dielectric constants.

It is assumed that a sensor electrode covers the whole relevant surface S of the transducer and that the polarization of the material is constant. The electric charge which appears on the electrodes of a piezoelectric sensor fixed to the surface of a vibrating thin plate or beam structure is computed as follows

$$Q = -\iint_S D_i n_i dS = -\iint_S D_3 dS \tag{8}$$

where n_i ($n_1 = n_2 = 0$, $n_3 = 1$, see figure 1) are the components of the unit vector normal to the surface of structure and at the same time identical with the direction of polarization of the piezoelectric sensor, while – in the absence of external electric field, and assuming that the piezoelectric transducers are made up of transversely-isotropic piezo-ceramics (which involves that: $e_{311} = e_{322}$ and it will be denoted by e_3, whereas $e_{312} = e_{321} = 0$) – the relevant component of the dielectric displacement vector (7) (having also noticed that $S_{33} \approx 0$) reads

$$D_3 = e_{3ij}S_{ij} = e_{311}S_{11} + e_{322}S_{22} + e_{333}S_{33} \simeq e_3(S_{11} + S_{22}). \tag{9}$$

It is assumed that (because of very good bonding) the in-plane deformation of piezoelectric element is consistent with the deformation of the underlying structure, thus, the relevant components depend on the corresponding curvatures and the distance between the mid-planes of the piezo-element and the structure, i.e.:

$$S_{11} = \frac{h_p + h_s}{2}\frac{\partial^2 w}{\partial x^2}, \qquad S_{22} = \frac{h_p + h_s}{2}\frac{\partial^2 w}{\partial y^2}. \tag{10}$$

Obviously, in the case of beam structures $S_{22} = 0$. Now, the electric charge induced on the shunted piezoelectric sensor attached to the surface of the plate structure can be expressed as follows

$$Q = \frac{-(h_p + h_s)}{2} e_3 \int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} dx dy, \tag{11}$$

which in case of beam structures (of width b) reduces to:

$$Q = \frac{-(h_p + h_s)}{2} b e_3 \int_{x_1}^{x_2} \frac{\partial^2 w}{\partial x^2} dx. \tag{12}$$

We would like to obtain the sensitivity function of piezoelectric sensor to specific structural modes. To do so, we first compute the amplitude of the electric charge induced on the transducer by substituting the time-harmonic form (3) into equation (11) to obtain:

$$Q = e^{\mathrm{i}\omega t} \tilde{Q} = e^{\mathrm{i}\omega t} \frac{-(h_p + h_s)}{2} e_3 \sum_{n=1}^{N} W_n \left[\int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 \Phi_n}{\partial x^2} + \frac{\partial^2 \Phi_n}{\partial y^2} dx dy \right]. \tag{13}$$

Here, $\tilde{Q}$ denotes the amplitude of the harmonically varied sensor charge.

It is assumed, that the piezoelectric sensors are connected to the charge-to-voltage transducers circuits. Hence, the resulting voltage signal which is at the input of the active control system is proportional to the charge given by equation (13) and the desired sensitivity function of a sensor to the structural mode m can be defined as follows:

$$\tilde{S}_m = R \frac{\tilde{Q}_m}{W_m} = R \frac{-(h_p + h_s)}{2} e_3 \left[\int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 \Phi_m}{\partial x^2} + \frac{\partial^2 \Phi_m}{\partial y^2} dx dy \right], \tag{14}$$

where $\tilde{S}_m \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$ is the sensitivity function of a sensor to the structural mode m, $\tilde{Q}_m$ is the electric charge amplitude induced by the mode m, and $R \left[\frac{\mathrm{V}}{\mathrm{C}}\right]$ is the gain of the signal conditioning circuit attached to the piezoelectric transducer.

The external loading introduced by the rectangle-shaped piezoelectric actuator situated in such a way that its edges are in parallel with the relevant axes of the global coordinate system (see figure 1) can be approximated by linear (i.e., per length) moments acting along these edges. The excitation function in equation (2) can be then expressed as follows [1]:

$$\begin{aligned} F_S(x, y) = EIK^f s_a \left[\delta'(x - x_1) - \delta'(x - x_2)\right] \left[H(y - y_1) - H(y - y_2)\right] \\ + EIK^f s_a \left[\delta'(y - y_1) - \delta'(y - y_2)\right] \left[H(x - x_1) - H(x - x_2)\right], \end{aligned} \tag{15}$$

where $\delta'(.)$ is the derivative of the Dirac delta function, $H(.)$ is the Heaviside step function, K^f is the material-geometric constant dependent of material properties of the piezo-ceramics and type of actuator (symmetric or antisymmetric) [1] and s_a is the strain of the actuator (in the x-, as well as in the y-direction, they are equal) caused by the applied driving voltage V which generates within the piezo-element a uniform electric field in the z-direction, $E_3 = V/h_p$, therefore:

$$s_a = \frac{d_3 V}{h_p} \tag{16}$$

where d_3 is the relevant piezoelectric material constant (from the strain-charge form). The effects of added mass and stiffness introduced by the actuator as well as a longitudinal strain of the structure (resulting from the transverse asymmetry of the actuator) are neglected in the present considerations.

While considering response of a structure to an external harmonic excitation, it is very convenient to perform the decomposition of the loading force into the eigenmodes of the structure. To obtain the amplitude of mode number m excited by an external force F_S both sides of equation (2) are multiplied by the shape function Φ_m and then integrated over the surface of the plate. Due to the orthogonality property of the mode shape functions, the following result is obtained:

$$W_m = \frac{\iint_S F_S \Phi_m dS}{\rho_s h_s \left(\omega_m^2 - \omega^2\right) \iint_S \Phi_m^2 dS}, \tag{17}$$

where ρ_s is the density of the structure and ω_m is the eigenfrequency of the considered mode m. To compute the modal decomposition ceofficients of the excitation introduced by the actuator driven with the harmonic voltage V relations (15) and (16) are used for equation (17) to get the following result:

$$\hat{A}_m = \frac{EIK^f d_3 V}{h_p \rho_s h_s \left(\omega_m^2 - \omega^2\right) \iint_S \Phi_m^2 dS} \left[\int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 \Phi_m}{\partial x^2} + \frac{\partial^2 \Phi_m}{\partial y^2} dx dy \right]. \tag{18}$$

When this result is compared with the relation sensor sensitivity (14) an important remark should be made. The parts of the both equations that depend on the coordinates of the placement of the sensor and actuator are exactly the same. This means that the efficiency of a piezoelectric transducer for sensing or actuating some structural vibration mode are similar with a proportional coefficient resulting from the other parts of the considered equations. These considerations of course lead in the formal way to the result consistent with the reciprocal principle and direct and inverse piezoelectric effects. These conclusions – as it will be shown later on – are of a great importance for the process of development of the active control system.

2.3 Active vibroacoustic control

It is assumed that the system that the considered control system for active reduction of vibroacoustic emission of beam, plate and panelled structures consists of a single piezoelectric sensor with a signal conditioning circuit (as described in the previous section), an amplifier with the gain equal $-G$ ($G \geq 0$) and a piezoelectric actuator. All of these elements are connected in a feedback control loop. Vibrations of the structure induce electric charge on the sensor, which – after being transformed into the voltage signal – is fed to the input of the amplifier and, after amplification, fed back to the actuator which in that way affects structure vibrations. The goal is to minimize the acoustic pressure in the air, in the far-field from the structure.

It is assumed that the considered electro-mechanical system is linear. There are two sources of vibrations. The primary source is an external disturbance, acting with angular frequency ω, which may be decomposed into the eigenmodes of the considered structure:

$$F_S = e^{\mathrm{i}\omega t} \sum_{n=1}^{N} \hat{F}_n \Phi_n, \tag{19}$$

where $\hat{F}_n$ are the modal amplitudes of the excitation described by the following formula:

$$\hat{F}_n = \frac{\iint_S F_S \Phi_n dS}{\iint_S \Phi_n^2 dS}. \tag{20}$$

The amplitudes of the structural modes excited by this source are equal:

$$F_n = \frac{\hat{F}_n}{\rho_s h_s \left(\omega_n^2 - \omega^2\right)} = \frac{\iint_S F_S \Phi_n dS}{\rho_s h_s \left(\omega_n^2 - \omega^2\right) \iint_S \Phi_n^2 dS}. \tag{21}$$

The other source of vibrations of excited in the structure is the influence of the actuator (to be used for active control). Modal decomposition of the introduced loading is given by equation (18). The driving, harmonic voltage V is equal:

$$V = -Ge^{\mathrm{i}\omega t} \sum_{n=1}^{N} S_n W_n. \tag{22}$$

where $\tilde{S}_n$ is the sensor sensitivity (14). For the sake of brevity, the following denotation is introduced: $A_n = \frac{\hat{A}_n}{V}$; accordingly:

$$\hat{A}_n = VA_n = -Ge^{\mathrm{i}\omega t} A_n \sum_{m=1}^{N} \tilde{S}_m W_m. \tag{23}$$

Due to the assumption of system linearity the transverse vibrations of the structure may be written as a sum of the responses to both mentioned excitation sources:

$$\begin{aligned} w(x,y,t) &= e^{\mathrm{i}\omega t} \sum_{n=1}^{N} \Phi_n(x,y) W_n = e^{\mathrm{i}\omega t} \left[\left(\sum_{n=1}^{N} F_n \Phi_n \right) + \left(\sum_{n=1}^{N} \hat{A}_n \Phi_n \right) \right] \\ &= e^{\mathrm{i}\omega t} \left[\left(\sum_{n=1}^{N} F_n \Phi_n \right) - G \left(\sum_{n=1}^{N} \tilde{S}_n W_n \right) \left(\sum_{n=1}^{N} A_n \Phi_n \right) \right]. \end{aligned} \tag{24}$$

We would like to find the relation between the amplitudes of specific structural modes and the parameters of the external excitation and the active control system. To compute the amplitude W_a of the mode a we compare the corresponding elements of equation (24), multiply both sides by the shape function Φ_a, divide by the proper proportional coefficient and integrate over the surface of structure:

$$\begin{aligned} &\iint_S \frac{\sum_{n=1}^{N} \Phi_n(x,y) \Phi_a(x,y) W_n}{\iint_S \Phi_a^2 dS} dS \\ &\qquad = \frac{1}{\iint_S \Phi_a^2 dS} \left[\sum_{n=1}^{N} \iint_S F_n \Phi_n \Phi_a dS - G \left(\sum_{m=1}^{N} \tilde{S}_m W_m \right) \sum_{n=1}^{N} \iint_S A_n \Phi_n \Phi_a dS \right]. \end{aligned} \tag{25}$$

After using the orthogonality property of the modal shape functions, the following relation is eventually obtained:

$$W_a = \frac{F_a}{1 + GS_a A_a} + \frac{GA_a}{1 + GS_a A_a} \sum_{\substack{m=1 \\ m \neq a}}^{N} S_m W_m. \tag{26}$$

Equation (26) implies some important remarks that should be taken into account while developing the active vibroacoustic control system. First part of the right-hand side of the equation $\frac{F_a}{1+GS_aA_a}$ represents the well-known relation describing the resultant gain of the closed-loop feedback controller. If we would be able to create a single-mode in-phase sensor/actuator pair, the system would remain unconditionally stable and the amplitude of the selected mode reaches zero as the feedback gain reaches infinity. Method of creating modal sensors/actuators has been described by Lee and Moon [4]. However, the practical implementation of such transducers is limited to simple one-dimensional beam structures and only to few lowest-order structural modes. Another important disadvantage of single-mode sensors/actuators is the fact, that we would need one separate pair of transducers for every mode we would like to control, which would lead to very complex, multi-layered structure.

Another remark, that can be concluded from equation 26 is that one of the conditions of the stability of the active control system is $GS_aA_a \neq -1$ for every mode number a in the whole considered frequency range. The sensor should also be sensitive to the structural modes excited by the corresponding actuator. To provide the described features the collocated sensor/actuator pairs can be used. There are two ways of implementing this solution. First, we can use a single piezoelectric transducer, working simultaneously as a sensor and an actuator. Second method requires two piezoelectric elements, mounted symmetrically on the both sides of the controlled structure. The examples of implementations of the first method have been described i.e. by Dosch [7], Anderson and Hagood [6] and Vipperman [8, 9]. One of the main disadvantages of this solution is the presence of the high actuator driving signal and the low sensing signal simultaneously at the input of the signal conditioning circuit. This implies the requirement of very high range of linear operation of the first stage of the amplifier. The second described solution is much less complicated and commonly used, but it requires the access to the both sides of the structure, which may not always be possible.

The presented considerations and equations describing the active control system clearly indicate on the high importance of the positioning of the piezoelectric transducers at the surface of the controlled structure on the efficiency of the control process.

3 Numerical simulations

3.1 Vibroacoustic emission

The far-field acoustic pressure distribution generated by vibrating structures is investigated here on the example of a thin aluminum plate of rectangular shape for which some numerical simulations of the radiation beam patterns were executed. The plate is 40 cm long, 25 cm wide and 2 mm thick. It is assumed that the structure is clamped at some part of the one of its shorter edges and all the other boundaries are free.

The computations were carried out in two stages. First, the eigenfrequencies and the shapes of the corresponding structural vibration modes had been determined using the finite element analysis. To this end, COMSOL MULTIPHYSICS software had been used and the solutions obtained for the two-dimensional plate model had been saved in a file. Basing on these computed vibrational characteristics, the far-field acoustic pressure distribution was determined using the relations (4) and (5). The results from the file were imported by a MATLAB script and numerical integration over the relevant regions were carried out for different structural modes and vibration frequencies. Some examples of the results obtained in that way are presented in figure 2.

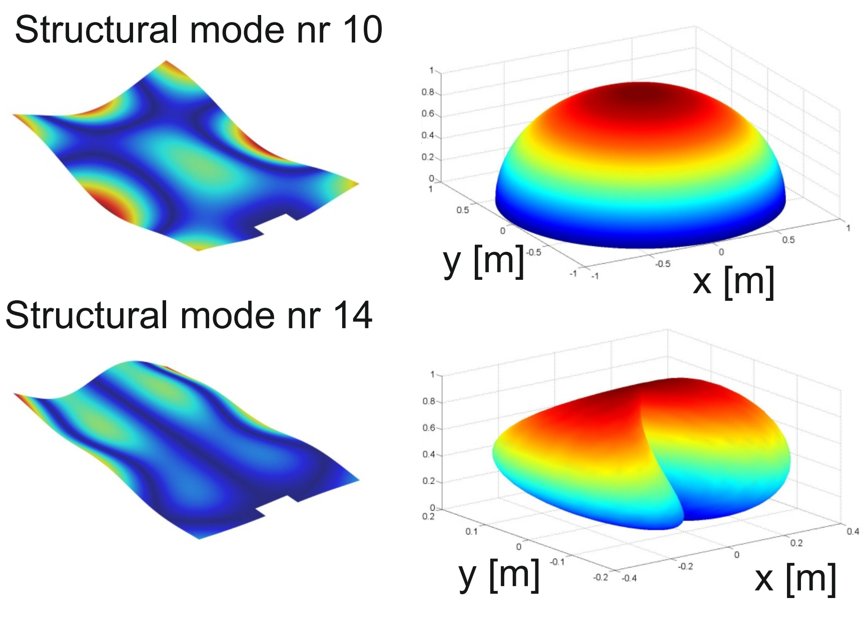

Figure 2: Three-dimensional acoustic beam patterns of two exemplary structural modes of the thin plate structure.

It is worth noticing that due to the relatively small dimensions of the considered plate structure, in the low-frequency region all modes reveal either acoustic monopole- or dipole-like radiation beam patterns and the “dipole” modes are weak acoustic radiators. This observation may suggest not to take into account such modes in the process of developing the active control system. However, due to the remarks made in section 2.3 and according to equation (26), it can be shown that depending on the actuator and sensor location, modes can excite each other in the feedback loop. Hence, in an extremely unfavorable case, omitting such “quiet” mode may result in a significant amplification of the noise generated by the structure, caused by the developed active control system.

Another important observation, made on the basis of the obtained radiation beam patterns, is that in some frequency around to the specific eigenfrequency of the considered plate – and thus within the bandwidth where the amplitude of corresponding structural mode is significant – the shapes of the radiation characteristics change very slightly. Therefore, the mean acoustic pressure can be estimated by the value computed for the central point of the considered half-sphere. Thus, the resulting pressure distribution arising as an interaction between different structural modes can be estimated by a linear combination of the modal amplitudes with proper constant coefficients. However, in case of larger or stiffer structures, such assumption cannot be made; the coefficients will be frequency-dependent and the determination of their values will be much more complicated, taking into account interaction between various, complex beam patterns.

3.2 Simulations of the vibroacoustic behaviour of beam structures

According to the considerations discussed in section 2.2, piezoelectric sensors and actuators are bounded with reciprocal relation, which implies that the sensitivity and ability of exciting specific structural modes depend only on the location of the transducer on the surface of the considered structure. Thus, we define the modal sensitivity function of a transducer as depending on the specific mode number and the location coordinates of the piezoelectric element; in case of beam structures:

$$\mathfrak{S}_{\mathfrak{b}}\left(n, x_b\right) = \left.\frac{\partial \Phi_n}{\partial x}\right|_{x=x_1+l} - \left.\frac{\partial \Phi_n}{\partial x}\right|_{x=x_1}, \quad x \in [0, L-l], \tag{27}$$

where n is the number of the considered mode, x_b is the position of the edge of the piezo-transducer l is the length of the piezo-element and L is the length of the beam. Using this function, the sensor and actuator modal coefficients can now be expressed as follows:

$$S_n = R\frac{-(h_p + h_s)}{2} e_3 \mathfrak{S}_{\mathfrak{b}}(n, x_b), \tag{28}$$

$$A_n = \frac{EIK^f d_3}{h_p \rho_s h_s \left(\omega_n^2 - \omega^2\right) \iint_S \Phi_n^2 dS} \mathfrak{S}_{\mathfrak{b}}(n, x_b). \tag{29}$$

The vibration mode shapes of a thin beam can be computed analytically, as the sum of harmonic and hyperbolic functions, with coefficients depending on the boundary conditions [12]. Basing on such a formula, the modal sensitivity function was computed for a piezoelectric sensor (of known dimensions) attached to the clamped beam structure. Some results of these computations, obtained for a 3 cm long piezo-element on a 58 cm long beam (with one end clamped and the other free) are shown in figure 3.

The presented results were used for positioning piezoelectric transducers on thin beams made of aluminum and glass-fiber, which were examined during further experimental research. For homogeneous beams the modal shape functions do not depend on the material; they are the same for every thin beam of the same length and boundary conditions and the material properties affect only the eigenfrequencies. In the presented case, the piezo-element location that allows to sense or excite every mode is close to the clamped end of the beam. The transducers may be positioned such that they will not respond or induce some specific structural modes, but they still will be sensitive to most of the modes in the considered low-frequency range.

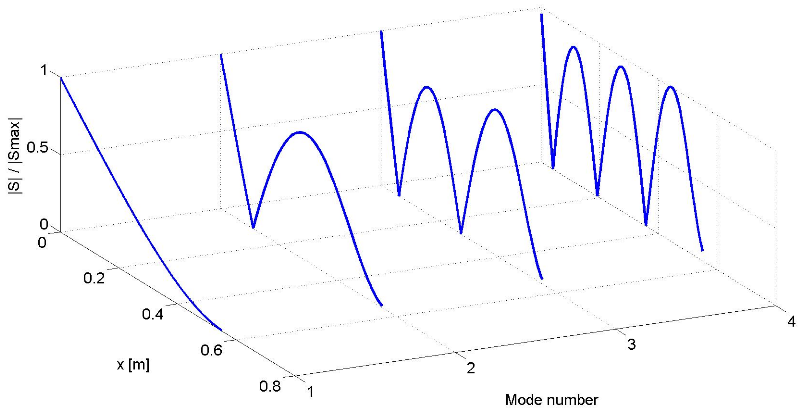

Figure 3: Normalized sensitivity functions for the rectangle-shaped piezoelectric sensor attached to a cantilevered beam of length 58 cm as a function of the structural mode number and the distance of the sensor from the clamped end of the beam.

3.3 Simulations of the vibroacoustic behaviour of plate and panelled structures

According to the considerations presented in the previous section, we define the modal sensitivity function for a rectangle-shaped piezoelectric transducer attached to the surface of a plate structure:

$$\mathfrak{S}_{\mathfrak{p}}(n, x_1, y_1, x_2, y_2) = \int_{x_1}^{x_2} \int_{y_1}^{y_2} \frac{\partial^2 \Phi_n}{\partial x^2} + \frac{\partial^2 \Phi_n}{\partial y^2} dx dy, \tag{30}$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of the diagonal vertices of the piezoelectric transducer. For arbitrary boundary conditions the mode-shape functions Φ_n cannot be, in general, found analytically. Therefore, the finite element analysis was applied to determine the eigenfrequencies and the corresponding eigenvectors of the investigated plate or panelled structures. Then, the values of the integrand from equation (30) at the specific points of the structure were also computed numerically. Some of the results obtained for selected structural modes are presented in figure 4. As mentioned previously, the plate structure is clamped by some part at one of the shorter edges and all the other boundaries are free. The modal sensitivity functions of five rectangle-shaped piezoelectric transducers with dimensions 2 cm $\times$ 3 cm, positioned at five different locations on the surface were computed. The normalized values of the obtained results for the five first vibration modes of the plate are given in table 1. The value 1 in a cell of the table indicates that the specific transducer is the most sensitive to the specific mode of all the piezo-elements (thus, the sensitivities are relative with respect to the result of the "best" sensor), while the value 0 indicates that it is not sensitive to this mode at all.

Mode number	Sensor 1	Sensor 2	Sensor 3	Sensor 4	Sensor 5
1	1	0.678	0.468	0.28	0.055
2	0.008	0.329	0.01	1	0
3	0.317	0.846	1	0.969	0.28
4	0.001	1	0.09	0.29	0
5	0.6	0.105	1	0.72	0.68

Table 1: Values of the normalized sensitivity function of piezoelectric sensors attached to the plate structure – the results of the numerical simulations.

Once again, it can be seen that relatively small, rectangle-shaped piezo-elements can be placed in locations that ensure very high or, in other case, negligible sensitivity to one or two selected structural modes, but that transducer will also respond to most of the other modes in the considered low-frequency range.

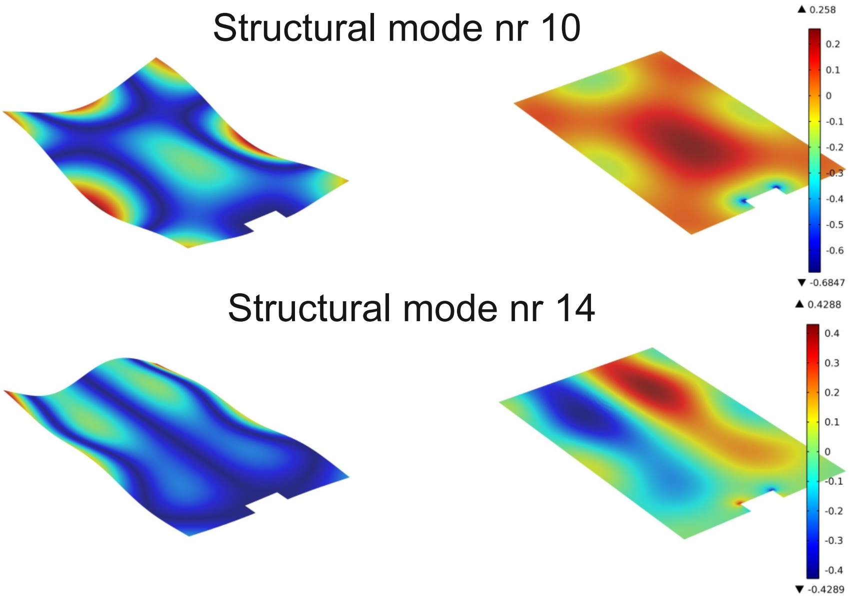

Figure 4: The shapes of two selected structural modes of the plate structure (left) and the corresponding distribution of electric charge (right) that would be induced on a piezoelectric sensor positioned at a specific point on the structure.

4 Experimental investigations

The experimental investigations were performed using various beam, plate and panelled structures with piezoelectric transducers attached to their surfaces, see figure 5. The locations of the piezo-elements were chosen basing on the results of the numerical simulations, described in the previous section. The purpose was to ensure negligible or high sensitivity to the selected structural modes. Such procedure may, of course, concern only a small number of modes taken into consideration.

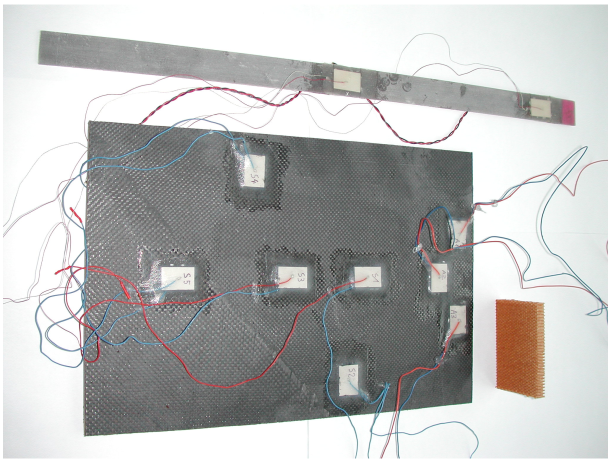

Figure 5: Some of the examined structures: the glass-fiber beam and the sandwich composite of carbon-fiber faces and a honeycomb core (and a piece of *Nomex*-honeycomb material used for the core)

Examination of the vibrations of the beam structures revealed an excellent agreement with the theoretical predictions. The vibrations were excited by a single piezoelectric actuator positioned close to the clamped end of the beam, while the other piezo-elements, fixed at different distances along the beam structure, were used as sensors. The electrodes of the sensors were connected to a simple charge-to-voltage converters. The results of the measurements confirmed that the sensitivity of the sensors was as intended. It is also worth to notice that for aluminum beams, for which the material constants are known, the predicted and measured eigenfrequencies agreed with an accuracy better than 1 Hz. That observation justifies also the assumption to neglect the stiffness and mass influence of the attached piezoelectric elements to the vibration characteristics of beam structures.

In case of the beam made of glass-fiber composite (see figure 5), no material constants were known. The beam was 58 cm long, 3 cm wide and 2.3 cm thick. Two rectangle-shaped piezoelements were attached to the surface of the structure: the first one, fixed 4 cm from the clamped end, served as an actuator simulating external source of vibrations. The second transducer was located 29 cm from the clamped end and it was used as a sensor. Due to the results of the numerical simulations, the sensor should be insensitive to structural modes No. 3 and 5. The resonant frequencies were found experimentally and the mode shapes were identified using a laser vibrometer. The results are presented in table 2; the modes No. 3 and 5 were not sensed by the sensor which agrees with the theoretical predictions.

Frequency [Hz]	Number of nodes in the mode shape function	Identified mode number
95,8	3	4
235,5	5	6
318,6	6	7
448,5	7	8
562,1	8	9

Table 2: Measured resonant frequencies and corresponding parameters of the structural mode shapes for glass-fiber composite beam

If a thin beam (of length L and rectangular cross-section of height h_s) is elastic, isotropic and homogeneous – or can be approximately treated as such – its eigenfrequencies can be calculated using the following formula [12]:

$$f_n = \frac{\beta_n^2 h_s}{2\pi L^2 2\sqrt{3}} v_b, \tag{31}$$

where $v_b = \sqrt{E_b/\rho_b}$ is the velocity of plane wave in the (supposedly elastic and isotropic) material of the beam (E_b and ρ_b are the Young's modulus of the material and its density, respectively) and β_n is the coefficient dependent on the boundary conditions and the mode number [12]. Equation (31) and the results of measurements given in table 2 were used to estimate the ("effective", average) speed of sound for the composite material of which the examined beam was made. The mean value found using the measured eigenfrequencies listed in table 2 was 2553 m/s. Now, it was used in equation 31 with the coefficient $\beta_2^2 = 22,034$ [12] appropriate for the 2nd mode (not used in the previous calculations) to estimate the eigenfrequency of this mode. The computed result of 17.7 Hz agrees well with the resonant frequency of 18.2 Hz measured for this mode.

Experimental investigations were carried out on plate structures, namely, thin aluminum plates and a sandwich composite plate made up of two carbon-fiber liners and a *Nomex*-honeycomb core, see figure 5. Here, only the results for the sandwich composite plate will be discussed. The plate was 40.2 cm long, 27.2 cm wide and 0.5 cm thick. It was clamped by a part of its shorter edge and all the other edges were free. Eight rectangle-shaped piezoelectric transducers with dimensions 2 cm $\times$ 3 cm were attached to the plate surface. Three of them were fixed close to the clamped boundary and served as actuators which simulated the external excitation sources. The other five acted as sensors. The problem is consistent with the one described in section 3.3. The values of modal sensitivity functions found experimentally for every transducer for the first five structural vibration modes are presented in table 3.

The shapes of the structural modes were determined using the laser vibrometer and compared to the corresponding shape functions computed using FEM. This comparision confirmed a good agreement between the simulations and the experiment. It also demonstrated that the low-frequency vibrations of the sandwich panel composite may be well approximated by a simple two-dimensional plate model.

Mode number	Sensor 1	Sensor 2	Sensor 3	Sensor 4	Sensor 5
1	1	0.5	0.634	0.2	0.062
2	0.089	0.523	0.178	1	0.103
3	0.662	0.625	1	0.6	0.185
4	0.465	0.922	1	0.52	0.367
5	0.052	0.091	0.973	1	0.445

Table 3: Values of the normalized sensitivity function of piezoelectric sensors attached to the plate structure – results of the experimental investigation.

The comparision of the results given in tables 1 and 3 reveals that the experimental and numerical results are in general similar, though some significant discrepancies between predicted and measured values are observed too. For example, the sensors 3 and 5 were in fact sources of the electric signal at the *all* considered resonant frequencies, although their locations were deliberately chosen in such way that the transducers should be – theoretically – insensitive or almost insensitive to some selected modes. As a matter of fact, none of the sensitivity values in table 3 is close to zero. It seems obvious that one of the most important reasons for the observed discrepancies between the measured and simulated results is that damping was completely neglected in numerical simulations and thus only one vibrational mode was considered, whereas due to damping effects present in the real structure the amplitudes of non-resonant modes compared to the amplitude of the considered resonant mode usually had non-negligible values and had to be taken into account.

5 Conclusions

The relation between the placements of piezoelectric transducers on the surfaces of beam, plate and panelled structures and the capability of sensing or exciting specific vibration modes was investigated. It has been shown that relatively small, rectangle-shaped piezo-elements will be sensitive enough to most of the forms of vibrations in the considered low-frequency range.

The radiation efficiency of various structural modes may differ significantly from each other. However, in the process of developing an active vibroacoustic control system, all of the forms of vibrations should be taken into account due to the complex, multimodal interaction in the closed feedback loop.

Due to the fact, that the amplitude of the excited mode is inversely proportional to the difference of the squared values of the corresponding eigenfrequency and excitation frequency, the number of considered modes can be significantly reduced in case of harmonic vibrations. This dependence can be used to limit the number of piezoelectric sensors necessary in adaptive control systems to determine the parameters of the external loading.

The best agreement of the results of the numerical simulations and experimental investigations was obtained for the beam structures. Both, the eigenfrequencies and the structural mode shapes were predicted precisely and the behavior of the piezoelectric transducers attached to the surfaces of the considered structures was like expected. In case of the plate and panelled structures some significant discrepancies between the theoretical predictions and the results of measurements were observed, probably due to some damping effects neglected in the modelling and some numerical errors in determining the mode shape functions.

Acknowledgements

The authors would like to express their sincere gratitude to Mr. Tomasz Szczepanik from the Institute of Aviation in Warsaw, Poland, for providing the samples of glass-fiber and carbon-fiber/Nomex composites. The samples have been prepared in the Institute of Aviation as one of the results of the project „Implementation technology to business practice of a new type of gyrodyne airplane" (No. UDA-POIG.1.3.1-14-074/09).

Financial support of the National Science Centre – Project "Adaptive Composite Noise Absorbers" (No. UMO-2011/01/N/ST8/07755) is gratefully acknowledged.

Financial support of Structural Funds in the Operational Programme Innovative Economy (IE OP) financed from the European Regional Development Fund – Project "Smart technologies for Safety Engineering SMART and SAFE" (No. TEAM/2008-1/4), operated within the Foundation for Polish Science Team Programme, is gratefully acknowledged.

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A Some technical aspects on the preparation of the composite structures for active control systems

The piezoelectric transducers used in the active vibration or vibroacoustic control systems, usually take the form of very thin and small plates (patches) with electrodes sputtered on their both faces. Due to the fact, that one of those faces needs to be attached to the surface of a controlled structure, the problem of ensuring electrical connectivity arises. Most of the kinds of glues typically used in bonding the piezo-elements are very good insulators, while the structures are typically made up of excellent conducting materials: metals (like aluminum) or carbon-fiber composites. A few solutions to cope with this issue are described in literature. Some of them require drilling a hole in the structure through which a wire is connected to the piezo-element. However, such a violation of the controlled element is often not possible and creates additional problems in the case of collocated sensor/actuator pairs. Another way to ensure the access to the bottom electrode of a piezo-element is to use additional pads between the transducer and the structure, but then the mass, thickness and mechanical properties of the attached system change significantly. Yet another approach suggests to use some kind of conductive glue providing both, a very good bonding and electrical contact; however, one must be very careful during manufacturing, since the squeezed-out glue may cause a short-circuiting of the electrodes of transducer. To avoid this situation, a combined method using two kinds of glue can be used, as it is illustrated in figure 6. This technique was successfully applied and tested by the authors during the experiments carried out on various aluminum beams and plates.

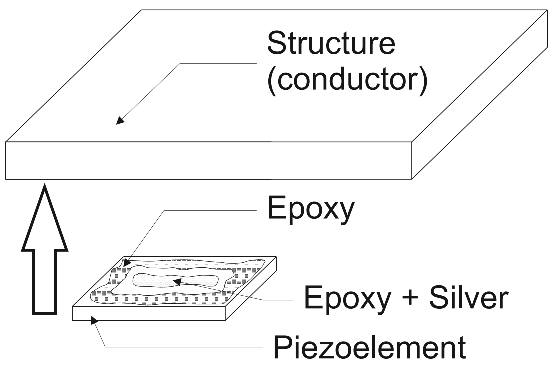

Figure 6: Technique of attaching a piezoelectric element to a structure made of conductive material, ensuring the electric contact between these elements and avoiding short-circuiting the electrodes.