SUBSTRUCTURAL DAMAGE IDENTIFICATION USING LOCAL PRIMARY FREQUENCY

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Abstract: This paper presents a substructuring method on damage identification using Local Primary Frequency (LPF).When a local excitation is applied on a concerned substructure, if the caused vibration mainly consists of only one single modal which represents most of the substructural distortion, then the corresponding frequency is defined as the substructural LPF. LPF reflects more information of the substructure and hence is more sensitivity to the substructural damage. Therefore, LPF can be used for substructural model updating and identification. However, generally substructures don't own LPF. In this case, virtual supports constructed by Substructure Isolation Method are applied on the substructural boundary, such that it can enhance the constraint on the boundary, and decrease the influence from elements outside the substructure. In this way, the substructure sensitivity is enhanced and correspondingly the LPF of the substructure can be constructed. Numerical simulation of a three-story space frame structure testifies that substructural damages are identified effectively by this method.
Keywords: Structure Health Monitoring (SHM), Damage Identification, substructuring method, Substructure Isolation Method, Local Primary Frequency (LPF), Virtual Supports

1 INTRODUCTION

In recent years, Structure Health Monitoring (SHM) has become a hot researched field in civil engineering [Ou, 2005; Kołakowski, 2007]. Sometimes, it is not easy to perform damage identification of large and complex structures entirely and precisely. In fact, usually only small substructures are crucial and concerned. Therefore, substructure monitoring would be sufficient using only local measured responses in real application. Existing substructuring methods usually separate the equation of motion of the concerned substructure from global structure, and estimate the parameters of substructure including stiffness, damping, mass, and sometimes the exposed interface forces [Yun and Lee, 1997; Koh and Shankar 2003; Yang and Huang, 2006]. To increase the optimization efficiency and avoid unnecessary estimation of the interface forces, the Substructure Isolation method (SIM) is proposed in [Hou et al., 2010] using the local impulse response. In the SIM method the interface force doesn't need to be estimated, and moreover the substructural damage can be identified precisely via a virtual, small and independent Isolated Substructure. Via this method, all existing classical global identification methods can be used for local damage identification, such as mode-based method etc..

However, for substructures with complex boundary, the existing methods usually require many sensors be placed for measuring boundary responses or estimating the interface force. Aiming at improving the drawbacks, Local Primary Frequency (LPF) Method is proposed for identifying the substructure, when the substructure is comparatively independent from global structure. However, generally substructures don't have such character. So Virtual supports can be applied on the primary Dof of substructural boundary using the SIM method to increase the independence of the substructure.

A three-story space frame structure verifies that damage is identified effectively by this method.

2 PRIMARY LOCAL FREQUENCY

Assume linear structure with *n* degree of freedoms (Dofs) contains *m* substructures. The stiffness matrix and mass matrix is *K* and *M* respectively. Let α_i is the damage extent of the *i*th extension substructure stiffness matrix K_i, then there exist

$$K = \sum \alpha_i K_i \ , \quad K_i = N_i^T K_{i,s} N_i \qquad (1)$$

where $K_{i,s}$ is the substructure matrix, N_i is the localization matrix linking the global Dofs to the *i*th local substructural Dofs.

Denote respectively the *r*th natural frequency and mode shape as ω_r and φ_r, *r=1,2,…,n*. The sensitivity of the *r*th natural frequency ω_r to the *i*th substructure can be expressed as

$$\frac{\partial \omega_r}{\partial \alpha_i} = \frac{\varphi_r^T K_i \varphi_r}{\omega_r} = \frac{\varphi_{r,i}^T K_{i,s} \varphi_{r,i}}{\omega_r} \qquad (2)$$

where $\varphi_{r,i} = N_i \varphi_r$ is the *r*th mode shape of the *i*th substructure. Then, the sum of the relative sensitivity of all the substructures can be got

$$\sum_i \frac{\partial \omega_r / \partial \alpha_i}{\omega_r} = \sum_i \frac{\varphi_r^T K_i \varphi_r}{2\omega_r^2} = \frac{1}{2} \qquad (3)$$

It can be seen from Eq.(2) and Eq.(3) that for one mode, if the substructural displacement is bigger than the rest substructures, then correspondingly, (1) the sensitivity of this mode to the substructure is higher, while (2) the sensitivity to the rest substructures should be lower.

Based on these two facts, the Local Primary Frequency (LPF) is proposed and defined. If the substructure has one mode of which the substructural displacements are much bigger than that of the others, then the corresponding frequency is defined as the substructural PLF. Therefore, the Primary Local Frequency is very easy to be exited by local excitation applied inside of the substructure. Similarly, when a local excitation is applied on concerned substructure, if the vibration mainly consists of only one single mode which represents most of the substructural distortion, then the corresponding frequency should be the substructural LPF. That is to say, the LPF is easy to be excited and identified. The PLF has high sensitivity to its substructure, and low sensitivity to the other substructure. Therefore only one PLF is enough to identify the corresponding substructure because of its high sensitivity.

However, PLF only belongs to the substructure which is high independent from the global structure. Usually, substructures don't have this characteristic, and sometimes they are high relative with other substructures. In this case, even local excitation can induce lots of modes which contain not only substructural modes but also global modes. Therefore, it is hard to pick the frequency with high sensitivity to substructure from the excited frequencies. In order to increase the independence and obtain the PLF of such kind of substructure, virtual supports are added by Substructure Isolation method on the substructure boundary. In the next section, Substructure Isolation Method is introduced.

3 VIRTUAL SUPPORTS

3.1 Substructure Isolation method

The core idea of Substructure Isolation method is to identify substructural damages based on *isolated substructure* model which is a virtual, independent and small structure constructed by adding virtual supports on the boundary of concerned substructure. After it is isolated from global structure, substructural damage identification can be then performed locally and precisely using the constructed responses of the isolated substructure by any of the existing methods which aim originally at global identification.

The sensors need to be placed on both boundary and inner substructure. Assume there are *l* degrees of freedom (Dofs) on the substructure interface, then *l* sensors needs to be placed in these Dofs. Furthermore, some sensors are placed inside the substructure. In this method, two kinds of responses are measured: *basic response* and *constraining response*. When applying excitation in the inner of substructure, the caused responses of all the sensors are defined as the *basic response*, and the corresponding excitation is the *basic excitation*. While the measured responses of all the sensors to the excitations applied on the boundary or outside the substructure are defined as the *constraining response* and the corresponding excitation is the *constraining excitation*.. To construct virtual supports for isolating the substructure, it needs to measure *l* groups of *constraining response*. The excitations should be applied on different positions of the boundary or outside the substructure.

Responses of the isolated substructure $\boldsymbol{d}_s$ can be constructed by *constraining function*, see Eq.(4). Then, the substructure is identified using the constructed response $\boldsymbol{d}_s$.

$$\boldsymbol{d}_s = \boldsymbol{d} - \boldsymbol{C}\boldsymbol{A}^{+}\boldsymbol{b} \qquad (4)$$

where $\boldsymbol{d}$ and $\boldsymbol{b}$ respectively consists of responses of inner seniors and boundary sensor which are from basic responses. While $\boldsymbol{C}$ and $\boldsymbol{A}$ are constraining matrix constructed by the l groups of constraining responses $\boldsymbol{c}_i$ and $\boldsymbol{a}_i$ (i=1,2,…,l) respectively, which consist of the corresponding responses of inner seniors and boundary sensor. $\boldsymbol{d}$, $\boldsymbol{b}$, $\boldsymbol{c}_i$ and $\boldsymbol{a}_i$ are vectors which collect the corresponding measured discrete responses of all time steps. Bath $\boldsymbol{C}$ and $\boldsymbol{A}$ are Toeplitz matrix.

3.2 Adding virtual support using Substructure Isolation method

Sometimes the boundary of substructure is too complex to place sensors on all the Dofs of boundary. Therefore the substructure can not be isolated completely from the global structure. In this case Substructure Isolation Method can be used to increase the sensitivity of substructure by adding virtual supports on the main Dofs of its boundary. The additional virtual supports can weaken connections between the substructure and global structure to some extent. When the substructure is constrained enough by the virtual supports, the substructure has the LPF.

The performance of the LPF method based on virtual supports should be noted:

1. Sensors needn't to be placed on all the Dofs of substructure boundary, but only along the main Dofs.
2. The global structure should be linear.
3. The constraining excitations needs to be applied along the sensors in order to guarantee that the constructed responses are only caused by the basic excitations.

3 NUMREICAL MODEL

3.1 Space frame model

A space frame model with three floors (see Figure 1) is taken as an example to describe and verify the substructure method based on LPF. The cross section of the column and beam respectively is 0.4m×0.4m and 0.4m×0.2m, and the thickness of plate is 0.12m. The Young's modulus is 0.345Gpa, and the density is 2600Kg/m^3. The damping ratios of first two orders are both 1%.

There are six plates in the frame model, denoted as B1~B6. The damage extents of plates are shown in Figure 2. The plate B2 is chosen as the substructure to be identified. The value of its damage extent is 1, i.e. it is intact.

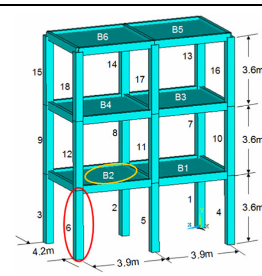


Figure 1 Frame model

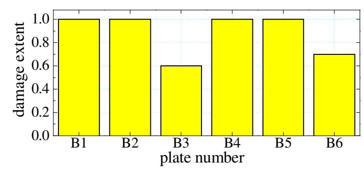


Figure 2 Damage extents of plates

There are 18 pillars in the frame model, of which the damage extents are shown in Figure 3. The pillar 6 is the substructure to be identified.

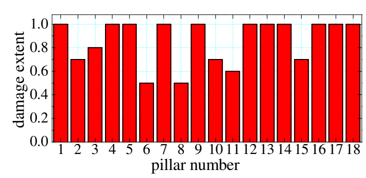


Figure 3 Damage extents of pillars

The next two sections will introduce how to identify the damages of the two substructures, plate B2 and pillar 6, using LPF method.

3.2 substructure identification of plate

Obviously, the plate is an independent substructure, so it has own LPF.

An accelerometer S_1 is placed in the middle of plate B2 to measure the vertical acceleration response of the plate, see Figure 4. The hammer excitation (Figure 5) is applied on the middle of the plate and perpendicular to its plane, see Figure 4, and the corresponding response is shown in Figure 6, which contains 5% Gaussian white noise.

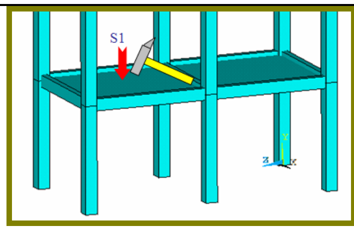


Figure 4 Placement of the sensor on substructure

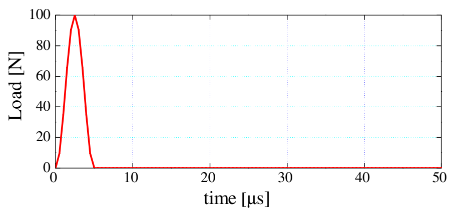


Figure 5 Local Excitation applied on substructure

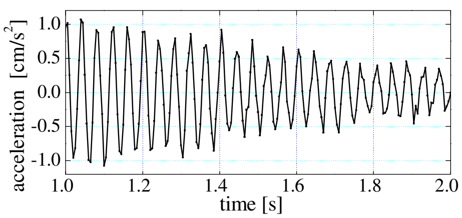


Figure 6 Vertical acceleration at the sensor location of the substructure

As it can be seen in Figure 6, the response contains only one main frequency, which is the LPF of plate B2. Therefore, its LPF can be identified easily from the free response, which is 26.39 Hz.

Assume damage extent of plate B2 is 0.2, 0.4, …, 1 respectively, and the rest members of the structure are intact. Repeat the identification procedure of LPF described above, the corresponding LPFs of the plate B2 can be computed through the FE model, which is shown in Table 1. From Figure 7, the damage of plate B2 could be identified precisely by interpolation method, which is 1.

Table 1 The mapping between damage extent and local primary frequency of plate B2

Damage extent	0.2	0.4	0.6	0.8	1.0
LPF (Hz)	16.8	21.1	23.4	25	26.4

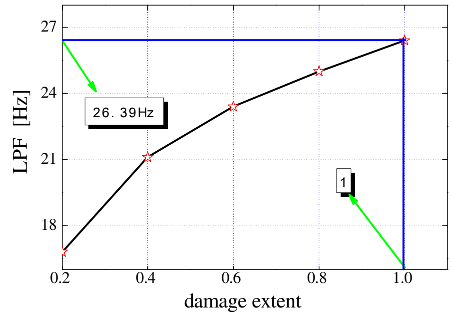


Figure 7 Identification of damage extent

3.3 substructure identification of pillar

The pillars have high correlation with each other. Therefore, the virtual supports should be used to increase the independence of the pillar.

3.3.1 Virtual supports

In order to compare the influence of the virtual supports, three cases are analyzed, shown in Figure 8, which are respectively:

a) no virtual support;
b) one virtual support;
c) fixed virtual supports.

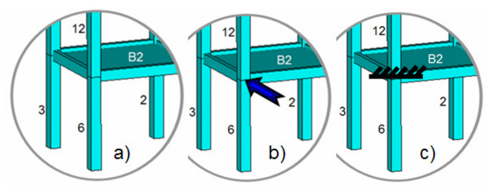
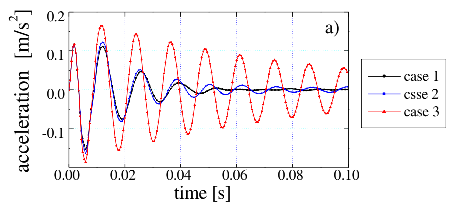


Figure 8 Virtual support applied on substructure boundary

Hammer excitation is applied on the middle of pillar 6 in each case, and the corresponding responses of the middle pillar are shown in Figure 9. When there is no virtual support on the boundary, the amplitude of response is damped rapidly, and several frequencies have been excited. Therefore, the pillar 6 without virtual support doesn't have LPF. When one support is applied to constrain the vertical Dof, a single frequency is mainly excited, which is corresponding LPF. Although the application of fixed supports, i.e. constraining both the vertical and rotation are the best, which can be seen clearly from the response, the rotation usually is not easy to measure. Therefore, considering the application and efficiency, adding one vertical virtual support is enough to make the pillar 6 have LPF. The next section will introduce how to construct vertical supports and obtain the LPF of pillar 6.



(b)

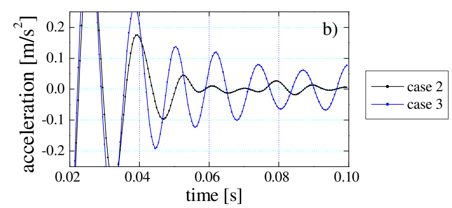


Figure 9 The comparison of responses of three kinds substructure boundary

3.3.1 Virtual supports

Two accelerometers S_1 and S_2 are placed on pillar 6, which are shown in Figure 10. Thereinto, sensor S_1 is transferred to vertical virtual support using Substructure Isolation method, see Figure 10.

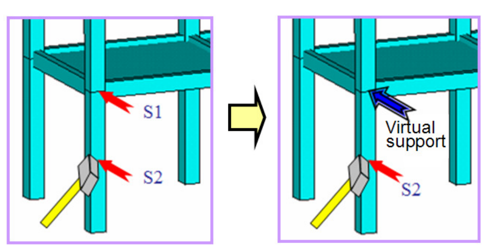


Figure 10 Adding virtual support

The hammer excitation (see Figure 5), denoted as F1 and F2, is applied on the placement of S_1 and S_2 respectively, Corresponding responses are show in Figure 11, which contain 5% Gaussian white noise.

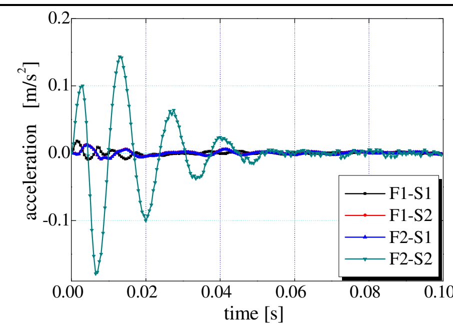


Figure 11 Responses of global structure tp hammer excitation F1 and F2

The response to hammer excitation F1 is the constraining response, which is used to construct the Toeplitz matrix A and B, see Figure 12.

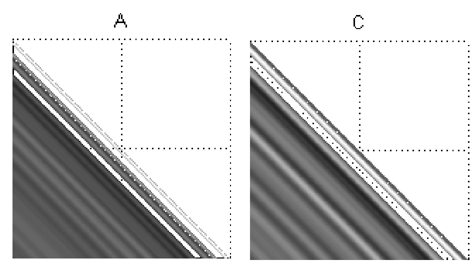


Figure 12 Constraining (*Toeplitz*) *matrix*

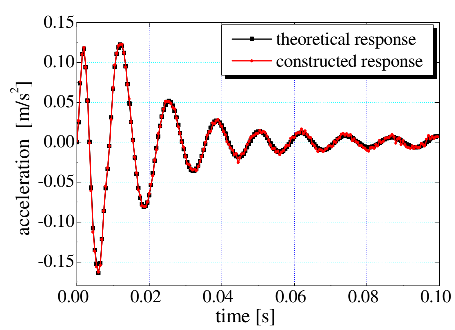


Figure 13 The constructed responses of the structure with virtual support

The response to hammer excitation F2 is basic response. Then the response of the structure with the additional vertical virtual support can be constructed using basic response and constraining matrix by *constraining function* Eq.(4). The constructed response is shown in Figure 13, which also gives the theoretical response computed by the FE model. The constructed and theoretical responses matched very well. Therefore, its LPF can be identified easily using the free response, which is 75.33 Hz.

Assume damage extent of pillar 6 is 0.2, 0.4, …, 1 respectively, and the rest members are intact, then the corresponding LPF of pillar 6 can be the computed using FE model. The relationship between given damage extent and the corresponding LPF is shown in Figure 14. Therefore, the damage of pillar 6 can be identified easily by interpolation of Figure 14 , which is 0.51, and very close to the actual value. It proves that the substructure damage can be identified precisely and easily via the LPF method.

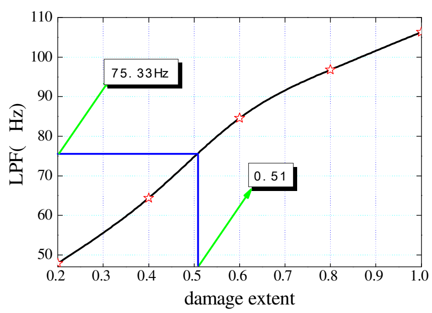


Figure 14 Identification of damage extent

4 CONCLUSIONS

The LPF method is presented for substructural damage identification only using local hammer exitated responses. A numerical example of a space frame has verified that the proposed method is efficient for local substructure monitoring. Conclusions are summarized as following:

(1) The LPF method is very easy to perform in real application: firstly, the hammer can be used as exciter, which is a common and simple tool; secondly, only very few responses need to be measured.

(2) When the substructure is dependent from the global structure, the Virtual supports can be applied on the main Dof of the substructural boundary using the Substructure Isolation Method (SIM) to increase the sensitivity of the substructure and to make the substructure have LPE.

It should be noted that it contains one single frequency, so the LPF method is mainly used efficiently for the simple substructure.

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