THE IDENTIFICATION PROCEDURE FOR THE CONSTITUTIVE MODEL OF ELASTO–VISCOPLASTICITY DESCRIBING THE BEHAVIOUR OF NANOCRYSTALLINE TITANIUM

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The main objective of the present paper is the development of identification procedure of the constitutive model of elasto-viscoplasticity describing the behaviour of nanocrystalline titanium. We intend to utilize the constitutive model presented by Perzyna (2010). The procedure is based on experimental observation data obtained by Jia et al. (2001) for ultrafine-grained titanium and by Wang et al. (2007) for nanostructured titanium. Hexagonal close-packed (hcp) ultrafine-grained (UFG) titanium processed by sever plastic deformation (SPD) has gained wide interest due to its excellent mechanical properties and potential applications as biomedical implants.

1. The constitutive model

We propose to introduce some simplification of the constitutive model developed by Perzyna (2010) by assuming that the internal state variable vector $\boldsymbol{\mu} = (\epsilon^p, d, \boldsymbol{\xi})$ consists of two scalars and one tensor, i.e. ϵ^p denotes the equivalent viscoplastic deformation, d defines the mean grain diameter and $\boldsymbol{\xi}$ is the microdamage second order tensor, with the physical interpretation that $(\boldsymbol{\xi} : \boldsymbol{\xi})^{1/2} = \xi$ defines the volume fraction porosity. The equivalent inelastic deformation ϵ^p describes the dissipation effects generated by viscoplastic flow phenomena, the microdamage tensor $\boldsymbol{\xi}$ takes into account the anisotropic intrinsic microdamage mechanisms on internal dissipation and d describes the dynamic grain growth during intensive deformation process. We postulate the plastic potential function in the form $f = f(J_1, J_2, \vartheta, \boldsymbol{\mu})$, where J_1, J_2 denote the first two invariants of the Kirchhoff stress tensor $\boldsymbol{\tau}$ and ϑ is absolute temperature. The evolution equations are assumed as follows

$$\mathbf{d}^p = \Lambda \mathbf{P}, \quad \mathrm{L}_{\boldsymbol{v}}\boldsymbol{\xi} = \boldsymbol{\Xi}, \quad \dot{d} = D \tag{1}$$

where

$$\Lambda = \frac{1}{T_m}\left\langle \Phi\left(\frac{f}{\kappa} - 1\right)\right\rangle, \qquad \mathbf{P} = \left.\frac{\partial f}{\partial \boldsymbol{\tau}}\right|_{\boldsymbol{\xi}=const}\left(\left\|\frac{\partial f}{\partial \boldsymbol{\tau}}\right\|\right)^{-1}, \tag{2}$$

$\mathbf{d}^p$ denotes the rate of inelastic deformation tensor, T_m denotes the relaxation time for mechanical disturbances, the isotropic work–hardening–softening function $\kappa = \hat{\kappa}(\epsilon^p, \vartheta, \boldsymbol{\xi}, d)$, Φ is the empirical overstress function, the bracket $\langle\cdot\rangle$ defines the ramp function, $\mathrm{L}_{\boldsymbol{v}}$ denotes the Lie derivative and $\boldsymbol{\Xi}$ and D denote the evolution functions which have to be determined.

Let us assume that the intrinsic microdamage process is generated by growth mechanism only. Based on the heuristic suggestions and taking into account the influence of the stress triaxiality and anisotropic effects on the growth mechanism we assume the evolution equation for the microdamage tensor $\boldsymbol{\xi}$ as follows

$$\mathrm{L}_{\boldsymbol{v}}\boldsymbol{\xi} = \frac{\partial g^*}{\partial \boldsymbol{\tau}}\frac{1}{T_m}\left\langle \Phi\left[\frac{I_g}{\tau_{eq}(\vartheta, \boldsymbol{\mu})} - 1\right]\right\rangle. \tag{3}$$

The tensorial function $\frac{\partial g^*}{\partial \boldsymbol{\tau}}$ represents the mutual micro(nano)crack interaction for growth process, $\tau_{eq} = \hat{\tau}(\vartheta, \boldsymbol{\mu})$ denotes the threshold stress function for growth mechanism, $I_g = b_1 J_1 + b_2\sqrt{J_2'}$ defines the stress intensity invariant, b_i $(i = 1, 2)$ are the material coefficients which can depend on d. In the

evolution equation (3) the function $g = \hat{g}\left(\boldsymbol{\tau}, \vartheta, \boldsymbol{\mu}\right)$ plays the fundamental role, and we introduce the denotation $\frac{\partial g^*}{\partial \boldsymbol{\tau}} = \frac{\partial \hat{g}}{\partial \boldsymbol{\tau}} \left(\left|\left| \frac{\partial \hat{g}}{\partial \boldsymbol{\tau}} \right|\right| \right)^{-1}$. Assuming that the dynamic grain growth is the rate dependent mechanism (cf. Perzyna (2010)) we postulate

$$\dot{d} = \frac{\hat{\mathcal{G}}(\vartheta, \boldsymbol{\mu})}{T_m} \left\langle \Phi \left[\frac{I_d}{\tau_d(\vartheta, \boldsymbol{\mu})} - 1 \right] \right\rangle, \tag{4}$$

where $\mathcal{G} = \hat{\mathcal{G}}(\vartheta, \boldsymbol{\mu})$ is the material function, $I_d = c_1 J_1 + c_2 \sqrt{J_2'}$ represents the stress intensity invariant for grain growth, c_i $(i = 1, 2)$ are the material coefficients which may depend on d, and $\tau_d = \hat{\tau}_d(\vartheta, \boldsymbol{\mu})$ denotes the threshold stress for dynamic grain growth mechanism.

2. The identification procedure

Let us introduce the particular form for the plastic potential function as follows $f = \left[J_2' + n\left(\vartheta, d\right) \left(\boldsymbol{\xi} : \boldsymbol{\xi}\right)^{1/2} \left(J_1^2\right) \right]^{\frac{1}{2}}$, where J_2' denotes the second invariant of the stress deviator of the Kirchhoff stress $\boldsymbol{\tau}$ and $n = n\left(\vartheta, d\right)$ is the material function. From $(1)_1$, $(2)_1$ and (2) we have the dynamical yield criterion in the form

$$\left[J_2' + n\left(\vartheta, d\right) \left(\boldsymbol{\xi} : \boldsymbol{\xi}\right)^{1/2} \left(J_1^2\right) \right]^{\frac{1}{2}} = \kappa \left[1 + \Phi^{-1} \left(\frac{\sqrt{3}}{2} T_m \dot{\epsilon}^P \right) \right]. \tag{5}$$

Taking advantage of the description of the microshear banding effects for nanocrystalline titanium we can propose the relation for the relaxation time (cf. Perzyna (2010))

$$T_m = T_m^0 \left[1 - f_{ms}^0 \frac{1}{1 + \exp\left(a - b\epsilon^P\right)} \right] \left(\frac{\dot{\epsilon}^P}{\dot{\epsilon}_s^P} - 1 \right)^{\frac{1}{p}}, \tag{6}$$

where T_m^0, f_{ms}^0, a, b, p and $\dot{\epsilon}_s^P$ are material function of d. We propose that the identification procedure consists of two parts. In the first part the determination of the material functions and the material constants involved in the description of the dynamic yield criterion (5) is presented. As an experimental base the results concerning experimental observation for ultrafine-grained titanium obtained by Jia et al. (2001) and for nanostructured titanium obtained from the compression tests at high strain rates $(10^3 - 10^4\ \mathrm{s}^{-1})$ by Wang et al. (2007). The second part is focused on the determination of the material functions and the material constants appeared in the evolution equations (3) and (4). To do that we consider a dynamic process of compression test (cf. Wang et al. (2007)).

3. Final comments

There is our hope that proposed identification procedure for the thermodynamical theory of elasto-viscoplasticity of nanocrystalline metals may be used as a base for the description of the behaviour of hexagonal close-packed ultrafine-grained titanium processed by sever plastic deformation and may allow to do the investigation of plastic strain localization and fracture phenomena in nano-mechanical processes. These coming results and excellent mechanical properties of this kind of titanium make potential applications possible as biomedical implants.

4. References

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