# ADAPTIVE DISCRETE/FINITE ELEMENT COUPLING FOR ROCK CUTTING PROCESS SIMULATIONS

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In the last years, the Discrete Element Method (DEM) has arisen as an interesting alternative to the continuum based methods for the simulation of problems characterized by strong discontinuities. Nevertheless, the high computational cost of this technology hinders its application to practical problems. For this reason new coupling techniques are developed in order to obtain efficient numerical models [4]. The possibility of the coupling between DEM and the Finite Element Method (FEM) can be a good solution in some cases, but the definition of each domain can be predefined in the generation of the model. A new adaptive coupling technique between DEM and FEM is presented, in order to avoid the definition of the DEM domain and minimizing the computational cost. The idea is to use DEM to analyze just the region of the solid wish presents fracture or damage, whereas FEM is used in the rest of the domain. These region or sub-domain can change during the simulation process, resulting in a progressive modification of the coupling definition. This technique is applied to geomechanical problems, such as rock cutting processes [5]. In this case, the rock is modelled using DEM/FEM coupling, and the tool as rigid body or FEM, depending of the analysis required.

## INTRODUCTION

The DEM based on spherical and cylindrical particles is widely recognized as a suitable tool for the modelling of geomaterials [1, 2], advantages of this method are clearly seen in modelling problems characterized by the strong discontinuities e.g. rock fracture during excavation. The main problem in the use of DEM for large simulations is the computational cost involved. In the present work, a coupling scheme of the DEM with the finite element method is presented.
The requirement of predefine the sub-domains with each methods is solved using an adaptive algorithm for the definition of the DEM sub-domain. This allows the use of DEM just in the areas whit higher values of stress, where fracture can occur.

## DISCRETE ELEMENT FORMULATION

The discrete element model assumes that material can be represented by an assembly of rigid particles interacting among themselves. The overall behaviour of the system is determined by the cohesive/frictional contact laws. The contact law can be seen as the formulation of the material model on the microscopic level. Cohesive bonds can be broken, which allows us to simulate fracture of material and its propagation. Basic formulation of the discrete element formulation using spherical or cylindrical particles was first proposed by Cundall and Strack [1].

The present work use the formulation developed in CIMNE by Rojek et al. in [3], where the translational and rotational movement is described by means of the standard equations of rigid body dynamic.

$$\begin{aligned} \mathbf{M}_D \ddot{\mathbf{x}}_D &= \mathbf{F}_D \\ \mathbf{J}_D \dot{\boldsymbol{\omega}}_D &= \mathbf{T}_D \end{aligned} \quad (1)$$

where $\mathbf{x}_D$ represent the position vector of the particle, $\boldsymbol{\omega}_D$ its angular velocity, MD a diagonal matrix with the particle mass in the diagonal, $\mathbf{J}_D$ a diagonal matrix with the particle moment of inertia, and $\mathbf{F}_D$ and $\mathbf{T}_D$ the resultant forces and moment about the particle central axes, respectively, considering the sum of all forces and moment applied to the particle due to external loads, contact interaction with the neighbour particles, and contact interaction with other obstacles. These equations are integrated considering the central difference scheme.

## DISCRETE/FINITE ELEMENT COUPLING SCHEME

One of the main problems of DEM is the computational cost of the simulations. In a real case, a large number of particles are required, and the analysis of each contact between them, wish leads to long computational time. In order to solve the situation different solutions can be considered. The combinations of DEM with other simulation technologies, as FEM, allow an efficient to create efficient models, minimizing the computational cost, taking advantages of each method, and avoiding their weak sides. The idea is to use the DEM in the sub-domain ($\Omega_D$) where the fracture occurs, and the FEM can be used in the other part, where the behaviour can be represented with a continuum based scheme.

**Finite element formulation.** In the case of the finite element method, the so-called explicit dynamic formulation is used. The explicit FEM is based in the solution of discredited equations of motion written in the current configuration in the following form where $\mathbf{M}_F$ is the mass matrix, $\mathbf{r}_F$ the vector of nodal displacements, $\mathbf{F}^{ext}$ and $\mathbf{F}^{int}$ the vectors of external loads and internals forces, respectively. Similarly to the DEM algorithm, the central difference scheme is used for the time integration of Eq.(2).

$$\mathbf{M}_F \ddot{\mathbf{x}}_F = \mathbf{F}_F^{int} + \mathbf{F}_F^{ext} \tag{2}$$

**Coupling Algorithm.** It is assumed that the DEM and FEM can be applied in different sub-domains of the same body. The DEM and FEM sub-domains overlap each other. The common part of the sub-domains ($\Omega_{DF}$) is the part where both discretization types are used with gradually varying contribution of each modelling method, similarly to the idea used by Xiao and Belytschko in [6]. The coupling of DEM and FEM sub-domains is provided by additional kinematical constraints. Interface discrete elements are constrained by the displacement field of overlapping interface finite elements. Additional kinematic relationships can be written jointly in the matrix notation as follows:

$$\chi = \mathbf{r}_D - \mathbf{N}\,\mathbf{r}_D \tag{3}$$

where $\mathbf{N}$ is the matrix containing adequate shape functions. The set of equation of motion for the coupled system with the penalty scheme is

$$\begin{bmatrix} \bar{\mathbf{M}}_F & 0 & 0 & 0 \\ 0 & \bar{\mathbf{M}}_{DU} & 0 & 0 \\ 0 & 0 & \bar{\mathbf{M}}_{DC} & 0 \\ 0 & 0 & 0 & \bar{\mathbf{J}}_F \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{r}}_F \\ \ddot{\mathbf{r}}_{DU} \\ \ddot{\mathbf{r}}_{DC} \\ \dot{\boldsymbol{\omega}}_D \end{Bmatrix} = \begin{Bmatrix} \bar{\mathbf{F}}_F^{\text{int}} - \bar{\mathbf{F}}_F^{\text{ext}} + \mathbf{N}^T \mathbf{k}_{DF} \chi \\ \bar{\mathbf{F}}_{DU} \\ \bar{\mathbf{F}}_{DC} - \mathbf{k}_{DF} \chi \\ \bar{\mathbf{T}}_D \end{Bmatrix} \tag{4}$$

where the subscript $DC$ and $DU$ are the constrained and unconstrained discrete elements, respectively, and $\mathbf{k}_{DF}$ the vector of the discrete penalty functions.

**Adaptivity of the DEM/FEM interface.** In a real case, the zone wish requires the use of DEM can change during the simulation process. With the standard DEM/FEM coupling, both sub-domains are predefined during the discretization process. This requires knowing where the fracture will appear and the use of DEM even when the solid have a linear elastic behaviour. In order to make more efficient the use of the DEM during the simulation, an algorithm for the change of the sub-domains is used. The idea is start the simulation using finite elements. When some area with a stress value higher than a predefined limit state is achieved, the FEM is replaced with DEM and a new DEM/FEM interface is defined. The progressive changes of the simulation method make more efficient these of both methods.

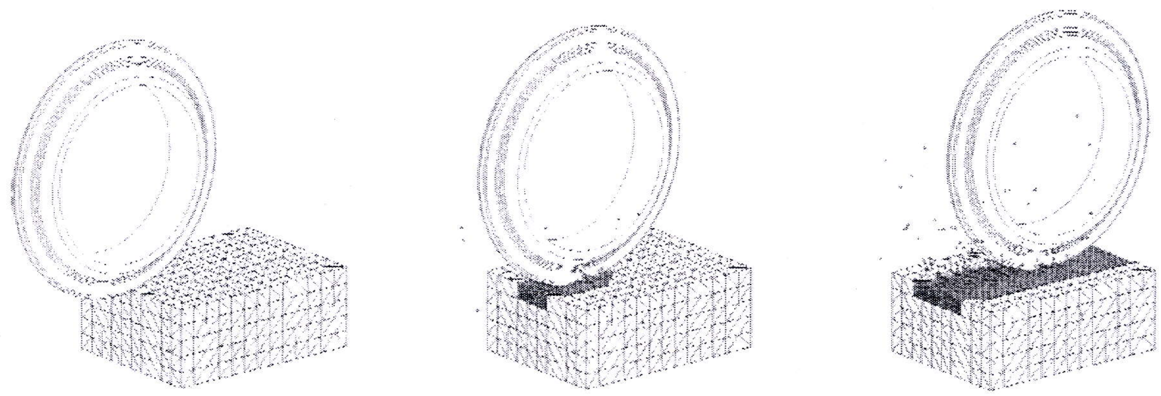
The criteria for the change of simulation method is defined with a predefined limit stress value ($\sigma^*$) in the finite elements, as shown Eq. (5).

$$\sigma > \sigma^* \tag{5}$$

When a change of domain occurs, the kinematic variables of the finite element should be projected into the new discrete elements, and the behaviours of both methods should be equivalents. In order to ensure the geometrical compatibility in the discrete elements used progressively during the simulation, a discretization of the full domain with cylindrical/spherical particles is generated and stored for its posterior use. The discrete elements activated in a change of domain are defined by the elements to be changed, and the particles in the new overlap sub-domain required in the coupled scheme. The particles in the new overlap region are selected using a distance function [7] based in the new boundary elements generated in the FEM sub-domain.

## SIMULATION OF ROCK CUTTING PROCESSES

The simulation of different rock cutting processes has been simulated using the adaptive discrete/finite elements coupling. The coupled model has been used in the rock subjected to fracture, while the cutting tool is discretized as a rigid body or with finite elements, depending of the analysis required.



(a) $t = 0.00\ s$ (b) $t = 0.06\ s$ (c) $t = 0.12\ s$

Fig 1. Rock cutting process with a disc cutter. Evolution of coupling interface.

In the Figure 1 the evolution of a rock cutting process with a disc cutter is shown. In the Figure 2, a comparison of the normal force obtained in the case of use full discrete elements in the rock, the fixed coupling scheme and the adaptive algorithm is depicted.

The difference of the average value is negligible, and the computational cost involved in the adaptive coupling case 40% lower than the standard coupling scheme.

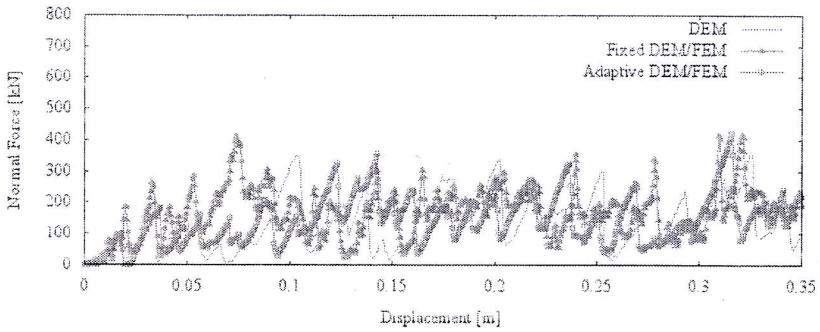



Fig 2. Comparison of normal force in rock cutting process with the coupling schemes.

## CONCLUSION

The coupling scheme using discrete and finite element methods can be an interesting approach for the simulation of fracture in geomaterials, minimizing the computational cost. The adaptively of the DEM/FEM interface can improve the efficiency in the use of both methods. The technique has been used in the simulation of rock cutting processes. The obtained results are consistent with the values obtained in a standard discrete elements simulation.

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